

Subject: Partial Pivoting For 2×2 Matrix

① solved problem - Scaled pivoting
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Scaled pivoting

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Example 2 Apply Gaussian elimination to the system

$$E_1: 0.003000x_1 + 59.14x_2 = 59.17$$

$$E_2: 5.291x_1 - 6.130x_2 = 46.78,$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution

$$\max S_1 = \max(|0.003|, |59.41|)$$

$$S_1 = 59.141$$

$$\max S_2 = \max(|5.291|, |6.130|) = 6.13$$

$$\text{Find } \frac{|a_{11}|}{S_1} = \frac{0.003}{59.141} = 5.07262 (10^{-5}) \\ = 0.507262 (10^{-4})$$

Scaled pivoting

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Solution

max $s_2 = \max(|5.291|, |-6.130|) = 6.13$

Find $\frac{|a_{21}|}{s_2} = \frac{5.291}{6.13} = 0.86313$

Swap rows

$$\left[\begin{array}{cc|c} 5.29 & -6.13 & 46.78 \\ 0.003 & 59.14 & 59.17 \end{array} \right]$$

since $\frac{|a_{21}|}{s_2} > \frac{|a_{11}|}{s_1}$
make swap

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Solution

Find m_{21}

$$5.291 x_1 - 6.13 x_2 = 46.78 \quad \text{I}$$

$$0.003000 x_1 + 59.14 x_2 = 59.17 \quad \text{II}$$

$$m_{21} = \frac{-0.003}{5.291} = -0.000567$$

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$$\left[\begin{array}{cc|c} 5.291 & -6.13 & 46.78 \\ 0.003 & 59.14 & 59.17 \end{array} \right] \quad m_{21} = -0.000567$$

$\frac{-m_{21}R_1 + R_2}{R_2}$

$$a_{21} = -\frac{5.67}{10000}(5.291) + 0.0030 = 0$$

$$a_{22} = -0.000567(-6.13) + 59.14 = 59.143$$

$$b_{21} = -0.000567(46.78) + 59.17 = 59.143$$

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$$\left[\begin{array}{cc|c} x_1 & x_2 & \\ 5.291 & -6.13 & 46.78 \\ 0 & 59.143 & 59.143 \end{array} \right]$$

$$x_2 = \frac{59.143}{59.143} = +1 \rightarrow \text{Back substitute}$$
$$x_1 = \frac{1}{5.291} (46.78 + 6.13(1)) =$$
$$x_1 = 10 \Rightarrow x_1 = 10, x_2 = +1$$
$$5.291x_1 = 46.78 + 6.13x_2$$