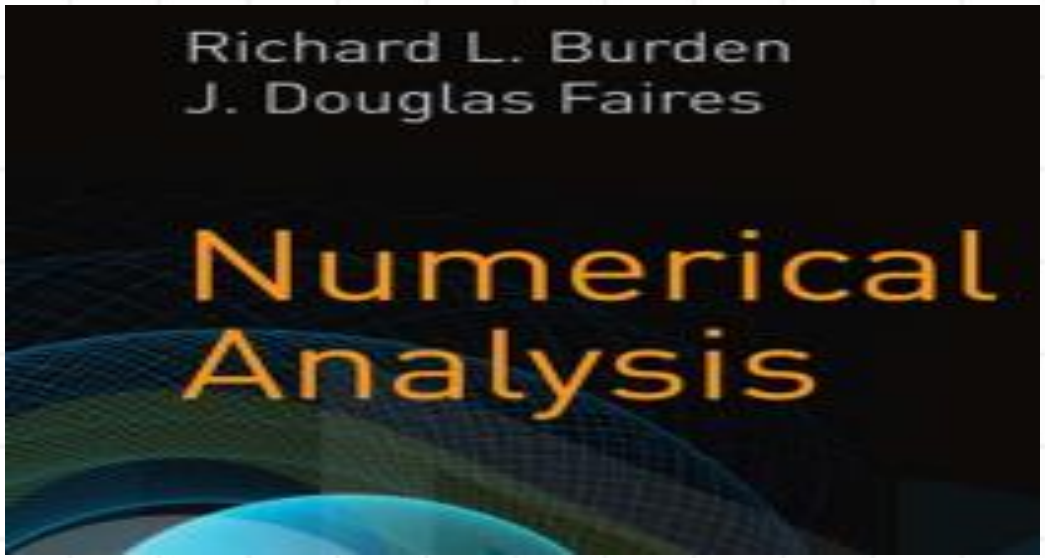
Subject: Partial Pivoting For 2x2 Matrix

ⓐ Solved problem - Complete pivoting

#2



Complete pivoting

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Example 2 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
$$E_2: \quad 5.291x_1 - 6.130x_2 = 46.78,$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

Find the max absolute value For all Elements

$|0.003000| = 0.0030$

$|59.141| = 59.14$ } First row

Complete pivoting

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Example 2 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
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using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

Find the max absolute value For all Elements

$|5.291| = 5.291$

$|-6.13| = 6.13$

} 2nd row $\Rightarrow$ max absolute value $= 59.14$

First row
2nd Column

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Example 1 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
$$E_2: \quad 5.291x_1 - 6.130x_2 = 46.78,$$

using four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Swap 2nd Column to First Column

$$\begin{array}{cc} x_1' & x_2' \\ \left[\begin{array}{cc|c} 59.14 & 0.003 & 59.17 \\ -6.13 & 5.291 & 46.78 \end{array}\right] & \end{array} \rightarrow -\frac{(-6.13)}{59.14} R_1 + R_2$$

$$m_{21} = 0.103652$$
$$\simeq 0.1037$$

Four digit rounding

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Example 1 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
$$E_2: \quad 5.291x_1 - 6.130x_2 = 46.78,$$

using four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Swap 2nd Column to First Column

x_1' x_2'

$$\left[\begin{array}{cc|c} 59.14 & 0.003 & 59.17 \\ -6.13 & 5.291 & 46.78 \end{array}\right] \rightarrow +0.1037\,R_1 + R_2$$

R_2

$a_{21} = 59.14(0.1037) - 6.13 = 6.133 - 6.13 = 0.003 \approx 0$

$a_{22} = 0.003(0.1037) + 5.291 = 5.29131 \approx 5.291$

$b_{22} = 59.17(0.1037) + 46.78 = 52.92$

Example 2 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
$$E_2: \quad 5.291x_1 - 6.130x_2 = 46.78,$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

$$\begin{array}{c} x_1' \quad x_2' \\ \left[\begin{array}{cc|c} 59.14 & 0.003 & 59.17 \\ 0 & 5.291 & 52.92 \end{array}\right] \end{array}$$

$$x_2' = \frac{52.92}{5.291} \simeq 10.002 \simeq 10$$

Back substitute $x_1' = \frac{1}{59.14}(59.17 - 0.003(10))$

$$= 1.0$$

Example 2 Apply Gaussian elimination to the system

$$E_1: \quad 0.003000x_1 + 59.14x_2 = 59.17$$
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using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

But $x_1' = x_2 = 1.00$

$x_2' = x_1 = 10.00$

Final Solution

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Complete Pivoting

Pivoting can incorporate the interchange of both rows and columns. **Complete** (or *maximal*) **pivoting** at the kth step searches all the entries a_{ij}, for $i = k, k+1, \ldots, n$ and $j = k, k+1, \ldots, n$, to find the entry with the largest magnitude. Both row and column interchanges are performed to bring this entry to the pivot position. The first step of total pivoting requires that $n^2 - 1$ comparisons be performed, the second step requires $(n-1)^2 - 1$ comparisons, and so on. The total additional time required to incorporate complete pivoting into Gaussian elimination is

$$\sum_{k=2}^{n} (k^2 - 1) = \frac{n(n-1)(2n+5)}{6}$$

comparisons. Complete pivoting is, consequently, the strategy recommended only for systems where accuracy is essential and the amount of execution time needed for this method can be justified.