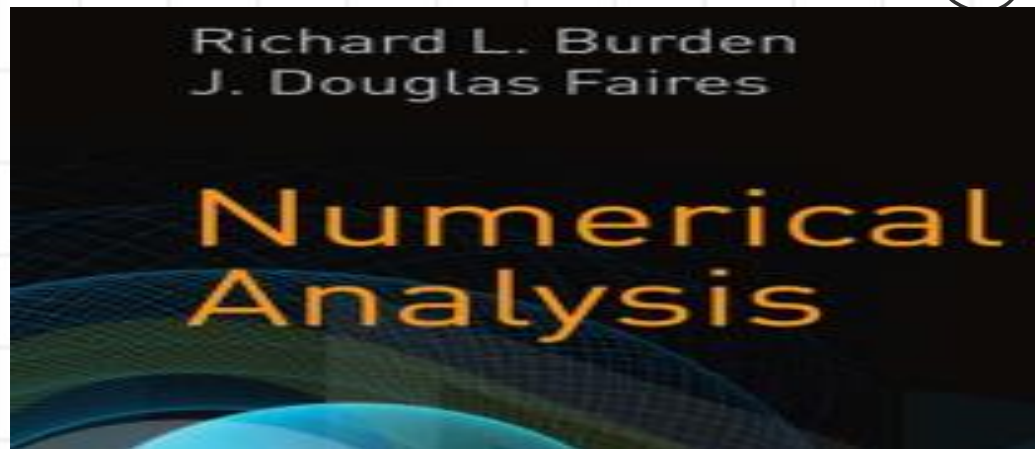


Subject: Partial Pivoting For 2×2 Matrix

① Solved problem - 1

Richard Burden Page 371



Prepared by Eng.Maged Kamel.

Example 1 Apply Gaussian elimination to the system

$$E_1: 0.003000x_1 + 59.14x_2 = 59.17$$

$$E_2: 5.291x_1 - 6.130x_2 = 46.78,$$

using four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

A

→ without Row Exchange

$$\begin{bmatrix} 0.003000 & 59.14 & 59.17 \\ 5.291 & -6.130 & 46.78 \end{bmatrix} \xrightarrow{\left(\begin{array}{l} -5.291 R_1 + R_2 \\ 0.003000 \end{array} \right) \rightarrow R_2}$$

$$M_{21} = -1763.66 = -1764$$

$$a_{21} = 0.003(-1764) + 5.291 = -0.001 \approx 0$$

$$a_{22} = +59.14(-1764) - 6.13 = -104329.09 \approx -104300$$

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$$b_{22} = (-1764)(59.17) + 46.78$$
$$= -104375.88 + 46.78 = -104400 + 46.78$$
$$= -104353.22$$
$$\Downarrow -104400$$

$$\left[\begin{array}{cc|c} x_1 & x_2 & \\ 0.003 & 59.14 & 59.17 \\ 0 & -104300 & -104400 \end{array} \right]$$

$$x_2 = +1.00095 \approx +1.001$$

Example 1 Apply Gaussian elimination to the system

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Back substitute

$$x_2 = +1.001$$

$$\Rightarrow x_{2,r} = 1.00$$

$$\left[\begin{array}{cc|c} x_1 & x_2 & \\ 0.003 & 59.14 & 59.17 \\ 0 & 104300 & -104400 \end{array} \right]$$

$$x_1 = \frac{1}{0.003} (59.17 - 59.14 (+1.001))$$

$$x_1^{\text{real}} = 10$$

$$= -0.02914 / 0.003 = -0.030 / 0.003 = -10.00$$

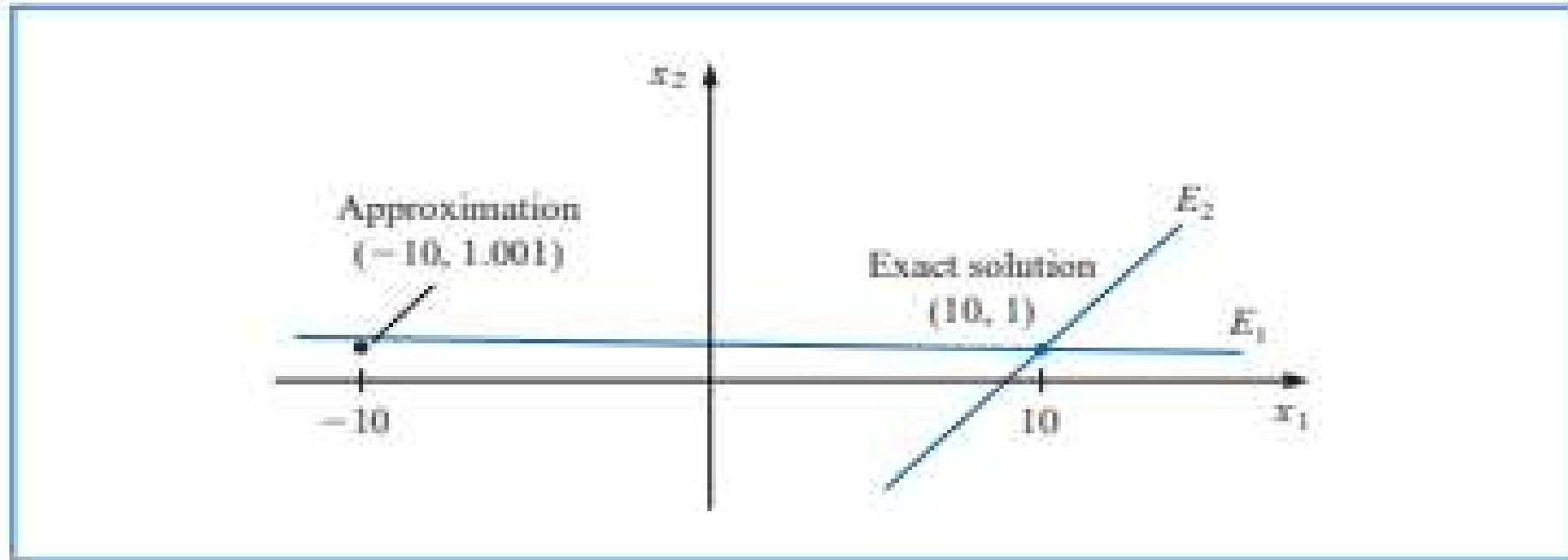
contains the small error of 0.001 multiplied by

$$\frac{59.14}{0.003000} \approx 20000.$$

This ruins the approximation to the actual value $x_1 = 10.00$.

This is clearly a contrived example and the graph in Figure 6.1. shows why the error can so easily occur. For larger systems it is much more difficult to predict in advance when devastating round-off error might occur. ■

Figure 6.1



Do Partial Pivoting

Example 2 Apply Gaussian elimination to the system

$$E_1: 0.003000x_1 + 59.14x_2 = 59.17$$

$$E_2: 5.291x_1 - 6.130x_2 = 46.78,$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

check $\max |a_{11}, a_{21}| = 5.291$
 $0.003, 5.291$

Make row Exchange

$$5.291x_1 - 6.13x_2 = 46.78 \quad \textcircled{\text{I}}$$

$$0.003000x_1 + 59.14x_2 = 59.17 \quad \textcircled{\text{II}}$$

$$m_{21} = -\frac{0.003}{5.291} = -0.000567$$

Example 2 Apply Gaussian elimination to the system

$$E_1: 0.003000x_1 + 59.14x_2 = 59.17$$

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using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

$$\left[\begin{array}{cc|c} 5.291 & -6.13 & 46.78 \\ 0.003 & 59.14 & 59.17 \end{array} \right] \quad m_{21} = -0.000567$$
$$-m_{21} \frac{R_1}{R_2} + R_2$$

$$a_{21} = -\frac{5.67}{10000}(5.291) + 0.0030 = 0$$

$$a_{22} = -0.000567(-6.13) + 59.14 = 59.143$$

$$b_{21} = -0.000567(46.78) + 59.17 = 59.143$$

Example 2 Apply Gaussian elimination to the system

$$E_1: 0.003000x_1 + 59.14x_2 = 59.17$$

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using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

$$\left[\begin{array}{cc|c} \overset{x_1}{5.291} & \overset{x_2}{-6.13} & \overset{x_3}{46.78} \\ 0 & 59.143 & 59.143 \end{array} \right]$$

$$x_2 = \frac{59.143}{59.143} = +1 \rightarrow \text{Back substitute}$$
$$x_1 = \frac{1}{5.291} (46.78 + 6.13(1)) =$$
$$x_1 = 10 \Rightarrow x_1 \text{ real} \quad x_2 = +1 = x_2 \text{ real}$$