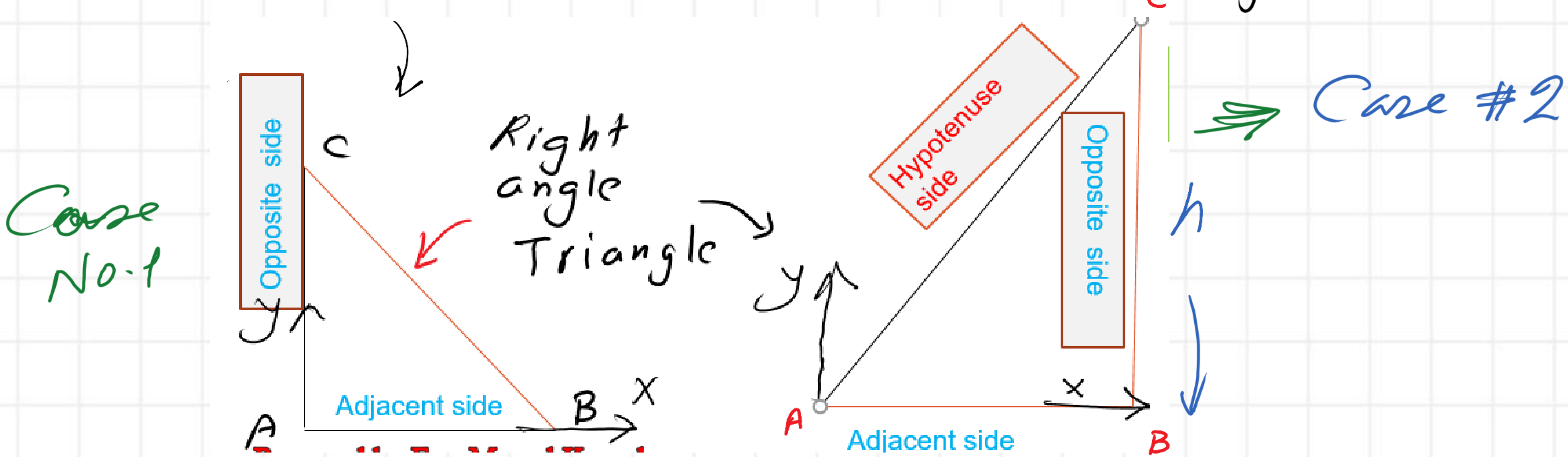
Objective of the lecture

1- How to determine the x bar and y bar for the right- angle triangle by using:

a- Horizontal strip parallel to the external x axis → at the adjacent base

b- Vertical strip perpendicular to the external x axis → at the adjacent base



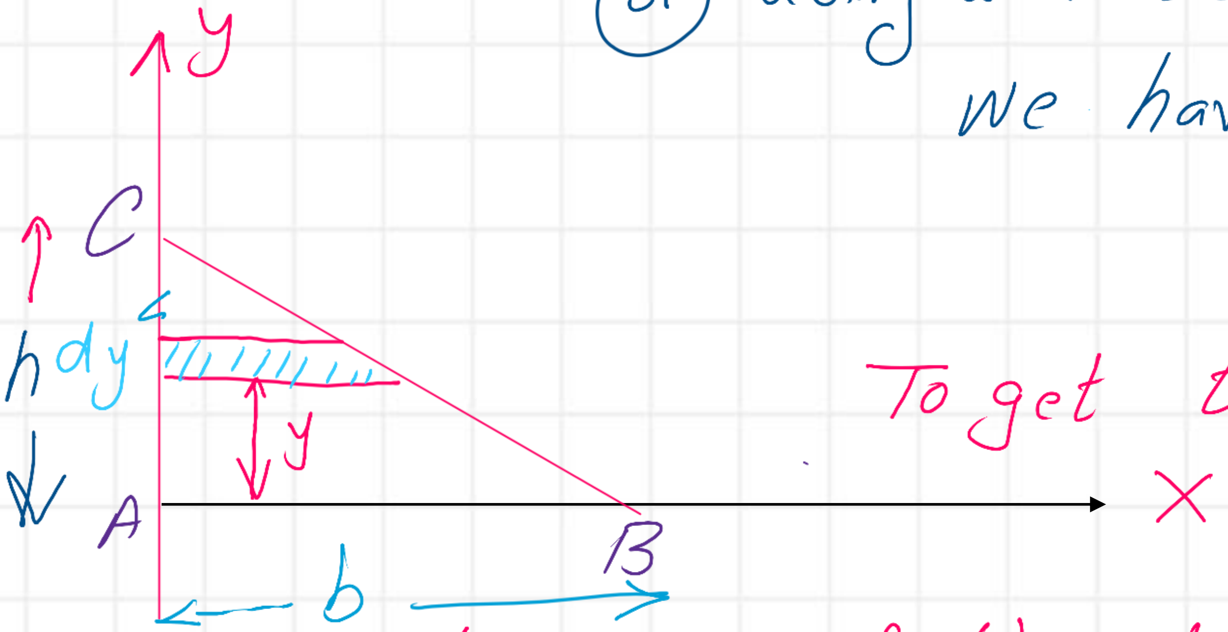
Prepared by Eng.Maged Kamel.

$\bar{y}$ bar for a right angle triangle   Case # 1

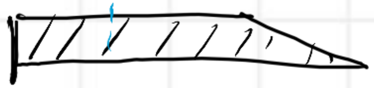
ⓐ using a horizonal strip

we have base = b of the triangle

heigh = h



To get the total area

use a strip 

What is the area of the strip ?

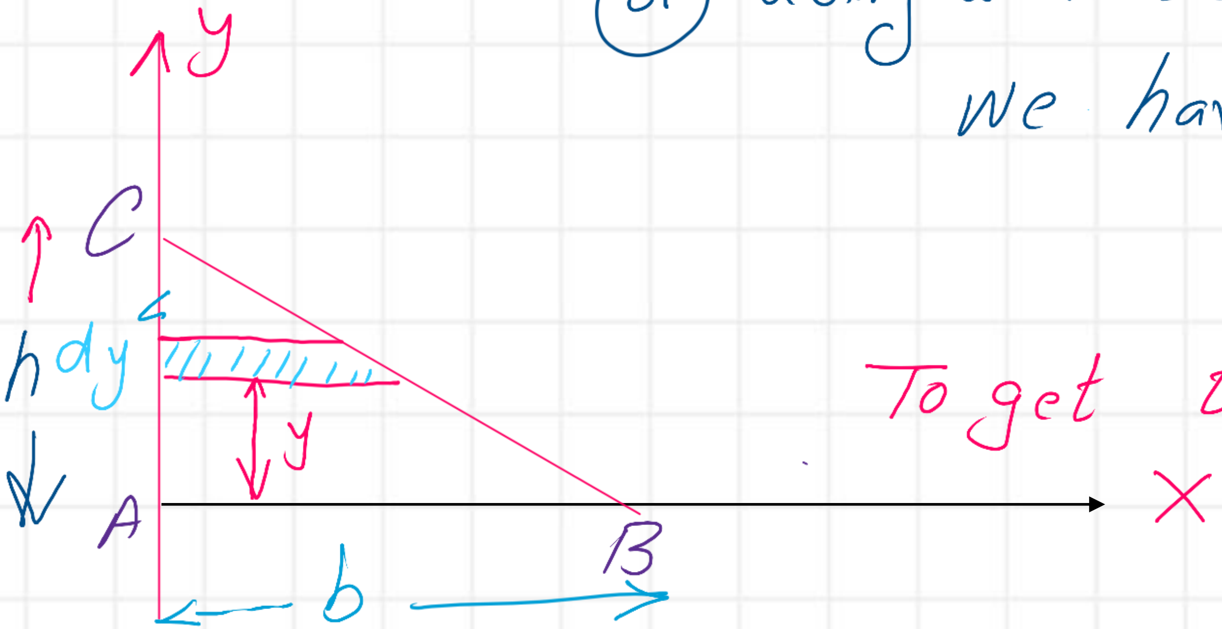
iF we Consider that strip width = dy

the breadth is = x.   dA → area of strip

$\bar{y}$ bar for a right angle triangle Case # 1

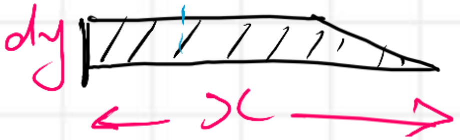
(a) using a horizonal strip

we have base $= b$ of the triangle
heigh $= h$



To get the total area

Use a strip



$$dA = x \cdot dy$$

→ From the slope equation of Line BC

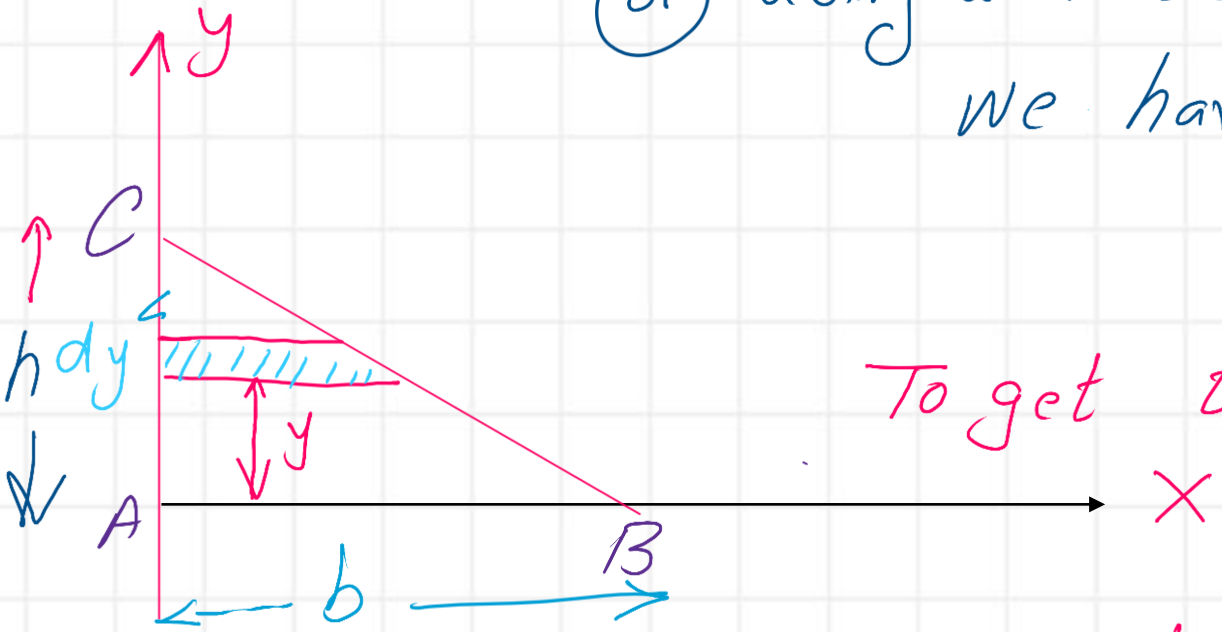
$y = mx + C$ where $m = -\frac{h}{b}$, $C = h$

$\bar{y}$ bar for a right angle triangle Case # 1

(a) using a horizonal strip

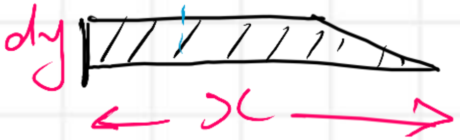
we have base = b of the triangle

heigh = h



To get the total area

Use a strip



$dA = x \cdot y$

$y = -\frac{h}{b}x + h$

check points C, B

L.H.S $= h$

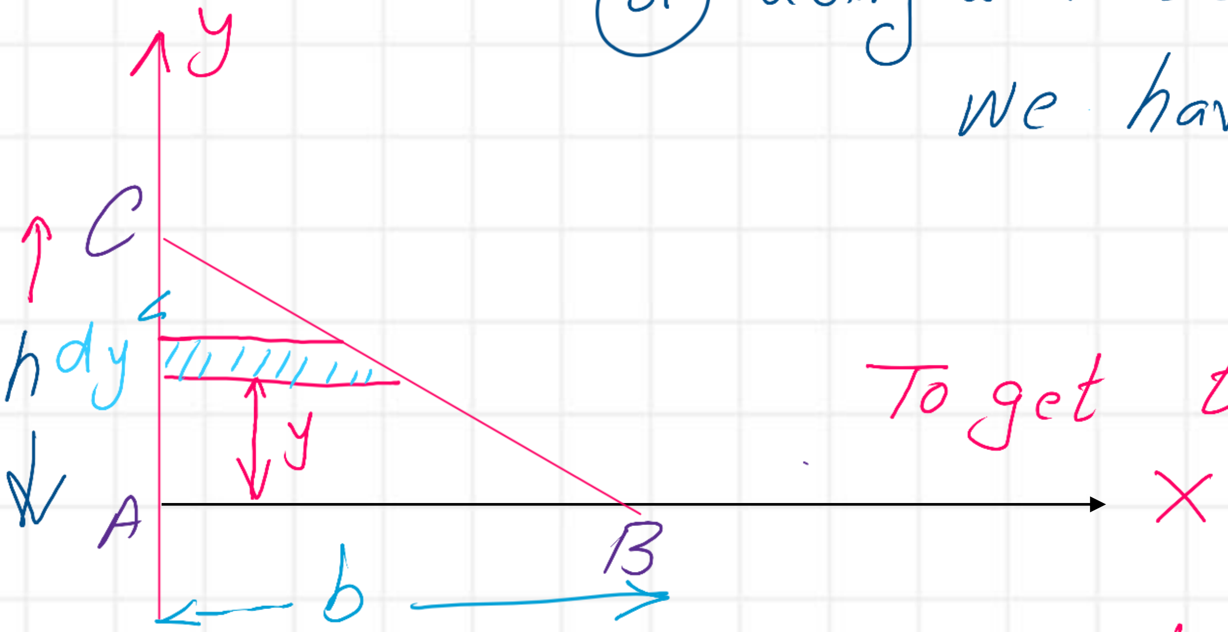
at $X = 0$, $y = h$ → R.H.S $= -\frac{h}{b}(0) + h = +h$

which is Correct

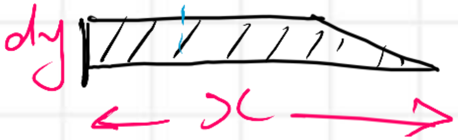
$\bar{y}$ bar for a right angle triangle — Case # 1

(a) using a horizonal strip

we have base $= b$ of the triangle
heigh $= h$



To get the total area

Use a strip 

$dA = l \cdot y$

$y = -\frac{h}{b}x + h$

check point B

at $x = b$

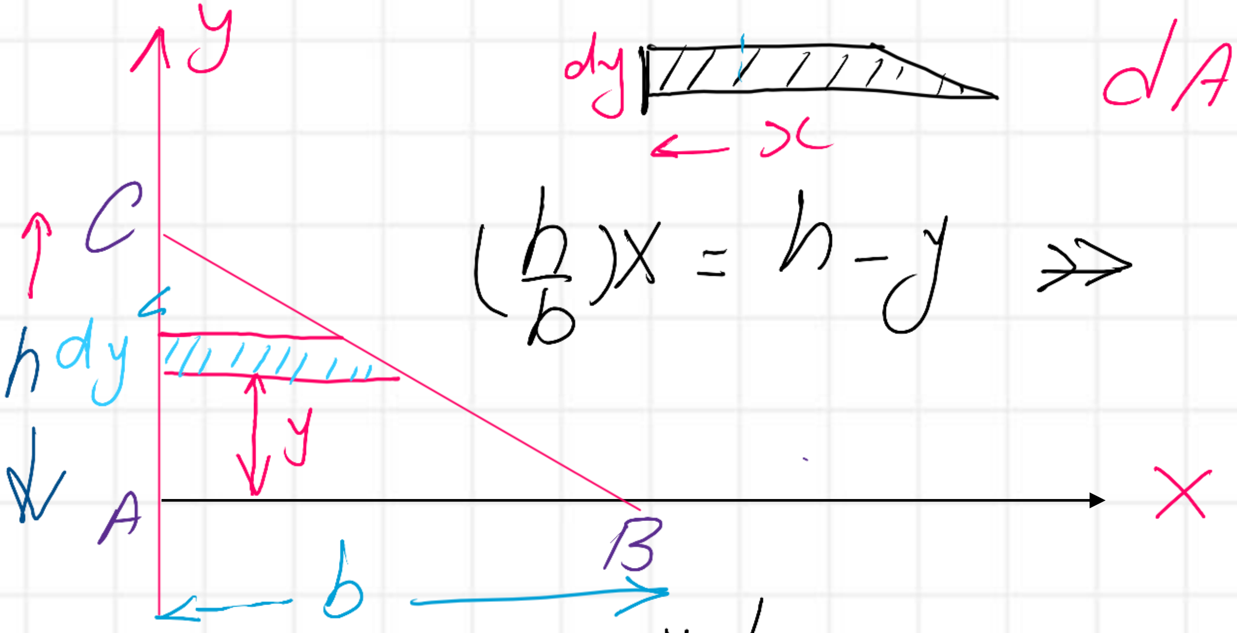
$y = 0$

L.H.S $= 0$

$\rightarrow$ R.H.S $= -\frac{h}{b}(b) + h = 0$

which is Correct

$\bar{y}$ bar for a right angle triangle   Case # 1



Calculate the triangle area by using

$$dA = x(dy)$$

$$\left(\frac{h}{b}\right)x = h - y \Rightarrow x = \left(\frac{h-y}{h}\right)(b)$$

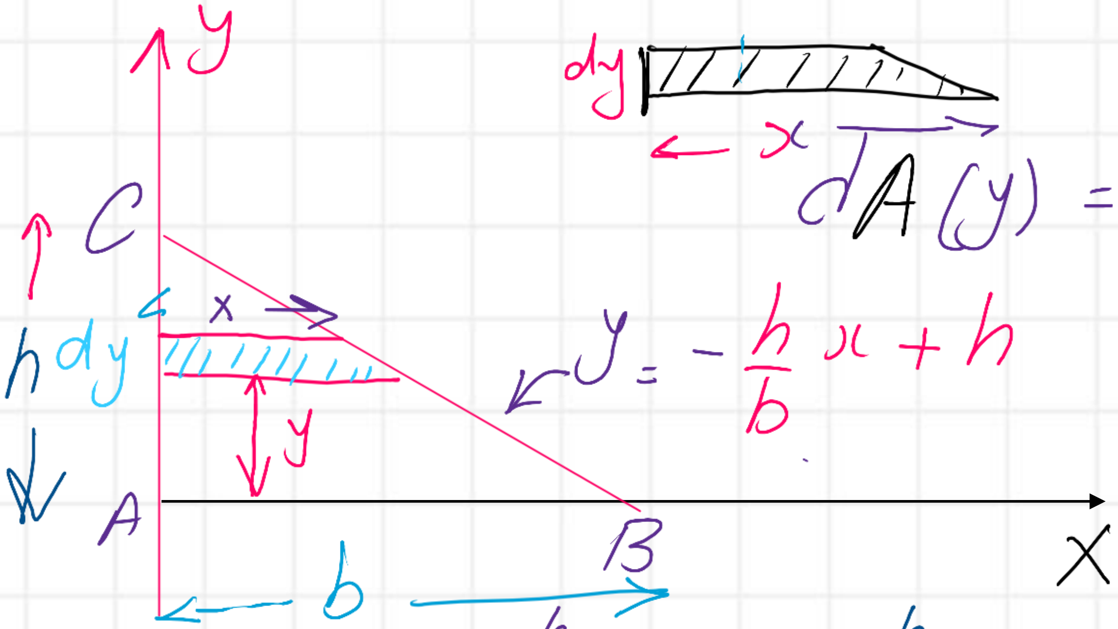
$$dA = \left(\frac{h-y}{h}\right) b\,(dy)$$

$$A = \int dA = \int_{y=0}^{y=h} \left(1 - \frac{y}{h}\right) b\,dy = b\left[\,y\Big|_0^h - \frac{y^2}{2h}\Big|_0^h\,\right]$$

$$A = b(h-0) - \frac{b}{2h}(h^2-0) = bh - \frac{bh}{2} = \frac{1}{2}bh$$

$\bar{y}$ bar for a right angle triangle   Case # 1

Calculate the First moment of Area



$$dA(y) = (x_y)\,dy$$

$$y = -\frac{h}{b}x + h$$

$$x = \left(\frac{h-y}{h}\right)(b)$$

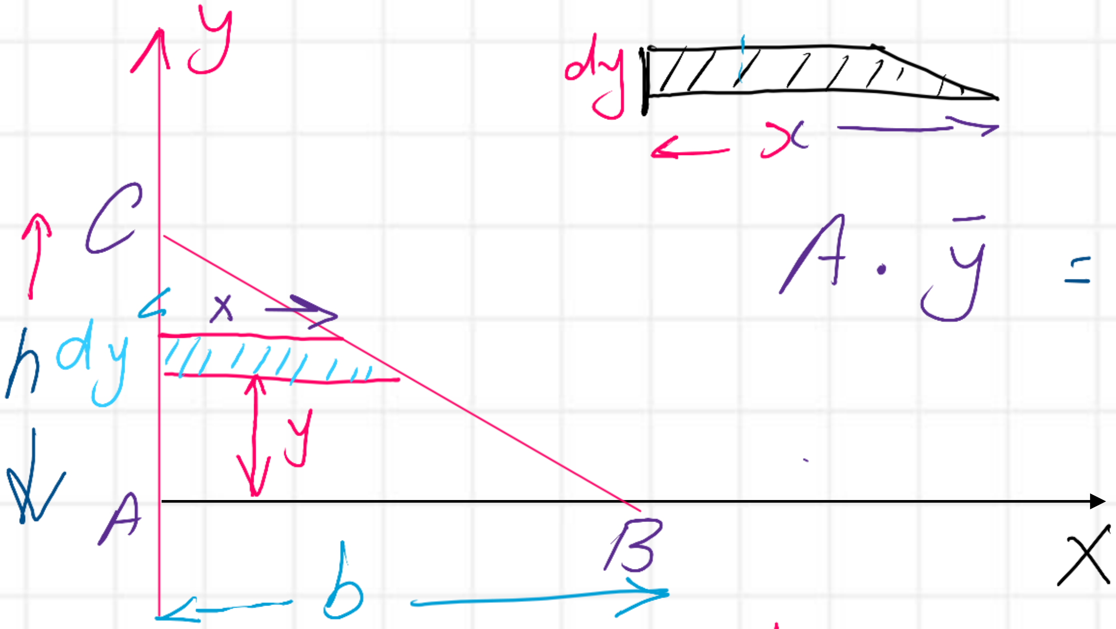
$$dA \cdot y = (y\,dy)\left(\frac{h-y}{h}\right)b$$

$$A\bar{y} = \int_0^h dA \cdot y = \int_0^h \left(\frac{b}{h}\right)\left(h y\,dy - y^2 dy\right)$$

$$A\bar{y} = \int_0^h \left(b y\,dy - \frac{b}{h} y^2 dy\right) = b\frac{y^2}{2}\Big|_0^h - \left(\frac{b}{h}\right)\left(\frac{y^3}{3}\right)\Big|_0^h = \frac{bh^2}{2} - \left(\frac{b}{h}\right)\left(\frac{h^3}{3}\right)$$

$\bar{y}$ bar for a right angle triangle   Case # 1

Calculate the First moment of Area



$$A\cdot\bar{y} = \frac{bh^2}{2} - \left(\frac{b}{h}\right)\left(\frac{h^3}{3}\right) = \frac{b\,h^2}{2} - \frac{bh^2}{3}$$

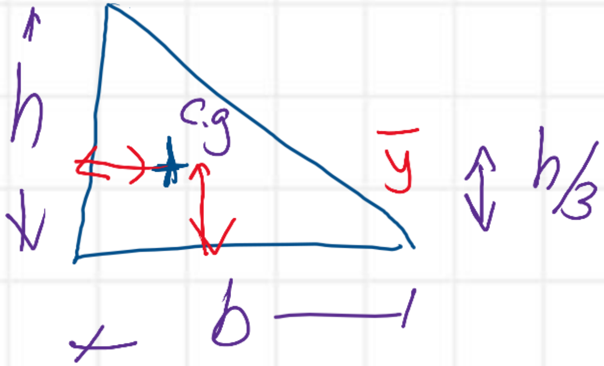
$$A\bar{y} = \frac{1}{6}bh^2$$

$$\bar{y} = \frac{A\cdot\bar{y}}{A} = \frac{1}{6}bh^2\left(\frac{1}{\frac{1}{2}bh}\right)$$

$$\bar{y} = \frac{1}{3}h$$

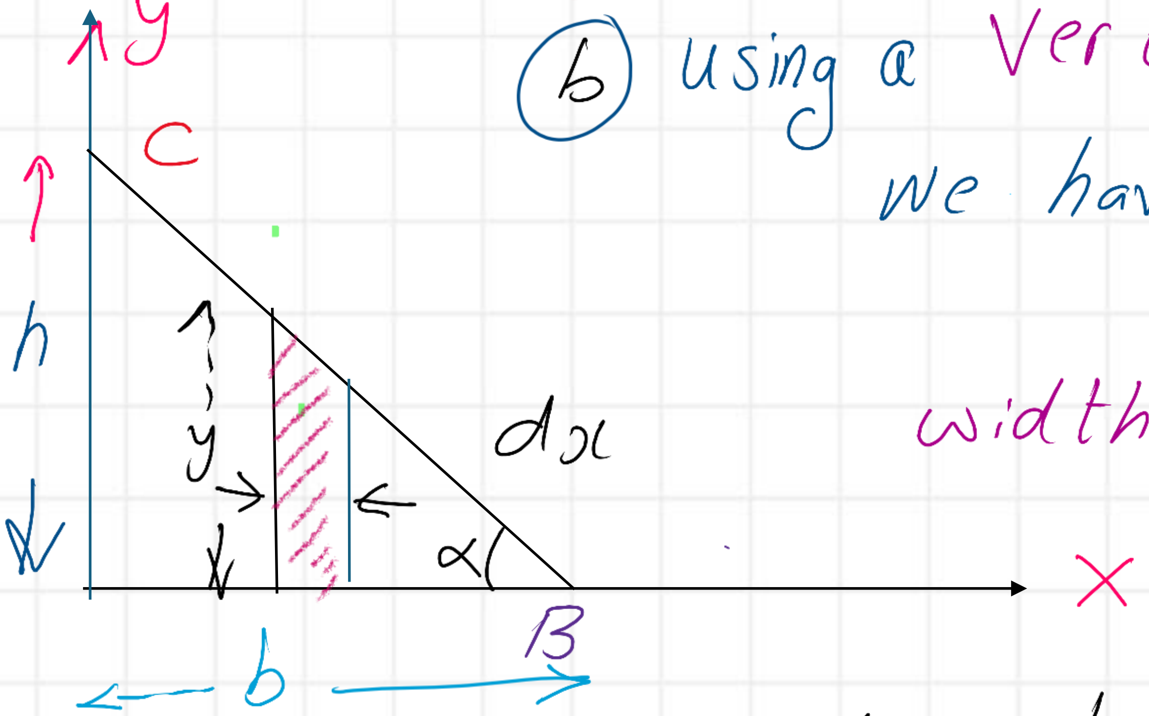
$\bar{x}$

$= b/3$



$\bar{y}$ bar for a right angle triangle Case # 1

(b) using a vertical strip



We have base = b of the triangle
heigh = h

width of strip = dx
heigh = y

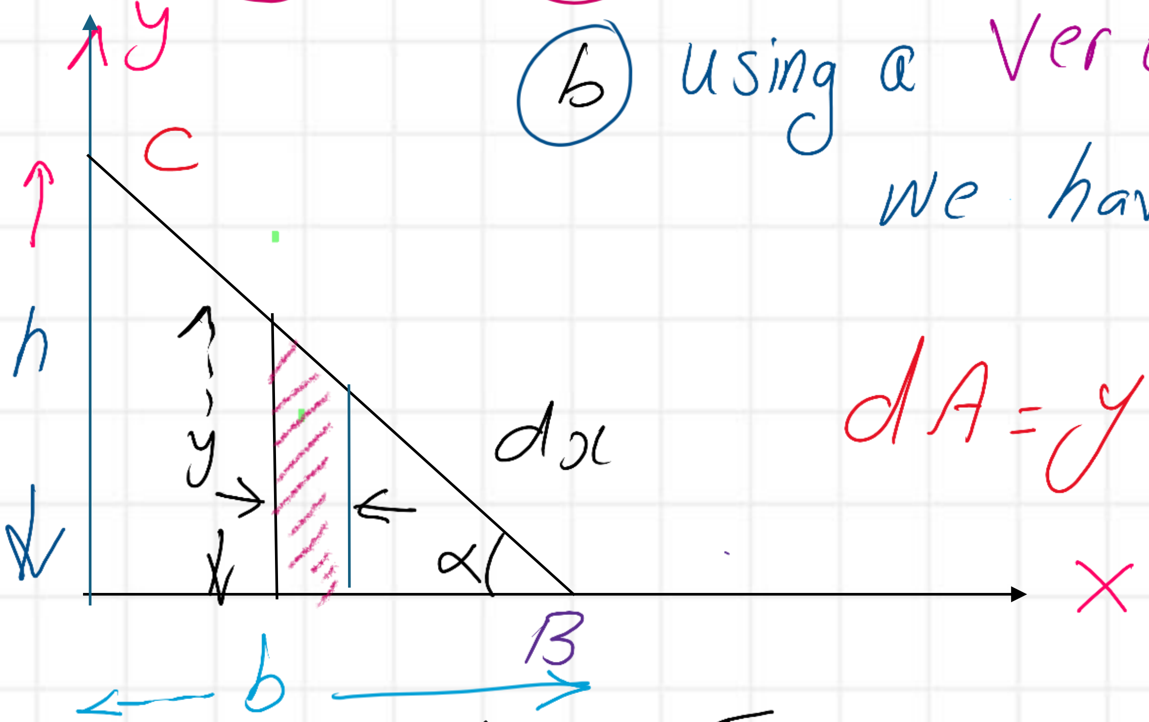
We have a relation between x & y as

$$y = -\frac{h}{b}(x) + h$$ For Line BC

$$dA = y \cdot dx$$

$\bar{y}$ bar for a right angle triangle   Case # 1

(b) using a Vertical strip



we have base = b of the triangle
heigh = h

$$dA = y \cdot dx = \left(\left(-\frac{h}{b}x\right) + h\right) dx$$

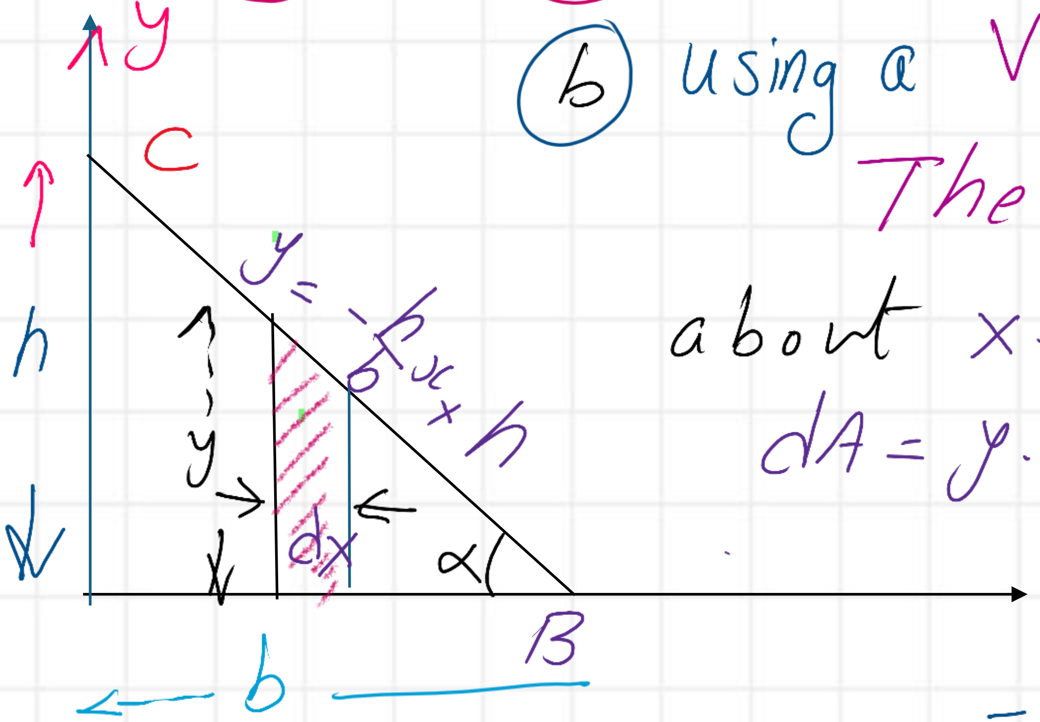
$A = \int dA$ From $x = 0 \rightarrow x = b$

$$A = \int_0^b -\frac{hx}{b}\,dx + h\,dx = -\frac{h}{b}\,\frac{x^2}{2}\Big|_0^b + h.x\Big|_0^b$$

$$= (h.b) - \frac{h}{2b}(b^2) = h.\frac{b}{2}$$

$\bar{y}$ bar for a right angle triangle Case # 1

(b) using a Vertical strip



The First moment of area about x-axis $\rightarrow dA \cdot y_{strip}$

$dA = y \cdot dx$, $y = -\frac{h}{b}x + h$

$\times \quad y_{st} = \frac{y}{2} = \left(-\frac{hx}{2b} + \frac{h}{2}\right)$

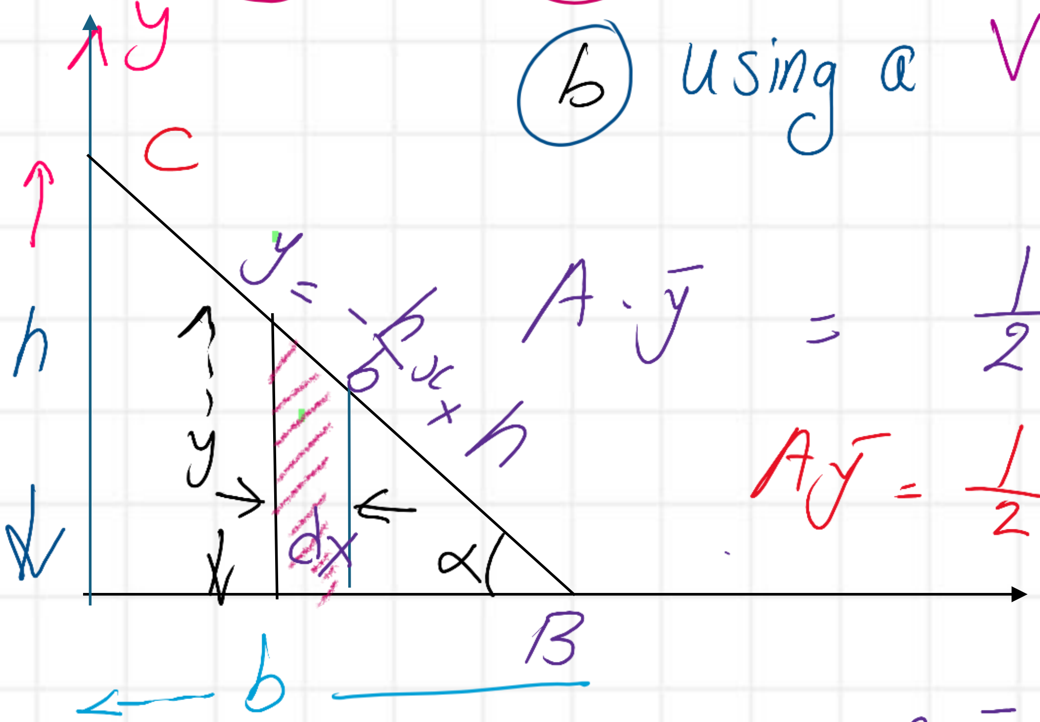
$dA \cdot y_{st} = y \cdot dx \; y_{str} = \frac{\bar{y}^2}{2} dx = \left(-\frac{h}{b}x + h\right)^2 \frac{dx}{2}$

$\int dA \cdot y_{st} = \frac{1}{2}\int_0^b \left(h^2 dx - \left(2\frac{h^2 x}{b} \cdot dx\right) + \frac{h^2 x^2 dx}{b^2}\right)$

$A \cdot \bar{y} = \frac{1}{2}\left[h^2 x\Big|_0^b - \frac{2}{b}h^2 \frac{x^2}{2}\Big|_0^b + \frac{h^2}{b^2}\frac{x^3}{3}\Big|_0^b\right.$

$\bar{y}$ bar for a right angle triangle Case # 1

(b) using a Vertical strip



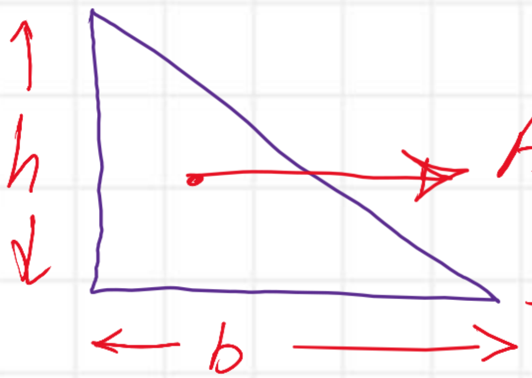
$$A \cdot \bar{y} = \frac{1}{2}\left[h^2 x\Big|_0^b - \frac{2}{b}h^2 \frac{x^2}{2}\Big|_0^b + \frac{h^2}{b^2}\frac{x^3}{3}\Big|_0^b\right]$$

$$A\bar{y} = \frac{1}{2}\left[h^2 b - bh^2 + \frac{bh^2}{3}\right]$$

$$A\bar{y} = \frac{1}{2}\left(\frac{bh^2}{3}\right) = \frac{bh^2}{6}$$

$A\bar{y}$

This is moment of area about X-axis



$A = \frac{1}{2}bh$

$\bar{y}$

x-axis

We have $A = \frac{1}{2}(b)(h)$

While $A\bar{y} = \frac{1}{6} bh^2$

Then

$$\bar{y} = \frac{A\bar{y}}{A} = \frac{bh^2}{6} \frac{1}{\left(\frac{1}{2} bh\right)} = \frac{h}{3}$$

Final Expression For $\bar{y}$

Which is the Same result as estimated by using a horizontal strip.