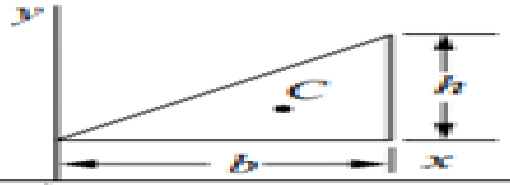
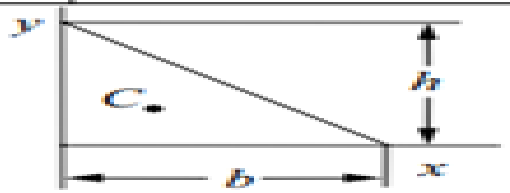
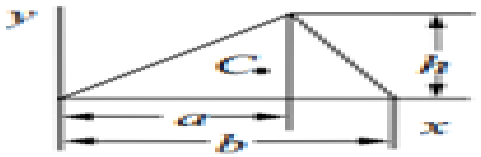
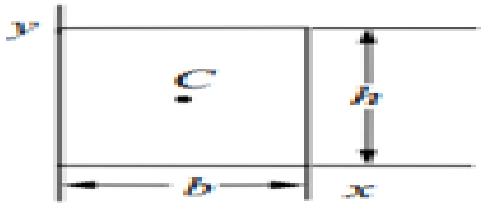
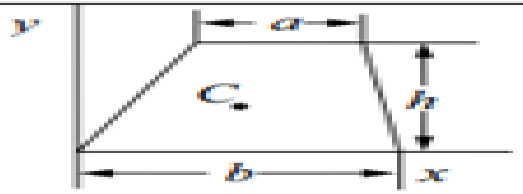
FE Reference Handbook 10.0

Figure	Area & Centroid
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$
	$A = bh/2$ $x_c = (a + b)/3$ $y_c = h/3$
	$A = bh$ $x_c = b/2$ $y_c = h/2$
	$A = h(a + b)/2$ $y_c = \frac{h(2a + b)}{3(a + b)}$

Area and Cg of a Triangle

How to get $\bar{X}$ value ?

Area of triangle abc

$$= \frac{1}{2}(\text{base})(\text{height})$$

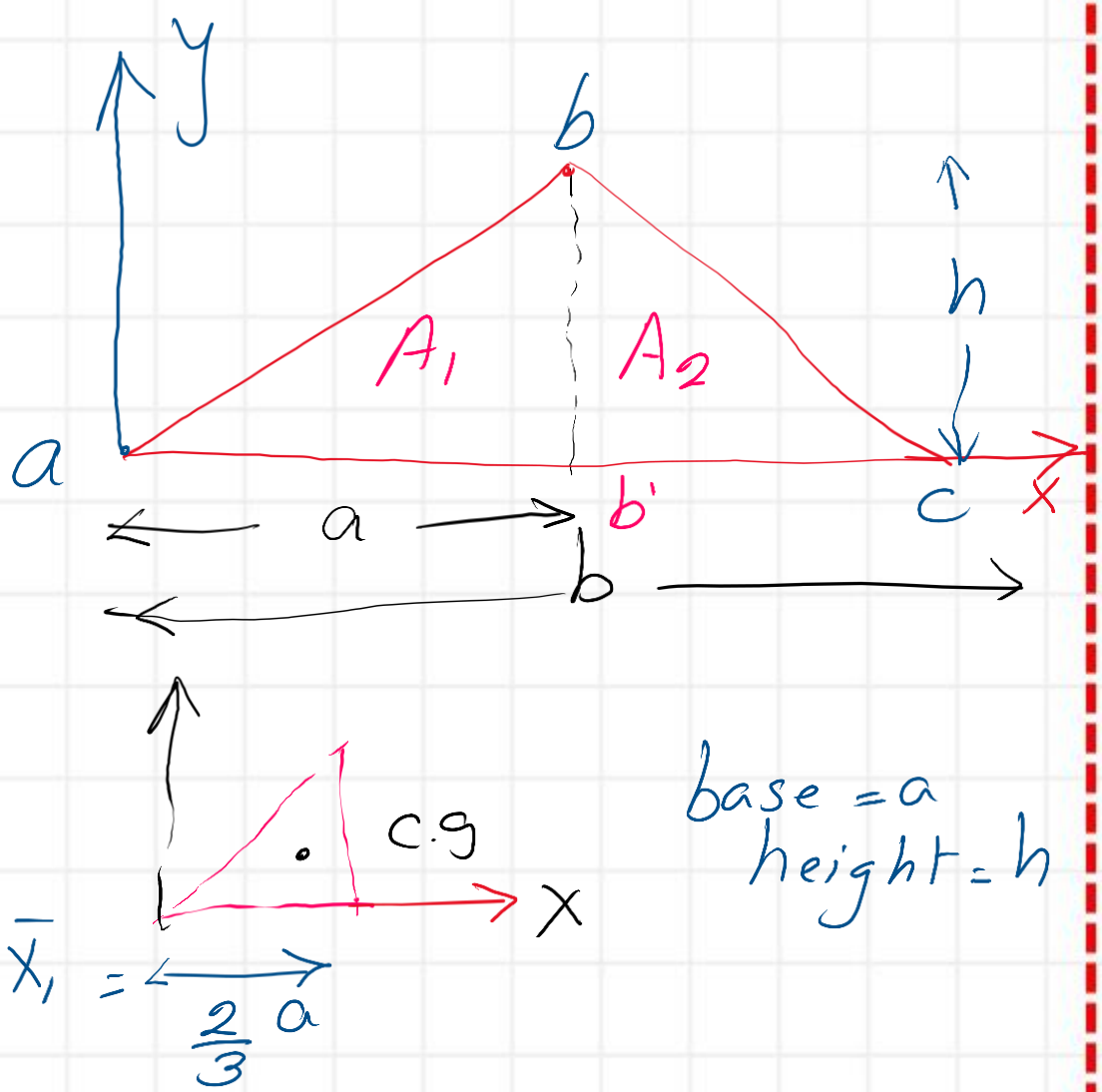
$$= \frac{1}{2}(b)(h)$$

For $\bar{X}$ > Consider triangle A_1

area of $A_1 = \frac{1}{2}a(h)$

Cg $\bar{X}_1 = \frac{2}{3}a$

distance to y-axis



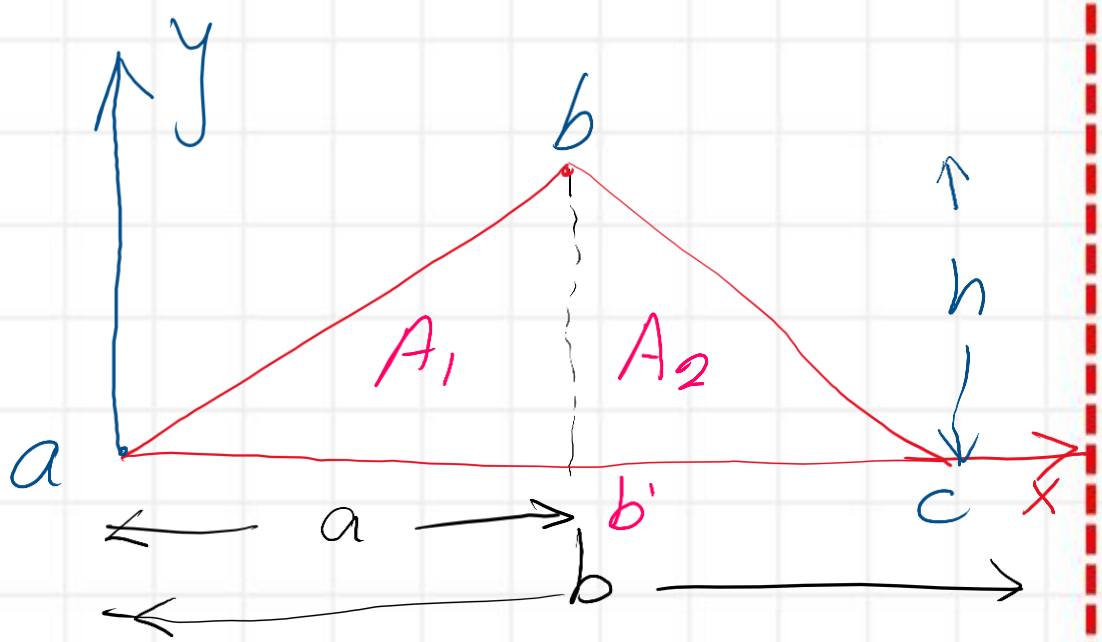
Area and Cg of a Triangle

For the big triangle

Area of triangle abc

$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(b)(h)$$

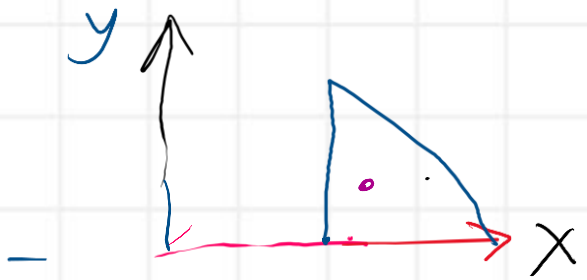


For $\bar{X}$ > Consider triangle (A_2)

area of $A_2 = \frac{1}{2}(b-a)\,h$

Cg $\quad \bar{X}_2 = a + \frac{1}{3}(b-a)$

distance to y-axis.



base $= (b-a)$

height $= h$

$$\bar{X}_2 = a + \frac{1}{3}(b-a)$$

$$\sum_{i=1}^{i=2} A_i \bar{x}_i = A \cdot \vec{\bar{x}}$$

A: area of whole triangle

↘ sum of product of triangle (its $\bar{x}$ distance of C.g)

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 = A \cdot \bar{x}$$

we have $A_1 = \frac{1}{2} a h$, $A_2 = \frac{1}{2}(b-a)h$

$\bar{x}_1 = \frac{2}{3} a$, $\bar{x}_2 = a + \frac{1}{3}(b-a)$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 = \frac{1}{2} a h \left(\frac{2}{3} a\right) + \frac{1}{2}(b-a)(h)\left(a + \frac{b}{3} - \frac{a}{3}\right)$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = \frac{1}{2}ah\left(\frac{2}{3}a\right) + \frac{1}{2}(b-a)(h)\left(a + \frac{b}{3} - \frac{a}{3}\right)$$

Simplify $\rightarrow$

$$= h\left(\frac{a^2}{3} + \left(\frac{b}{2} - \frac{a}{2}\right)\left(\frac{2a}{3} + \frac{b}{3}\right)\right)$$

$$= h\left(\frac{a^2}{3} + \left(\frac{2ab}{6} + \frac{b^2}{6} - \frac{2a^2}{6} - \frac{ab}{6}\right)\right.$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = h\left(\frac{a^2}{3} + \left(-\frac{a^2}{3}\right) + \left(\frac{ab}{6} + \frac{b^2}{6}\right)\right.$$

$$= \frac{h}{6}(ab + b^2)$$

$$A\bar{x} = \frac{1}{2}(bh)\,\bar{x} \qquad \rightarrow \bar{x} = \frac{1}{(1/2\,bh)}\left(\frac{h}{6}\right)b(a+b) = \frac{a+b}{3}$$

Area and Cg of a Triangle

How to get $\bar{y}$ value ?

Area of triangle abc

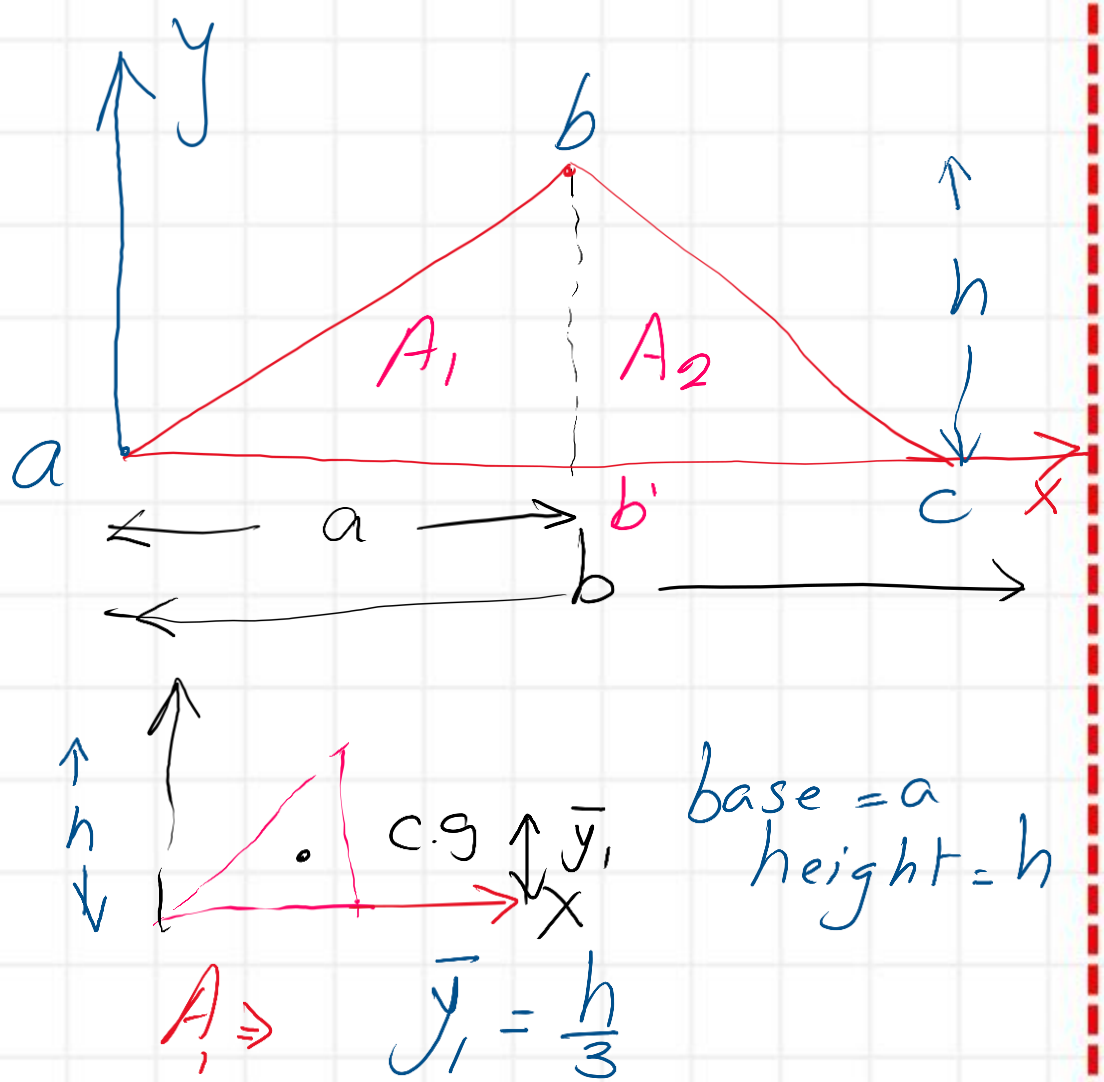
$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(b)(h)$$

For $\bar{y}$ > Consider triangle A_1

$$A_1 = \frac{1}{2}a(h)$$

$\bar{y}_1 = \frac{h}{3}$ From C.g to X_axis



$A_1 \Rightarrow \quad \bar{y}_1 = \frac{h}{3}$

Area and Cg of a Triangle

Area of triangle abc

$$= \frac{1}{2}(\text{base})(\text{height})$$

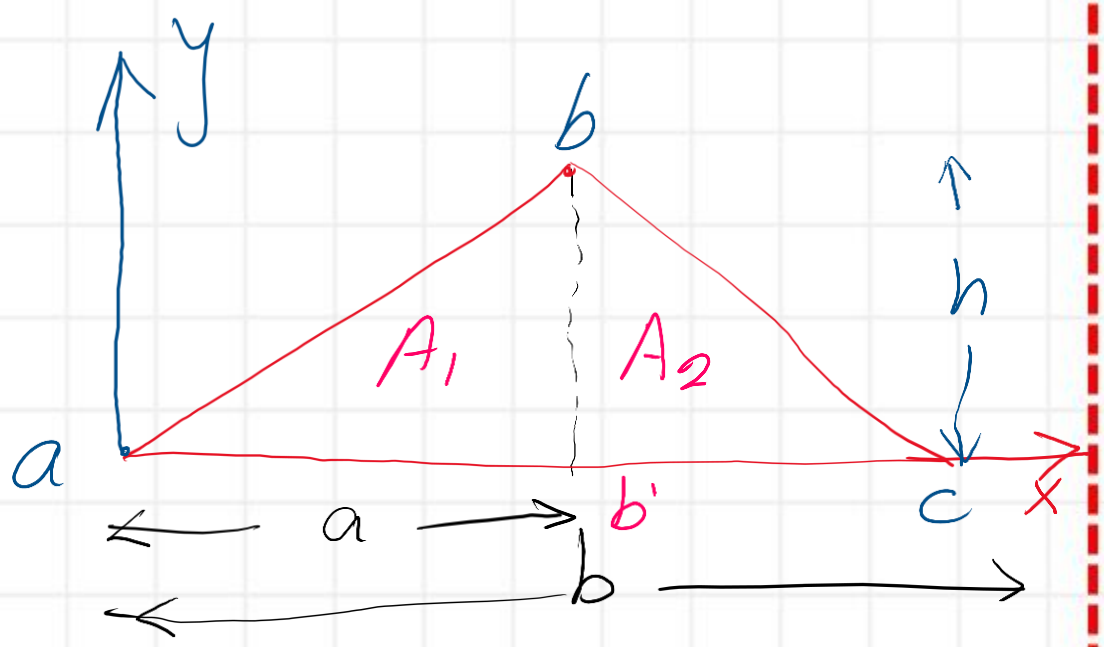
$$= \frac{1}{2}(b)(h)$$

For $\bar{y}$ > Consider triangle A_2

area of $A_2 = \frac{1}{2}(b-a)h$

$\bar{y}_2$ C.g to X – axis

$$\bar{y}_2 = \frac{h}{3}$$



base = (b-a)
height = h

$$\bar{y}_2 = \frac{h}{3}$$

$$\sum_{i=1}^{i=2} A_i \bar{y}_i = A \cdot \vec{\bar{y}}$$

A: area of whole triangle

↘ sum of product of triangle (its distance of C.g)

$$A_1\bar{y}_1 + A_2\bar{y}_2 = A \cdot \bar{y}$$

we have $A_1 = \frac{1}{2}(ah)$, $A_2 = \frac{1}{2}(b-a)h$ } $A = \frac{1}{2}bh$

$\bar{y}_1 = \frac{h}{3}$ & $\bar{y}_2 = \frac{h}{3}$ }

$$A_1\bar{y}_1 + A_2\bar{y}_2 = \frac{1}{2}(ah)\left(\frac{h}{3}\right) + \frac{1}{2}(bh)\left(\frac{h}{3}\right) - \frac{1}{2}ah\left(\frac{h}{3}\right)$$

$A_1\bar{y}_1 + A_2\bar{y}_2 = bh^2/6$, $\bar{y} = \frac{bh^2}{\frac{1}{2}bh}\left(\frac{1}{6}\right) = \frac{h}{3}$

Prepared by Eng.Maged Kamel.