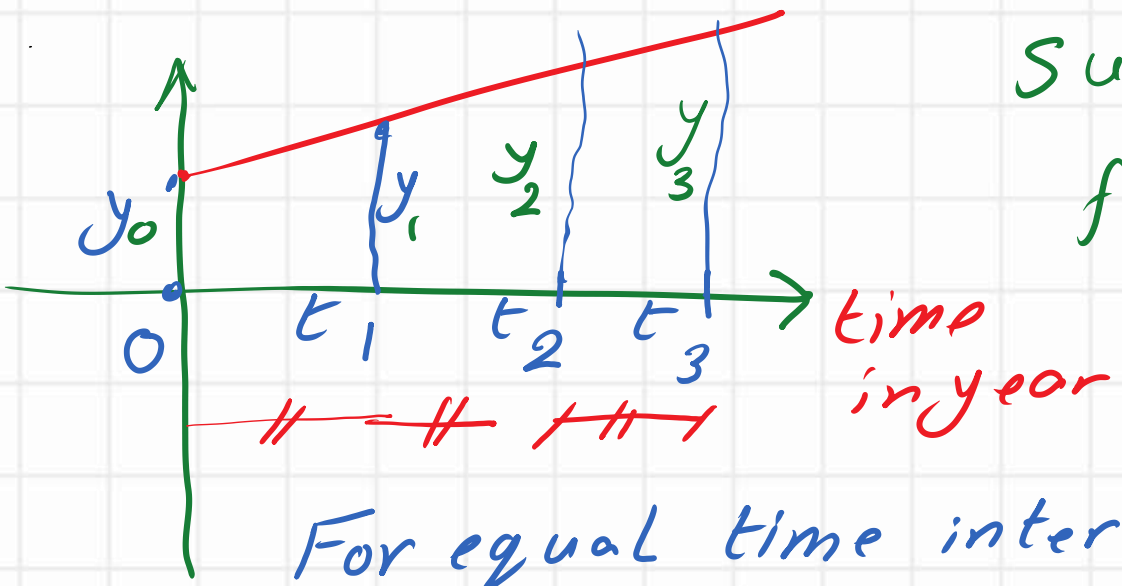
How to convert a Linear function to an exponential function



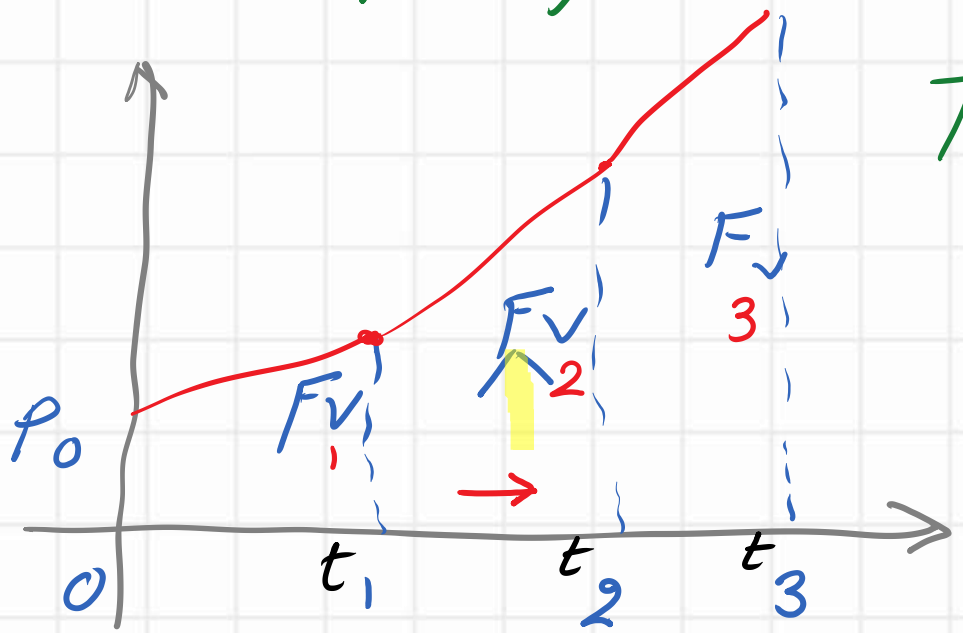
Suppose we have a Linear function, represented by the given shape.

For equal time intervals -

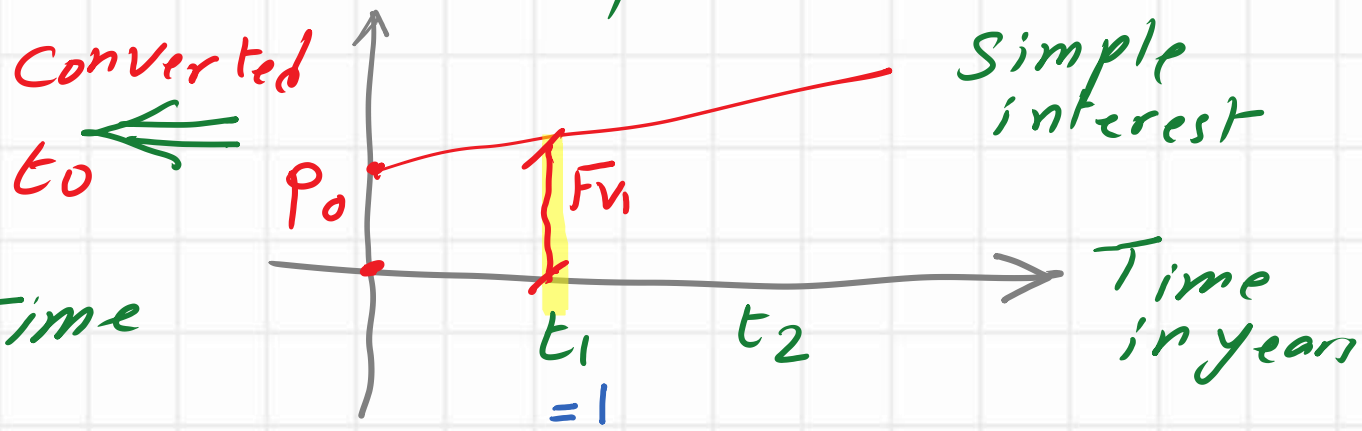
in Linear function we have $y_1 - y_0 = y_2 - y_1 = y_3 - y_1$

To convert to an exponential function let $\frac{y_1}{y_0} = \frac{y_2}{y_1} = \frac{y_3}{y_2}$

The shape of the Graph, will change

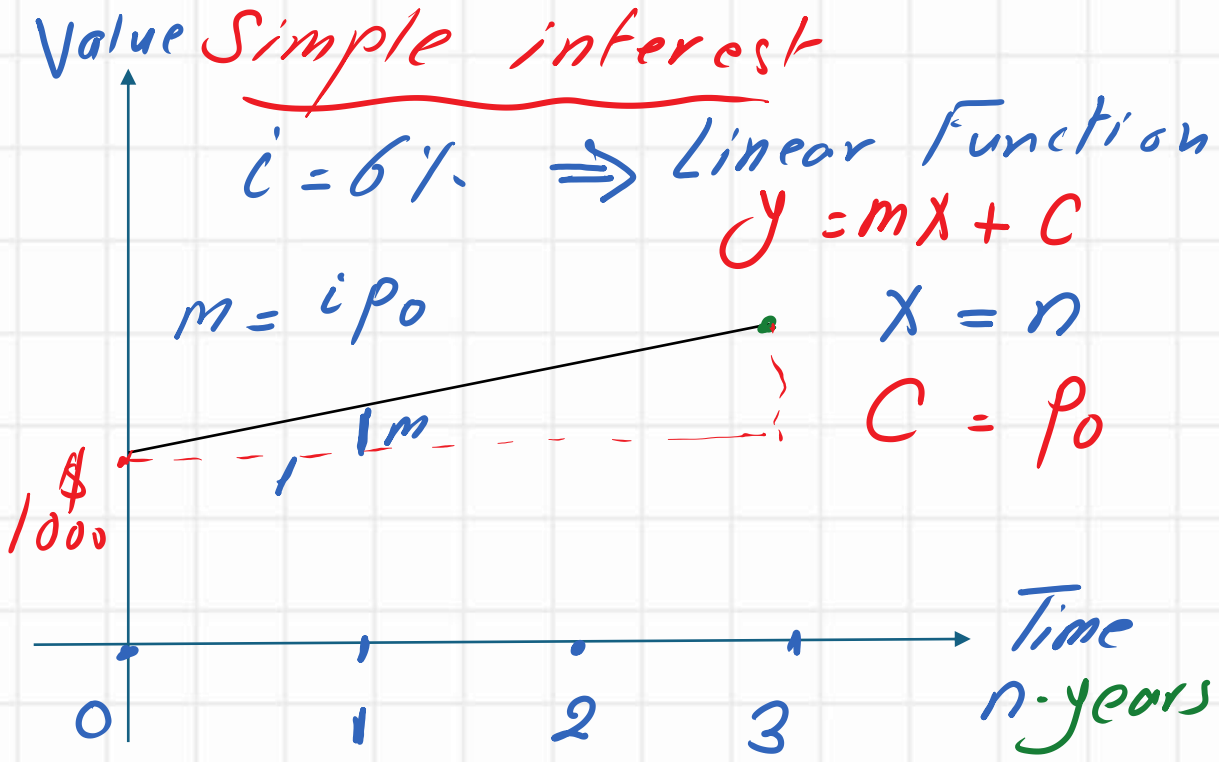


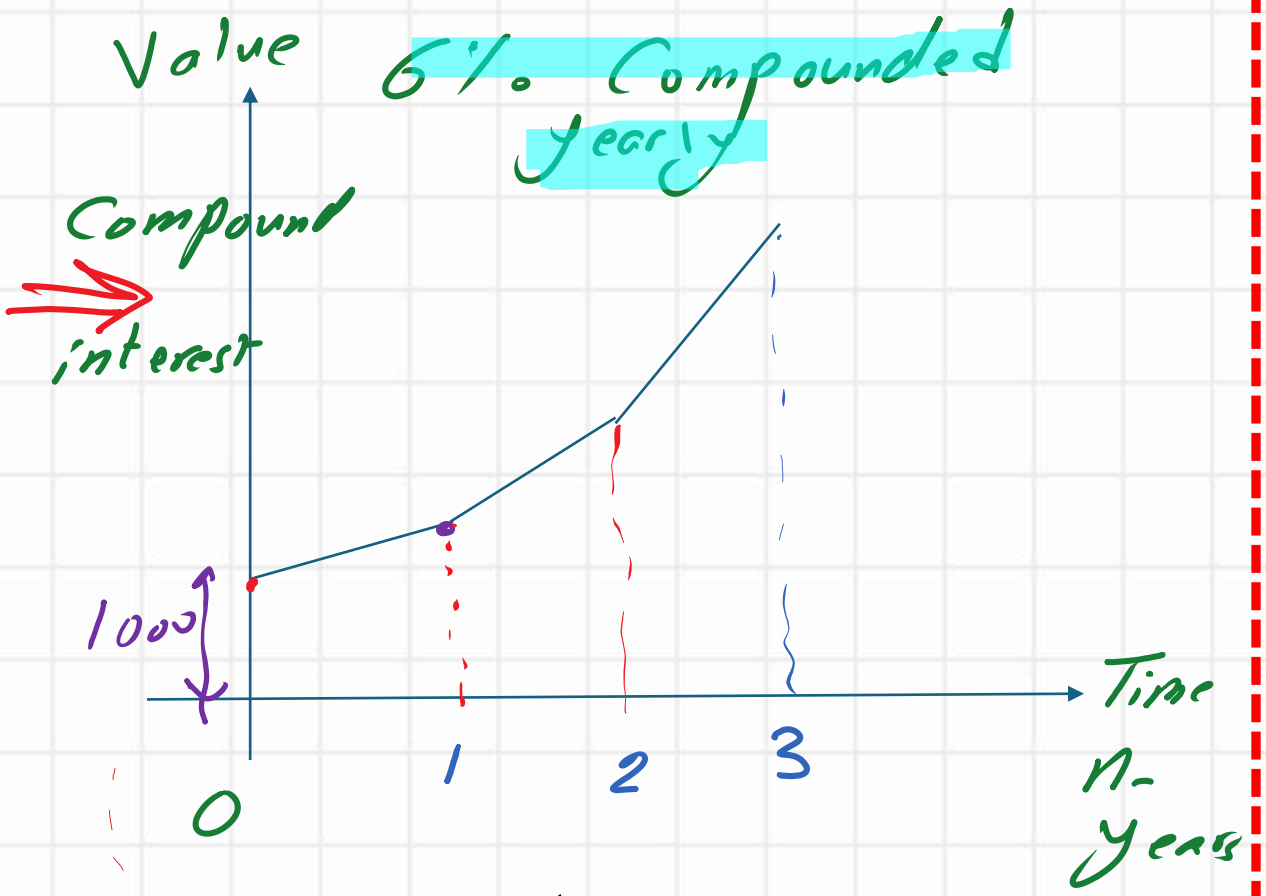
This what the Compound interest does



↳

① get the P_0 value at time (t_0)
② get the FV_1 value at time (t_1) = 1 year } From Simple interest
③ at $t_2 \rightarrow y_2 = FV_2 = \frac{FV_1}{P_0}(FV_1)$ → this is the new value





Deposit $1000 in a bank

For 3 years with 6%. Simple interest

Slope $m = ipo$

$FV = (ipo)n + Po$

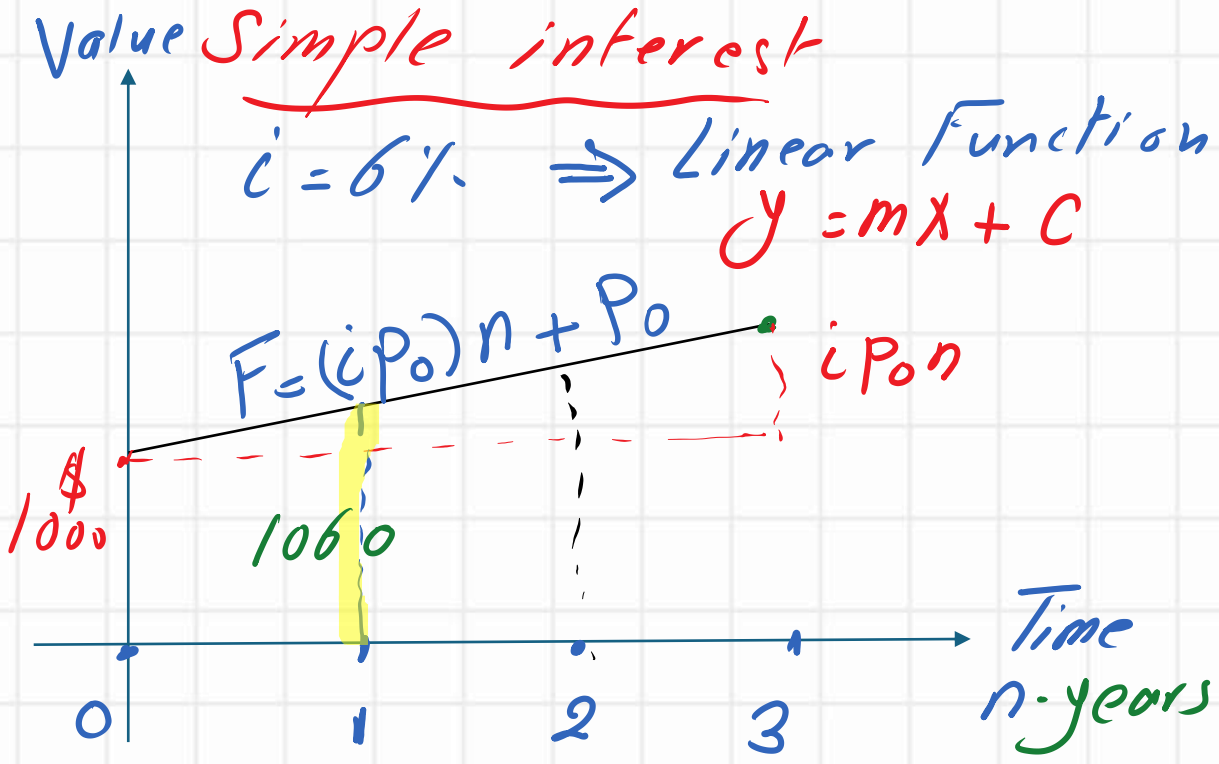
Deposit $1000

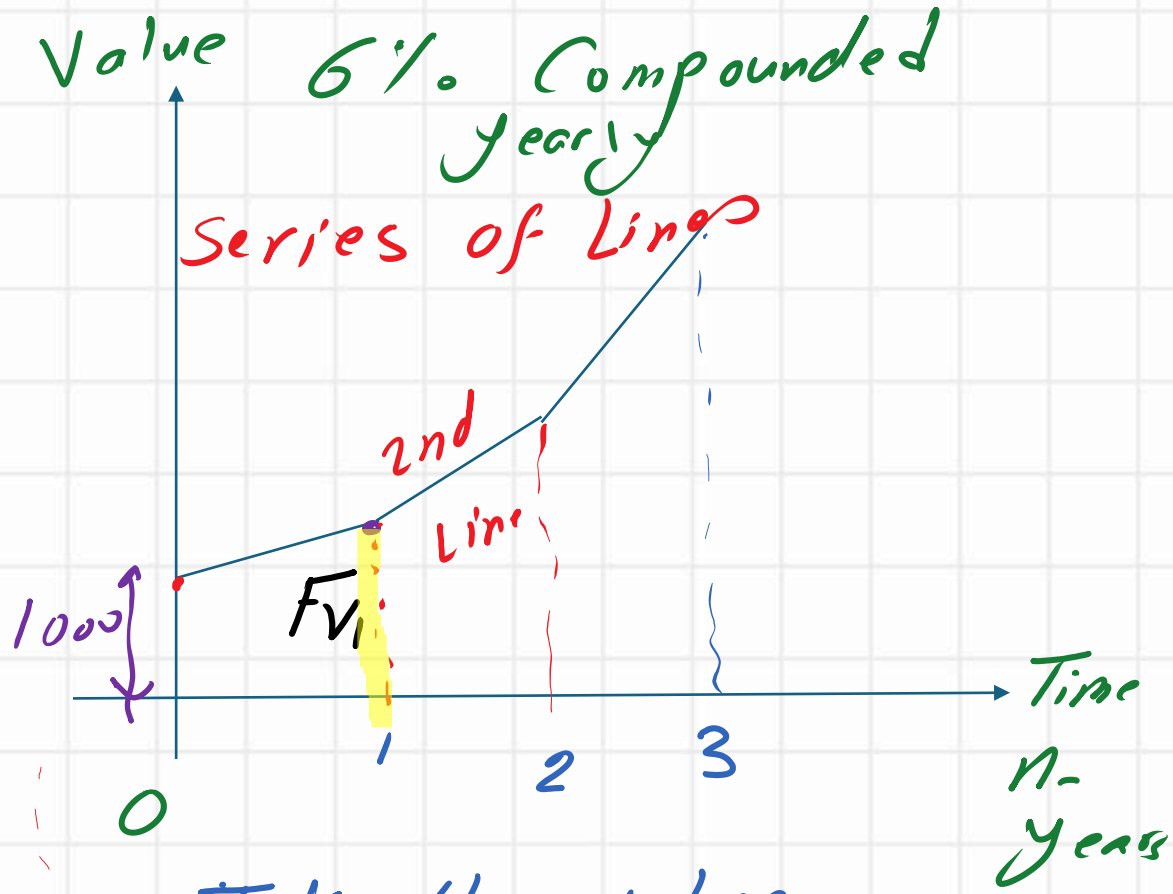
For three years

6%. Compounded yearly

$FV(n=1) = 1000 + 0.06(1000)$

$= 1060$





in a Form of Line equation

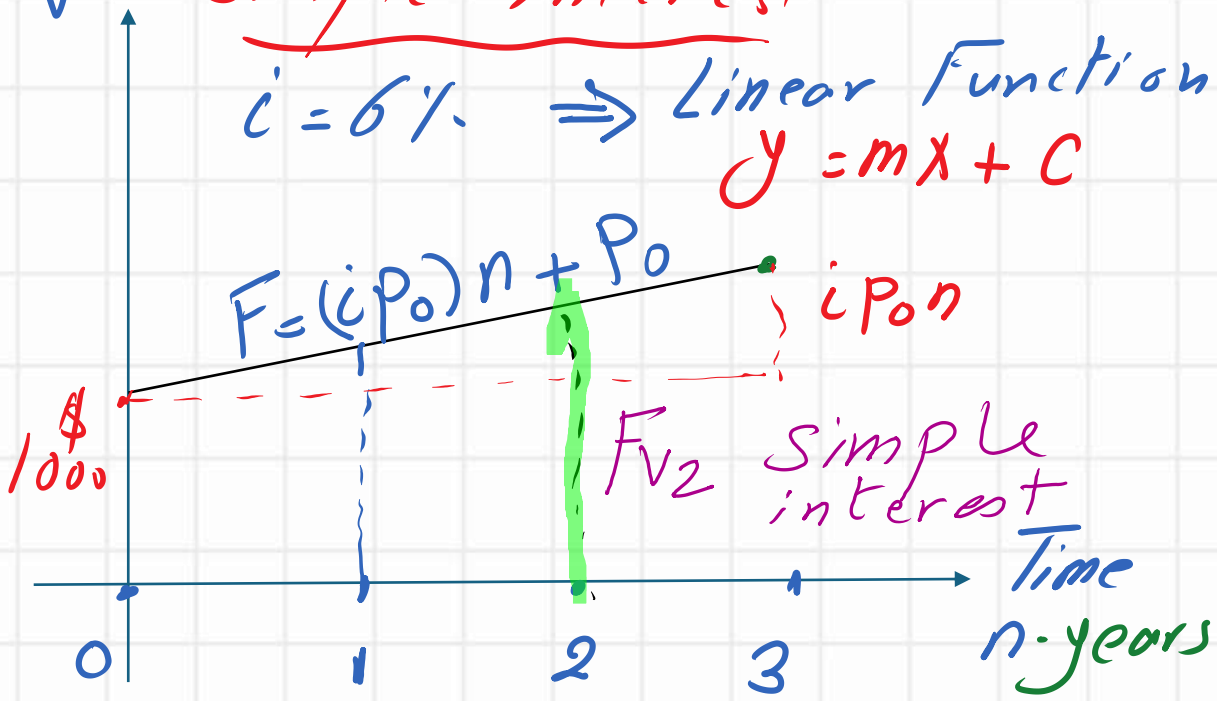
$F = iP_0n + P_0 \Rightarrow n = 1 \quad FV = 0.06(1000)1 + 1000$

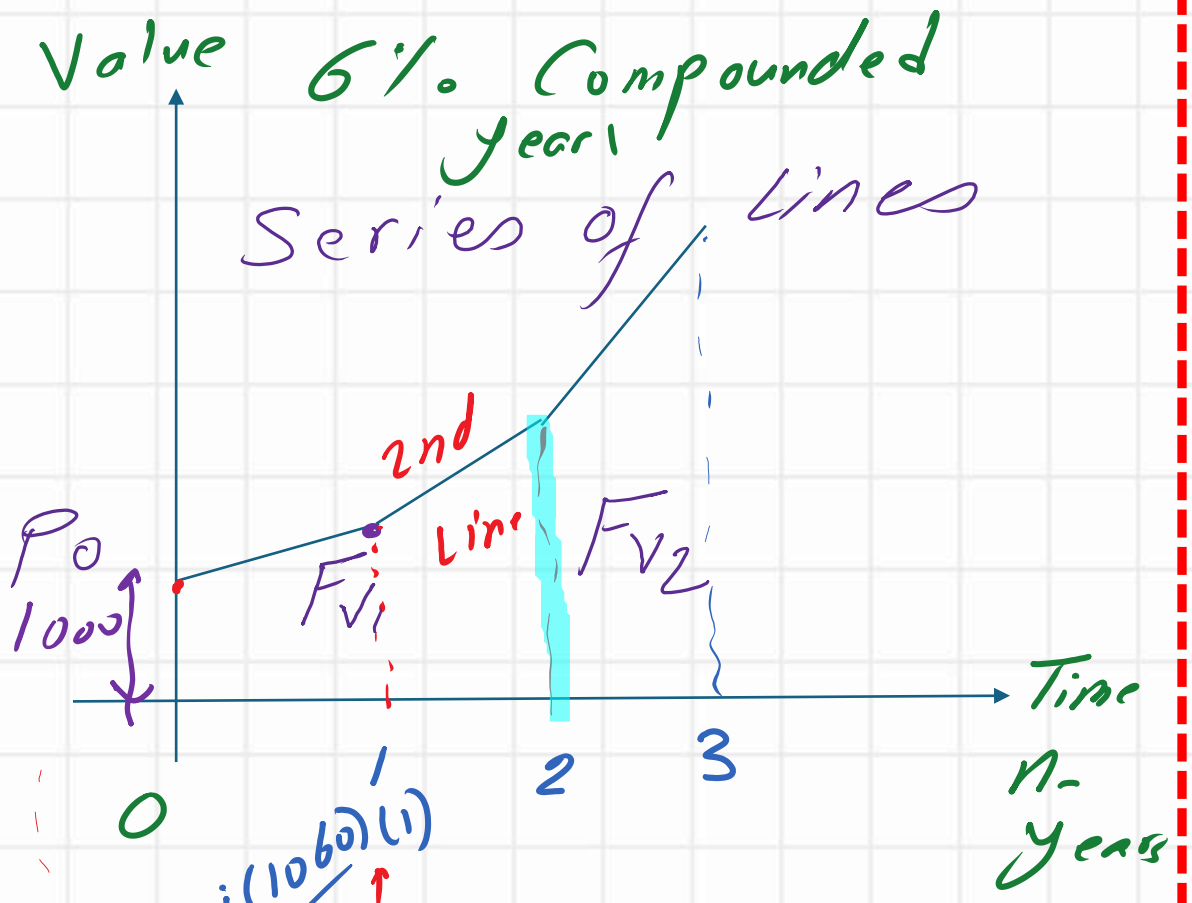
$FV(n=1) = 60 + 1000 = 1060$

Take the value

$FV_1 = 1060$ at $t_1 = 1$

the same value

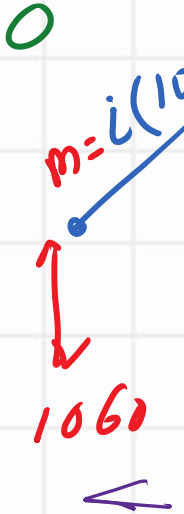




in a Form of Line equation as simple interest.

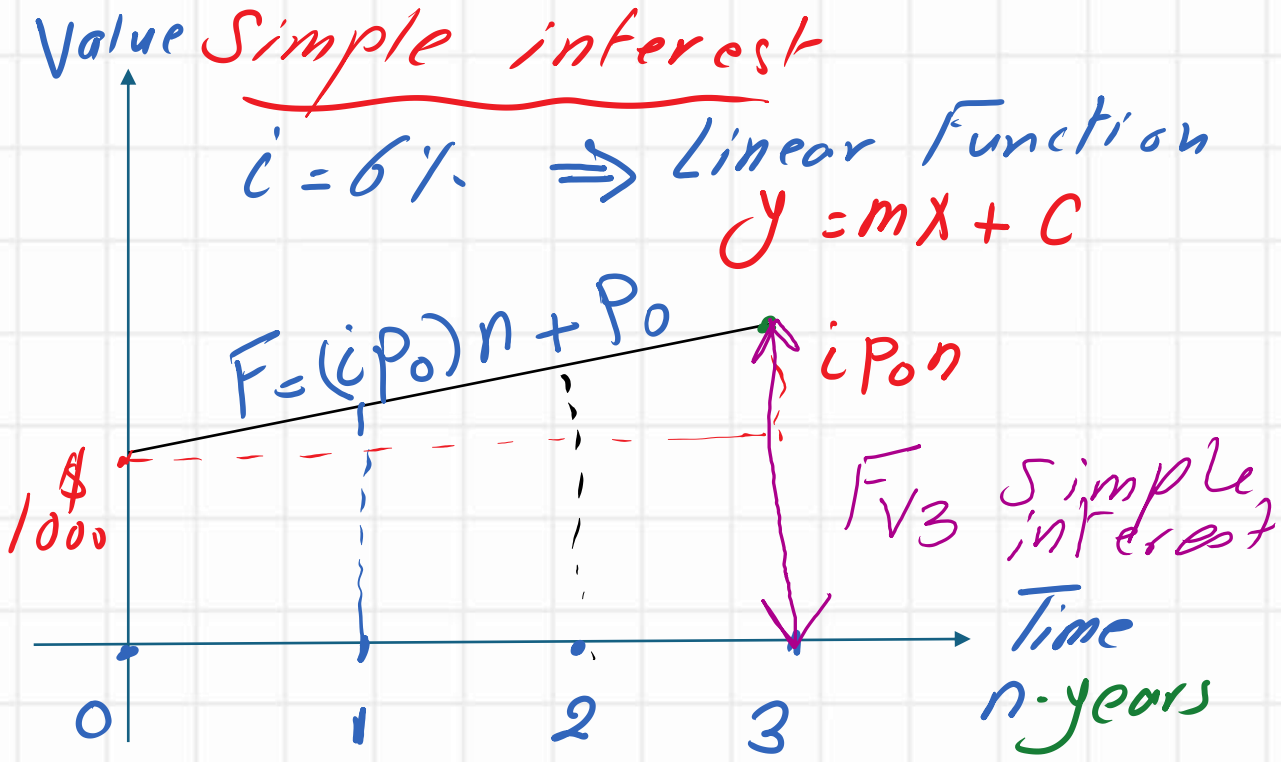
$FV(n=2) = 0.06(1000)(2) + 1000 = 1120$ #

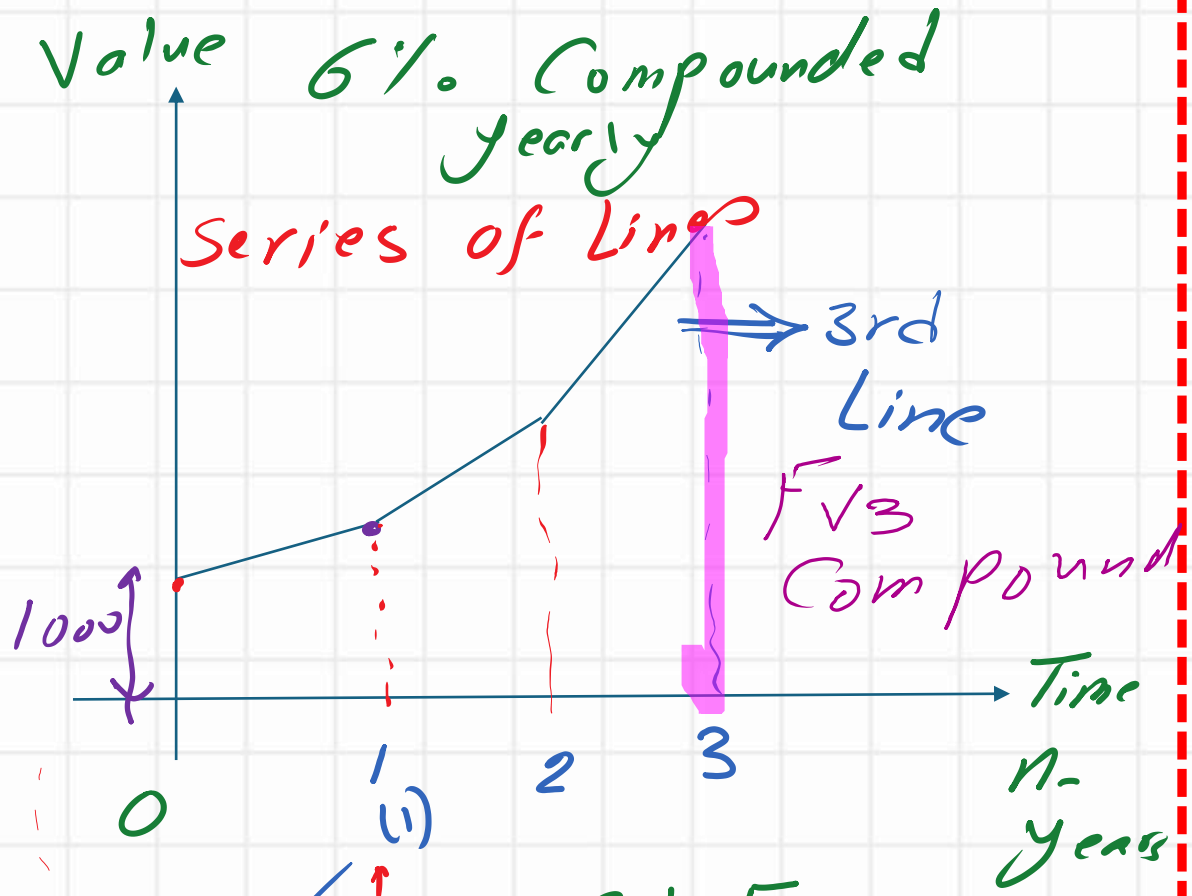
Use $\frac{FV2}{FV1} = \frac{FV1}{P0}$ ⟹



get the FV_2 as =

$\left(\frac{FV1}{P0}\right)(FV_1) = \frac{1060}{1000}^2 = 1123.6$ #





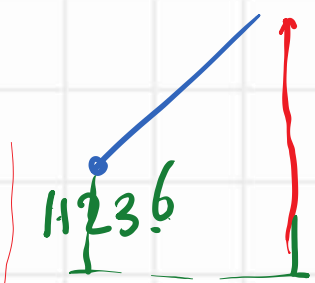
in a Form of Line equation

$F = iP_0n + P_0 \Rightarrow$

$FV(n=3) \Rightarrow (0.06)(1000)(3) + 1000$

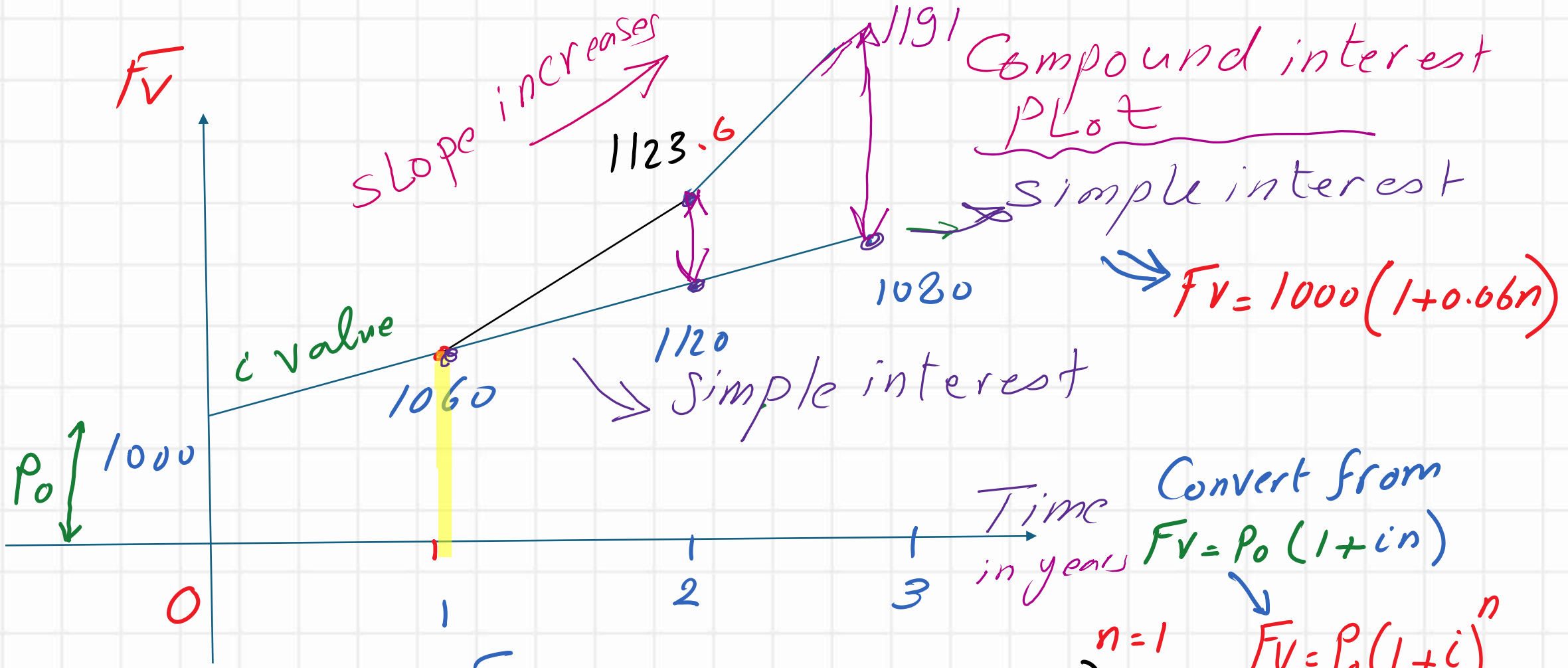
$= 180 + 1000 = 1180$ #

$\frac{FV_3}{FV_2} = \frac{FV_2}{FV_1} = \frac{FV_1}{P_0}$



get FV_3 as $(FV_2)^2 FV_2$ #

$FV_3 = \left(\frac{FV_2}{FV_1}\right) FV_2 = \frac{(1123.6)}{(1060)} = 1191.0$



Compound interest plot

simple interest

$FV = 1000(1+0.06n)$

Simple interest

Convert from $FV = P_0(1+in)$ to $FV = P_0(1+i)^n$

Compound interest : $FV(n=1) = 1000(1+0.06)^{n=1}$

$FV(n=2) = 1000(1+0.06)^{n=2}$

$FV(n=3) = 1000(1+0.06)^{n=3}$

For $1.00 $i = 100\%$ - Compounded annually

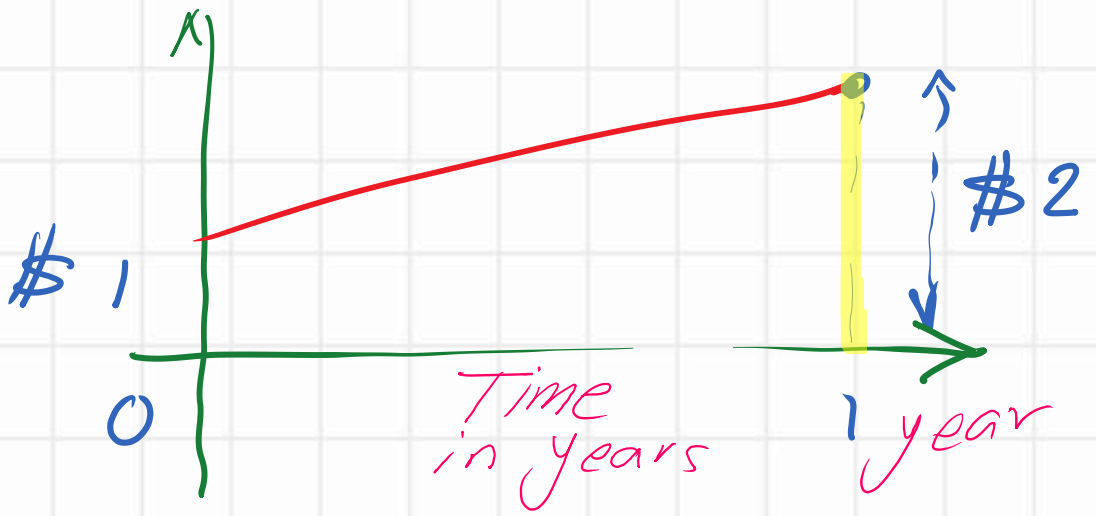


Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

Frequency	Approximate balance
1 (annually)	$2.00
2 (semi-annually)	$2.25
4 (quarterly)	$2.441406
12 (monthly)	$2.613035
365 (daily)	$2.714567
8760 (hourly)	$2.718127
525,600 (each minute)	$2.718279
31,536,000 (each second)	$2.718282

$$F = P(1+i)^n = 1\left(1+\frac{(100)}{100}\right)^1 = 1(1+1) = \$2$$

Source for Table 4.13

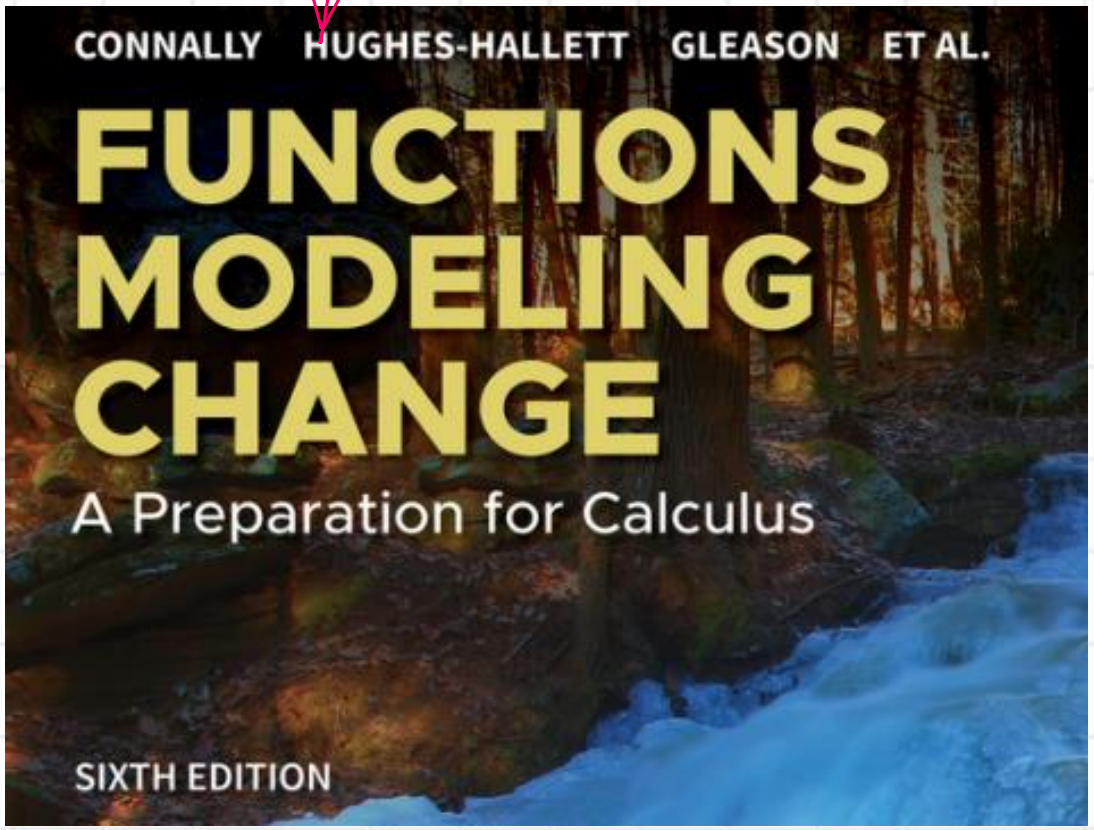


Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

Frequency	Approximate balance
1 (annually)	$2.00
2 (semi-annually)	$2.25
4 (quarterly)	$2.441406
12 (monthly)	$2.613035
365 (daily)	$2.714567
8760 (hourly)	$2.718127
525,600 (each minute)	$2.718279
31,536,000 (each second)	$2.718282