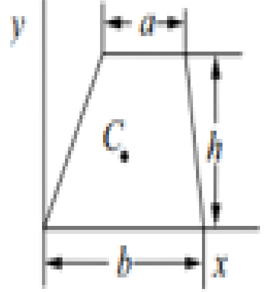
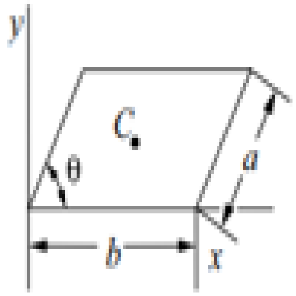
FE Reference Handbook 10.0

Figure	Area & Centroid	
	$A = h(a+b)/2$ $y_c = \dfrac{h(2a+b)}{3(a+b)}$	$I_{x_c} = \dfrac{h^3(a^2+4ab+b^2)}{36(a+b)}$ $I_x = \dfrac{h^3(3a+b)}{12}$
	$A = ab\sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a\sin\theta)/2$	$I_{x_c} = (a^3 b\sin^3\theta)/12$ $I_{y_c} = [ab\sin\theta(b^2 + a^2\cos^2\theta)]/12$ $I_x = (a^3 b\sin^3\theta)/3$ $I_y = [ab\sin\theta(b + a\cos\theta)^2]/3 - (a^2b^2\sin\theta\cos\theta)/6$

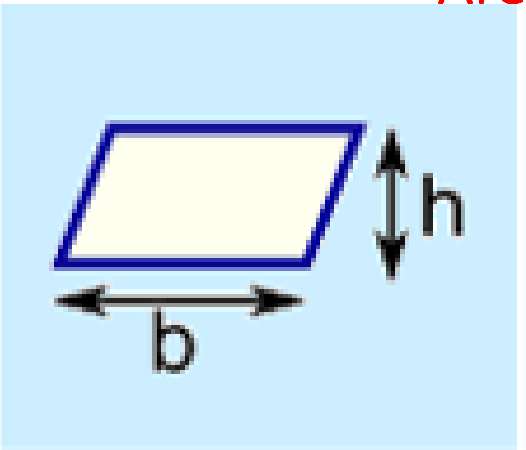
← Parallelogram

$$xc = \frac{(b + a\,\mathrm{Cos}\,\theta)}{2}$$

Area and Cg for Parallelogram.

Three areas : $A_1, A_2 \& A_3$

Area for Parallelogram.

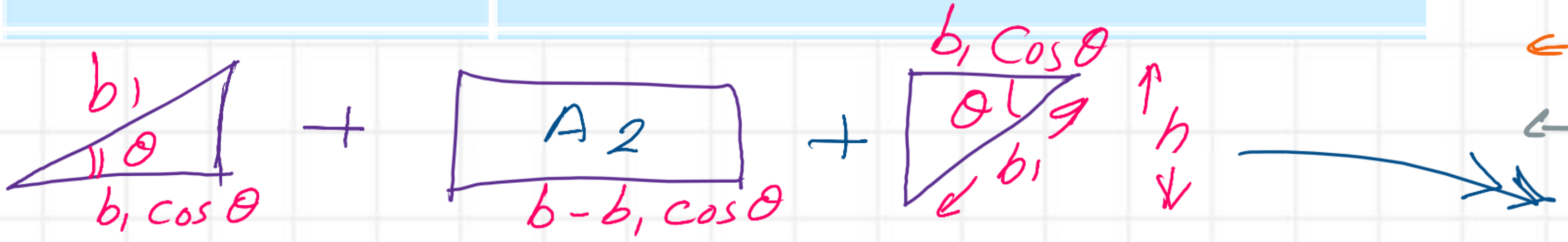


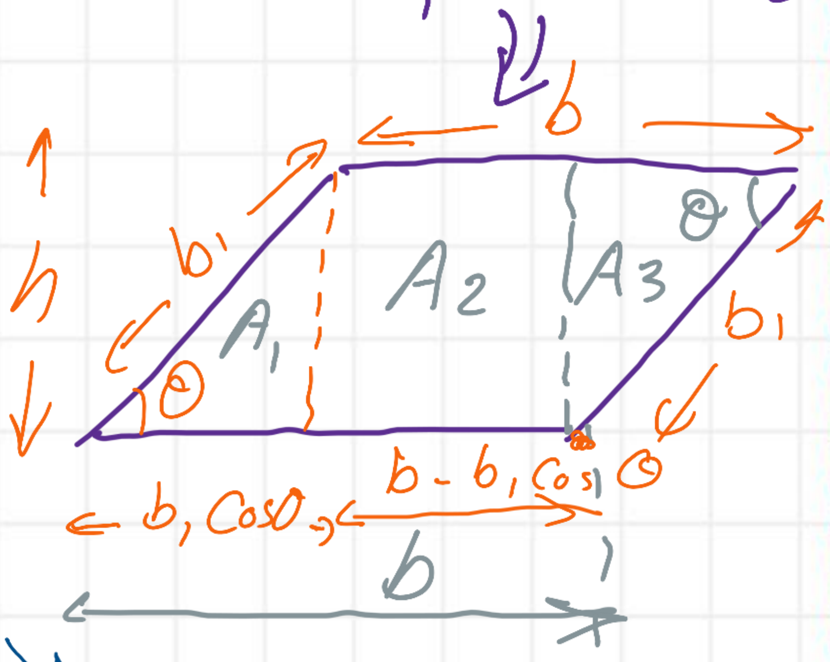
Parallelogram

Area = b × h

b = base

h = vertical height





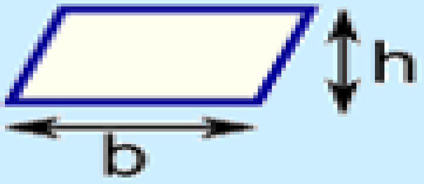
$$A = A_1 + A_2 + A_3$$

$$A = \frac{1}{2}(b_1 \cos\theta)\, h + (b - b_1 \cos\theta)\, h + \frac{1}{2} b_1 \cos\theta\,(h)$$

$$= b_1 \cos\theta\, h + b\, h - b_1 \cos\theta$$

$$= bh$$

Cg for Parallelogram.

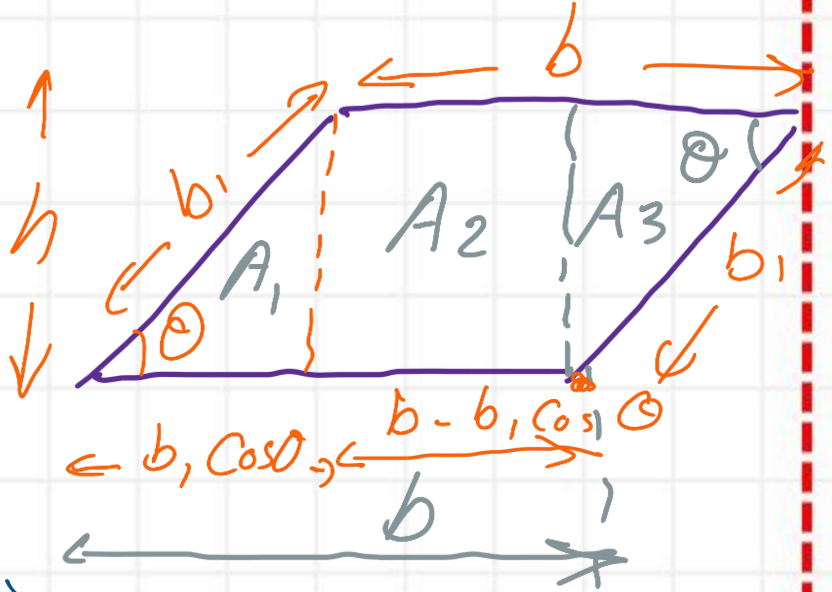


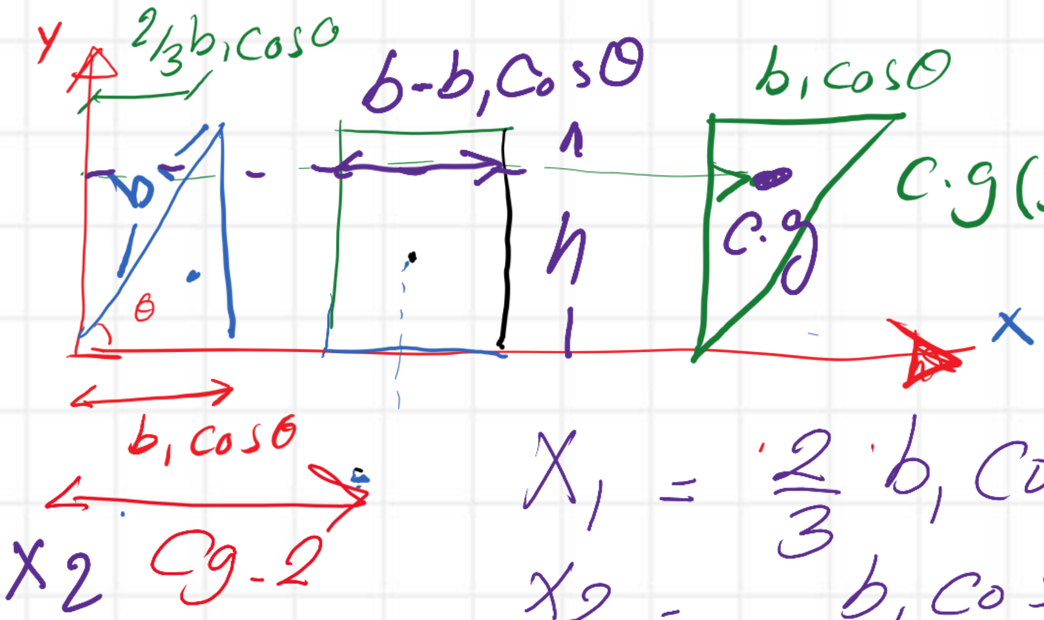
Parallelogram

Area = b × h

b = base

h = vertical height





$$c.g(3) = b + \frac{1}{3} b_1 \cos\theta$$

x Cg distances For A_1, A_2 & A_3

$$X_1 = \frac{2}{3} b_1 \cos\theta$$

$$X_2 = b_1 \cos\theta + \frac{1}{2}(b - b_1 \cos\theta) = \frac{1}{2}(b + b_1 \cos\theta)$$

$$X_3 = b + \frac{1}{3} b_1 \cos\theta$$

$A_1 = \frac{1}{2} b_1 \cos\theta (h)$ $\quad A_2 = (b - b_1 \cos\theta) h \quad A_3 = \frac{1}{2} b_1 \cos\theta \cdot h$

$\bar{X}_1 = \frac{2}{3} b_1 \cos\theta \quad \bar{X}_2 = \frac{1}{2} b_1 \cos\theta + \frac{b}{2} \quad \bar{X}_3 = b + \frac{1}{3} b_1 \cos\theta$

$$\sum_{i=1}^{i=3} A_i x_i = \frac{1}{\cancel{2}} b_1 \cos\theta\, h \left(\frac{\cancel{2}}{3} b_1 \cos\theta\right) = \frac{1}{3} b_1^2 \cos^2\theta (h)$$

$i=1$

$$+ (b - b_1 \cos\theta) h \left(\frac{1}{2} b_1 \cos\theta + \frac{b}{2}\right) = h\left[\cancel{\frac{1}{2} b b_1 \cos\theta} + \frac{b^2}{2} - \frac{1}{2} b_1^2 \cos^2\theta - \cancel{\frac{b b_1}{2} \cos\theta}\right]$$

$i=3$

$$+ \left(\frac{1}{2} b_1 \cos\theta\right) h \left(b + \frac{1}{3} b_1 \cos\theta\right) = h\left[\frac{1}{2} b b_1 \cos\theta + \frac{1}{6} b_1^2 \cos^2\theta\right]$$

$$A\bar{X} = h\left[\frac{1}{3} b_1^2 \cos^2\theta - \frac{1}{2} b_1^2 \cos^2\theta + \frac{1}{6} b_1^2 \cos^2\theta\right] = 0$$ b_1^2 Term

$h\left[\frac{1}{2} b b_1 \cos\theta\right]$ $\quad hb^2/2$ For b^2 term

b & b_1 Terms

Parallelogram

Exprension for $A \cdot \bar{x}$

$A\bar{x} = h\left[\frac{1}{3} b_1^2 \cos^2\theta - \frac{1}{2} b_1^2 \cos^2\theta + \frac{1}{6} b_1^2 \cos^2\theta\right] = 0$ b_1 Term

$+h\left[\frac{1}{2} b b_1 \cos\theta\right]$, bb_1 term

$= + h\left(\frac{b^2}{2}\right) = \frac{hb^2}{2} + \frac{h}{2}(b b_1 \cos\theta) = \frac{1}{2}h\left(b^2 + b b_1 \cos\theta\right)$

but $A = bh$

Then $\bar{x} = \frac{\cancel{h} \cancel{b}(b + b_1 \cos\theta)}{2\cancel{b}\cancel{h}} = \frac{1}{2}(b + b_1 \cos\theta)$

Final Exprension

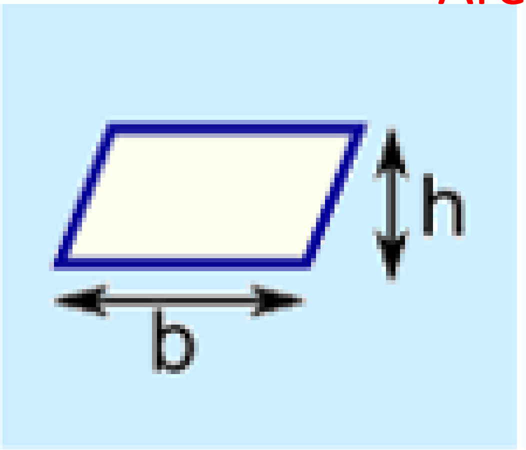
For $b_1 = a$ as in FE Reference

$\bar{x} = \frac{1}{2}(b + a\cos\theta)$

Area and Cg for Parallelogram.

Part_2 Estimate

Area for Parallelogram.

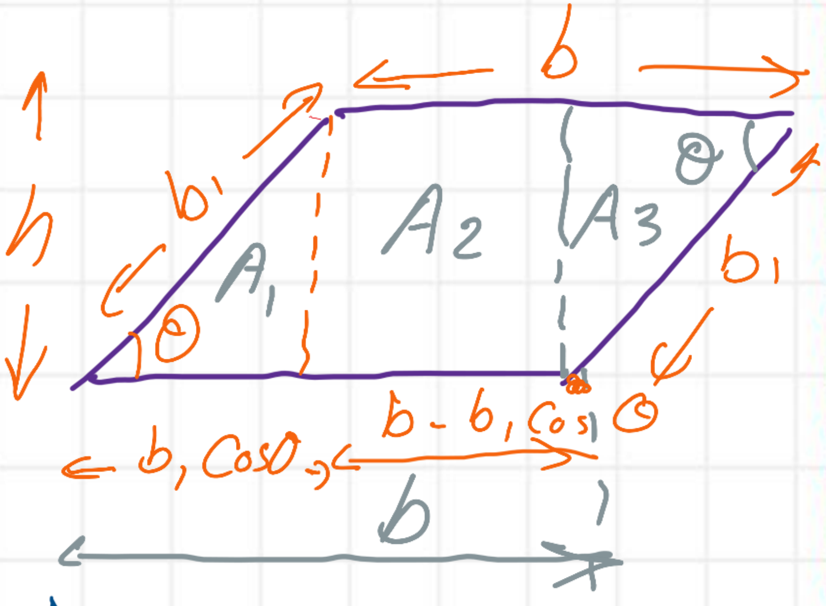


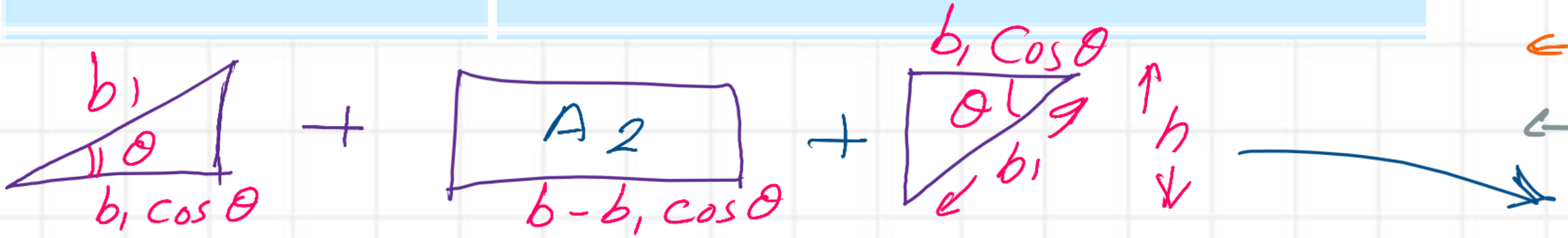
Parallelogram

Area = b × h

b = base

h = vertical height





$A = A_1 + A_2 + A_3$

$A = \frac{1}{2}(b_1 \cos\theta)h + (b - b_1\cos\theta)h + \frac{1}{2} b_1 \cos\theta (h)$

$= b_1 \cos\theta \, h + b h - b_1 \cos\theta$

$= bh$

$A_1 = \frac{1}{2} b_1 \cos\theta (h)$

$y_1 = \frac{1}{3} h$

VL distance c.g → x-axis

Area A_1

$A_1 \bar{y}_1 = \frac{1}{6} b_1 h^2 \cos\theta$

$A_2 = (b - b_1 \cos\theta) h$

$y_2 = \frac{h}{2}$ Area-2

$A_2 \bar{y}_2 = \frac{1}{2} h^2 (b - b_1 \cos\theta)$

$A_3 = \frac{1}{2} b_1 \cos\theta . h$

$y_3 = \frac{2}{3} h$ Area-3

$A_3 \bar{y}_3 = \frac{1}{3} h^2 (b_1 \cos\theta)$

$$\sum_{i=1}^{i=3} A_i \bar{y}_i = \frac{h^2}{6} \left[b_1 \cos\theta + 3b - 3b_1 \cos\theta + 2 b_1 \cos\theta \right]$$

$$= \frac{h^2}{6} [3b + 0] = \frac{3bh^2}{6} = bh^2/2$$

$$A = bh \longrightarrow \bar{y} = \frac{bh^2}{2} \times \frac{1}{bh} = \frac{h}{2} = \boxed{b_1 \sin\theta/2}$$

$b_1 \Rightarrow$ Considerd as a in Some Textbooks