

Second - Solved problem $\Rightarrow$ 3×3 Matrix

Find L & U values using option-2 $\Downarrow$

Turn matrix $A \rightarrow$ upper matrix

[Use row operations
Use Elementary matrices

Find inverse of L and check

$$L * L^{-1} = I$$

Example # 2 Write the lower and upper matrix.

For $A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \Rightarrow$

Check whether the Matrix is invertible and estimate its determinant.

Use Crowt's method

Check if matrix A is invertible

$$\begin{aligned} |A| &= +10 \begin{vmatrix} -10 & 3 \\ 2 & -10 \end{vmatrix} - 3 \begin{vmatrix} 2 & 3 \\ 3 & -10 \end{vmatrix} + 4 \begin{vmatrix} 2 & -10 \\ 3 & 2 \end{vmatrix} \\ &= 10(100 - 6) - 3(-20 - 9) + 4(4 + 30) = \\ &\quad 940 + 87 + 136 = 1163 \neq 0 \end{aligned}$$

Matrix is invertible has L & U

$$A = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \cdot \begin{bmatrix} +1 & U_{12} & U_{13} \\ 0 & +1 & U_{23} \\ 0 & 0 & +1 \end{bmatrix}$$

Crout's

$$U_{11} = U_{22} = U_{33} = 1$$

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12} + L_{22} & L_{21}U_{13} + L_{22}U_{23} \\ L_{31} & L_{31}U_{12} + L_{32} & L_{31}U_{13} + L_{32}U_{23} + L_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

Compare the first column of A and LU

Find U_{12} & U_{13} Values

$$L_{11} = a_{11} = 10$$

$$L_{21} = a_{21} = 2$$

$$L_{31} = a_{31} = 3$$

$$U_{12} = \frac{a_{12}}{a_{11}} = \frac{3}{10}$$

$$U_{13} = \frac{a_{13}}{a_{11}} = \frac{4}{10}$$

Prepared by Eng. Maged Kamel.

2nd Example

Write the lower and upper matrix for $A =$
 Crout's method

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

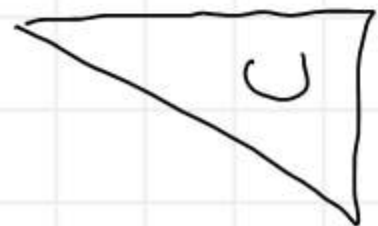
Solution: Convert $A \rightarrow$ an upper matrix

Eliminate a_{21}
 a_{31}

Use a_{11} as a pivot

$a_{11} \ a_{12} \ a_{13}$

$$\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$



multipliers $\frac{a_{21}}{a_{11}} = \frac{2}{10} \rightarrow -\frac{2}{10} R_1 + R_2$
 $\frac{a_{31}}{a_{11}} = \frac{3}{10} \rightarrow -\frac{3}{10} R_1 + R_3$

$$\begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 1.10 & -11.2 \end{bmatrix}$$

option - 2

From before,

$$\left. \begin{aligned} L_{11} &= 10 \\ L_{21} &= 2 \\ L_{31} &= 3 \end{aligned} \right\}$$

$$U_{12} = 3/10$$

$$U_{13} = 4/10$$

Ex: 2 $A_1 = \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 1.10 & -11.2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & L_{32} & L_{32}U_{23} + L_{33} \end{bmatrix}$

Find the values of L_{22} , L_{32} and U_{23}

Divide over

$L_{22} = -10.6$
 $L_{32} = 1.10$ } get $U_{23} = 2.2 / -10.60 = \frac{-1.1}{5.30}$

Convert $A_1 \rightarrow U_1$, use L_{22} as a pivot

multiplier = $\frac{L_{32}}{L_{22}} = \frac{1.10}{-10.60} = -\frac{1.10}{10.60} \Rightarrow$

$\rightarrow \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.60 & 2.2 \\ 0 & 1.1 & -11.20 \end{bmatrix} \rightarrow \text{No change same} \rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & 0 & L_{33} \end{bmatrix}$

$\rightarrow -10.60 \rightarrow -10.6$

$\rightarrow \frac{+1.1}{10.60} R_2 + R_3 \rightarrow \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 0 & -11.63 \\ & & 10.6 \end{bmatrix}$

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Example # 2

$$L_{11} = 10$$

Find the value of L_{33}

$$U_{12} = \frac{3}{10}$$

$$U_{13} = \frac{4}{10}$$

$$U_{23} = -\frac{2.2}{10 \cdot 60}$$

$$\begin{bmatrix} 1 & \frac{3}{10} & \frac{4}{10} \\ 0 & +1 & 11/53 \\ 0 & 0 & +1 \end{bmatrix} \rightarrow \text{U-matrix Final}$$

$$\begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & -2.2 \\ 0 & 0 & -\frac{1163}{106} \end{bmatrix} \xrightarrow{\frac{R_1}{L_{11}}} L_{22} = -10.60$$

$$\xrightarrow{\frac{R_3}{L_{11}}} L_{33} = -\frac{1163}{106}$$

$$\left\{ \begin{bmatrix} L_{11} & L_{11} U_{12} & L_{11} U_{13} \\ 0 & L_{22} & L_{22} U_{23} \\ 0 & 0 & L_{33} \end{bmatrix} \right. \begin{array}{l} \xrightarrow{\frac{R_1}{L_{11}}} \\ \xrightarrow{\frac{R_2}{L_{22}}} \\ \xrightarrow{\frac{R_3}{L_{33}}} \end{array}$$

Example #2

$$\begin{aligned} L_{11} &= 10 \\ L_{21} &= 2 \\ L_{31} &= 3 \end{aligned}$$

$$\begin{aligned} U_{12} &= \frac{3}{10} \\ U_{13} &= \frac{4}{10} \end{aligned}$$

$$\begin{aligned} U_{11} &= U_{22} = U_{33} = 1 \\ U_{23} &= \frac{-2 \cdot 20}{10 \cdot 60} \end{aligned}$$

$$L_{22} = -10.60$$

$$L_{32} = 1.10$$

$$L_{33} = \frac{-1163}{106}$$

$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.10 & -\frac{1163}{106} \end{bmatrix}$$

$\uparrow L$

$$\begin{bmatrix} 1 & \frac{3}{10} & \frac{4}{10} \\ 0 & 1 & -\frac{2 \cdot 2}{10 \cdot 6} \\ 0 & 0 & 1 \end{bmatrix}$$

$\uparrow U$

$$\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10.6 + 0.6 & 0.8 + 2 \cdot 2 \\ 3 & 0.9 + 1.1 & 1.2 - \frac{121}{530} - \frac{1163}{106} = -10 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

A

= A-matrix ok

Check the Product
of L By U

Pivot

A

→ Using Elementary matrices 2nd Example

Thru $A \rightarrow U$
 $(E_3 E_2 E_1) A$

We have Three elementary matrices E1, E2 and E3

Zero

$$\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{10} & 1 & 0 \\ -\frac{3}{10} & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 1.1 & -11.2 \end{bmatrix}$$

E_1 E_2 $E_1 A$

E1 is formed by using a11 as a pivot

E2 is formed by using a22 as a pivot

$$E_2 \cdot (E_1 A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1 \cdot 1}{10.6} & 1 \end{bmatrix} \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 1.1 & -11.2 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 0 & \frac{-1163}{10.6} \end{bmatrix}$$

E_2 $E_1 A$ $E_2 E_1 A$

still not $\downarrow$

$$E_3 \begin{bmatrix} \frac{1}{10} & 0 & 0 \\ 0 & -\frac{1}{10.6} & 0 \\ 0 & 0 & -\frac{10.6}{1163} \end{bmatrix} \begin{bmatrix} 10 & 3 & 4 \\ 0 & -10.6 & 2.2 \\ 0 & 0 & -\frac{1163}{10.6} \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{10} & \frac{4}{10} \\ 0 & 1 & \frac{-2.2}{10.6} \\ 0 & 0 & 1 \end{bmatrix}$$

E_3 $E_2 E_1 A$ U Final

E_3 is formed by making diagonals equal one

$$\overset{\rightarrow}{E_3} E_2 E_1 A = U$$

$$(E_3 E_2 E_1) L U = U$$

$$\overset{\rightarrow}{E_3} E_2 E_1 = L^{-1}$$

$$\begin{bmatrix} 1/10 & 0 & 0 \\ 0 & -1/10.6 & 0 \\ 0 & 0 & -10.6/1163 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1.1/10.6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2/10 & 1 & 0 \\ -3/10 & 0 & 1 \end{bmatrix}$$

E_3

E_2

E_1

How can we get L^{-1} ?

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

Find the inverse of matrix L

$$E_3 E_2 E_1 = L^{-1}$$

$$\begin{bmatrix} 1/10 & 0 & 0 \\ 0 & -1/10.6 & 0 \\ 0 & 0 & -10.6/1163 \end{bmatrix}$$

E_3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1.1/10.6 & 1 \end{bmatrix}$$

E_2

$$\begin{bmatrix} 1 & 0 & 0 \\ -2/10 & 1 & 0 \\ -3/10 & 0 & 1 \end{bmatrix}$$

E_1

E_1

$$\begin{pmatrix} 1/10 & 0 & 0 \\ 0 & -1/10.6 & 0 \\ 0 & 0 & -10.6/1163 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2/10 & 1 & 0 \\ -3/10 & 0 & 1 \end{pmatrix}$$

E_3

$E_2 \cdot E_1$

$$\begin{array}{c} E_3 \qquad E_2 \ E_1 \\
 \begin{pmatrix} \frac{1}{10} & 0 & 0 \\ 0 & \frac{-1}{106} & 0 \\ 0 & 0 & -\frac{106}{1163} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{10} & 1 & 0 \\ -\frac{17}{53} & \frac{11}{106} & 1 \end{pmatrix}
 \end{array}$$

$$= \begin{pmatrix} \frac{1}{10} & 0 & 0 \\ \frac{2}{106} & -\frac{10}{106} & 0 \\ \frac{34}{1163} & \frac{-11}{1163} & -\frac{106}{1163} \end{pmatrix} \Rightarrow L^{-1}$$

Check $L L^{-1} = I$

$$\begin{pmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & +1.10 & -\frac{1163}{106} \end{pmatrix} \begin{pmatrix} \frac{1}{10} & 0 & 0 \\ \frac{2}{106} & -\frac{10}{106} & 0 \\ \frac{34}{1163} & -\frac{11}{1163} & -\frac{106}{1163} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.10 & -\frac{1163}{106} \end{bmatrix}$$

↑ 2