Source for Table 4.13

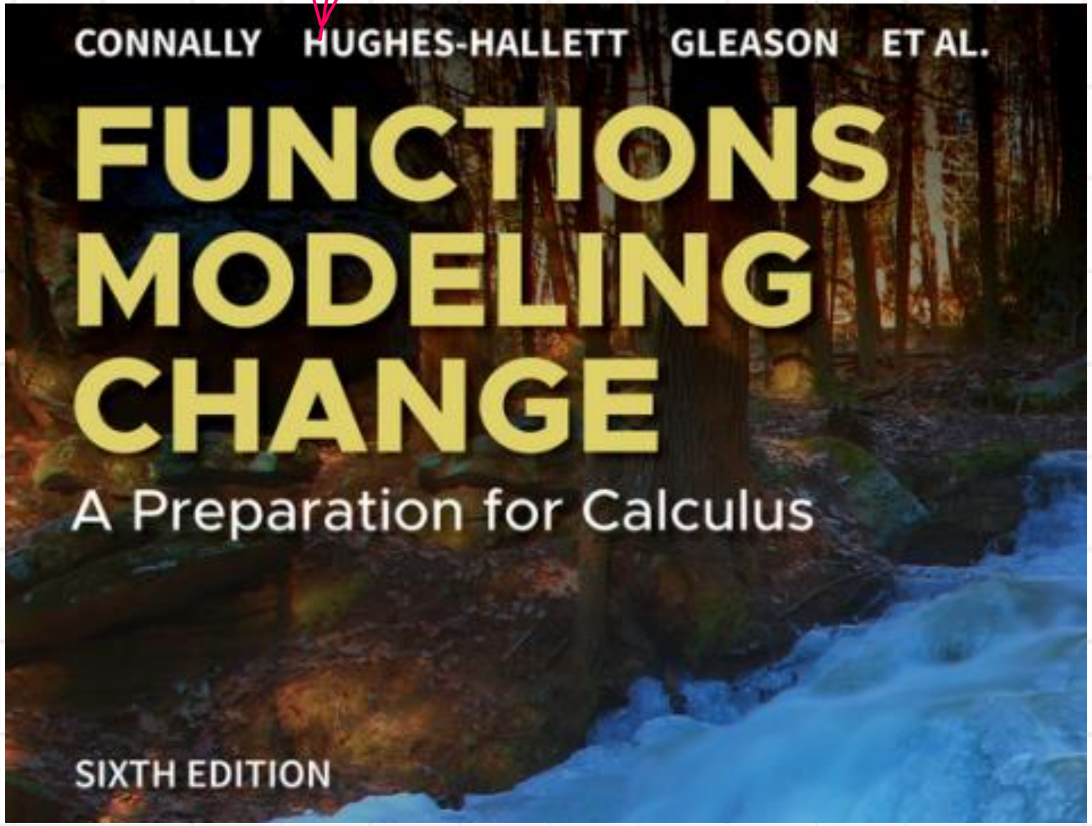

Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

Frequency	Approximate balance
1 (annually)	\$2.00
2 (semi-annually)	\$2.25
4 (quarterly)	\$2.441406
12 (monthly)	\$2.613035
365 (daily)	\$2.714567
8760 (hourly)	\$2.718127
525,600 (each minute)	\$2.718279
31,536,000 (each second)	\$2.718282

Content of Post 6a

ⓐ Future Value for \$1 Semiannually.

FV → $i = 100\%$

ⓑ Future Value For \$1 Quarterly

FV ⇒ $i = 100\%$

ⓒ Future Value For \$1 monthly

FV ⇒ $i = 100\%$ Frequency

ⓓ Future Value for \$1 → Frequency daily

FV ⇒ $i = 100\%$

ⓔ Future Value For \$1 Continuously

F_V of \$1 at $t = 1/2$ Semi-annually

$i = 100\%$

The value can be shown after one year as

$F_V(n=1\ \text{year}) = P_0 + i n (P_0)$

$P_0 = \$1$
$n = 1$ year
$i = 100\%$

$$F_V(n=1) = \underset{P_0}{1} + \underset{i}{\left(\frac{100}{100}\right)}\underset{n}{(1)}\underset{P_0}{(1)} = \$\,2$$

F_V at $t=1$

To change the Frequency to semi-annual

Get the F_V at $t = \frac{1}{2}$ year $= 1$-semi-annual

one semi-annual

$F_V(n = 1\ \text{semi-annual}) = (1+2)\frac{1}{2} = \$\,1.50$

Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

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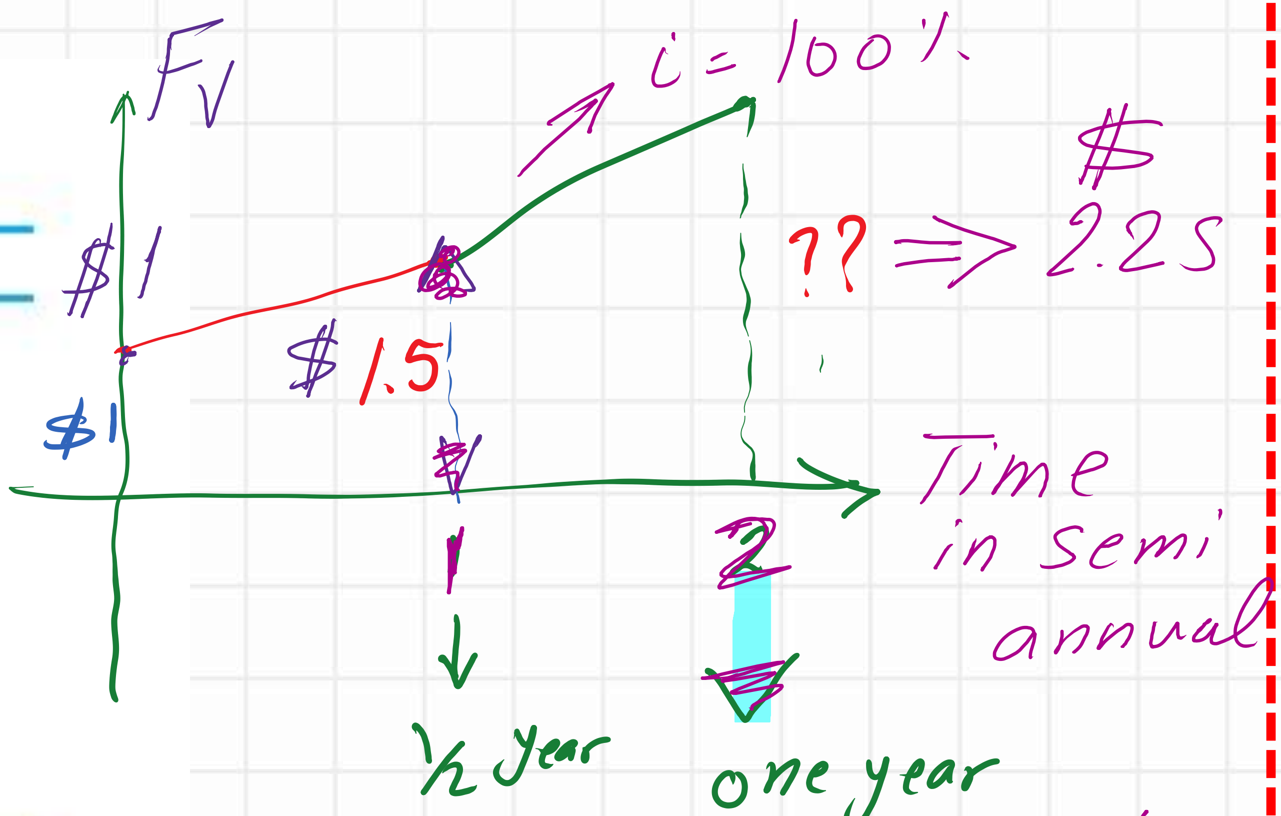


Fv (n = 2 semi-annual)

$$= \left(\frac{1.5}{1}\right)(1.5) = 2.25 \quad \#.$$

It appears that, correct to five decimal places, $e \approx 2.71828$; in fact, the approximate value to 20 decimal places is

$$e \approx 2.71828182845904523536$$

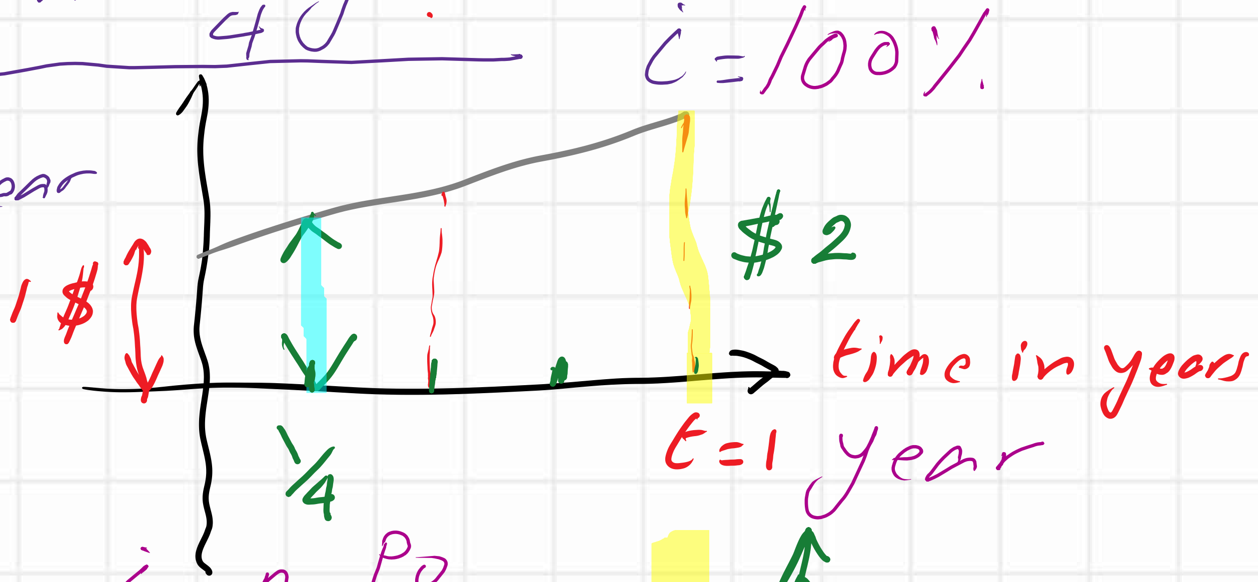
which is the same value at the table

$$Fv = \left(1 + \frac{i}{n}\right)^{nt} = \left(1 + \frac{(100/100)}{2}\right)^{2(1)} \Rightarrow 1.5^2$$

FV of 1\$ at quarter $n = \frac{1}{4}$ year

Estimate FV at $t = 1$ year

$F_V(n = 1 \text{ year}) = P_0 + in(P_0)$



$P_0 = \$1$

$n = 1$ year

$i = 100\%$

$$FV(n=1) = \overset{}{1} + \left(\overset{i}{\frac{100}{100}}\right)\left(\overset{n}{1}\right)\left(\overset{P_0}{1}\right) = \$2$$

To change the Frequency to Quarterly

Get the value at $t = \frac{1}{4}$ year = 1 - Quarter

$$FV(n = 1 \text{ quarter}) = P_0 + inP_0 = 1 + \left(\overset{i}{\frac{100}{100}}\right)\left(\overset{n}{\frac{1}{4}}\right)\left(\overset{P_0}{1}\right) = \$1.25$$

Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

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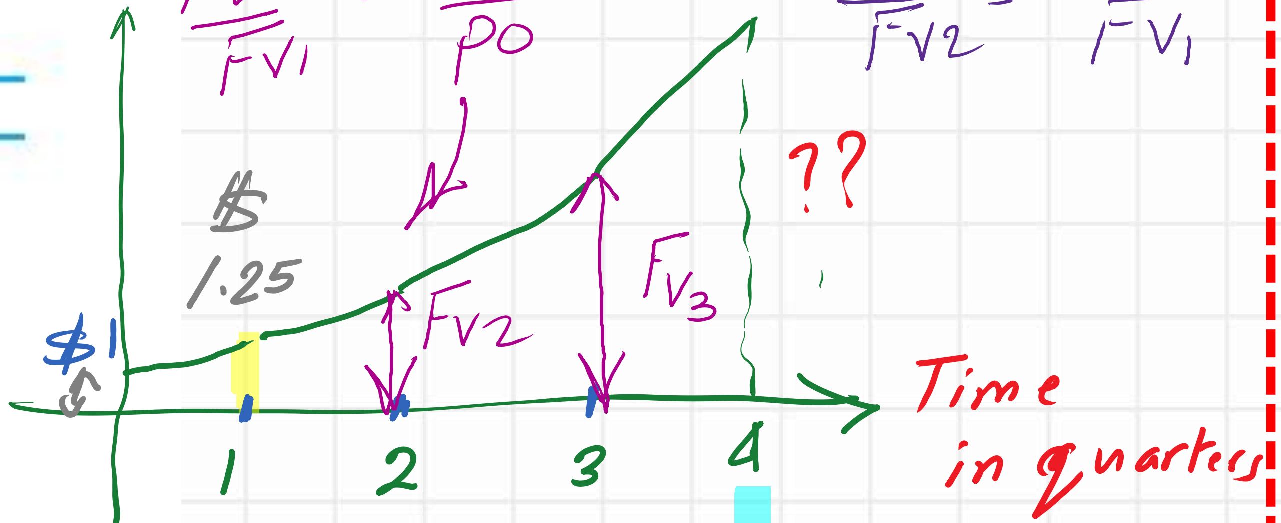
31,536,000 (each second)	\$2.718282

$$\frac{FV_2}{FV_1} = \frac{FV_1}{P_0} \qquad \frac{FV_3}{FV_2} = \frac{FV_2}{FV_1}$$



one year

$FV(n = 1\text{ quarter}) = \frac{1.25}{1} = \$1.25$

$FV(n = 2\text{ Quarters}) = \left(\frac{1.25}{1}\right)(1.25) = 1.5625$ & $FV(n=3q) = \frac{FV_2^{\,2}}{FV_1} = 1.953$

$FV(n = 4\,qu) = \left(\frac{1.25}{1}\right)^4 = 2.4414$

It appears that, correct to five decimal places, $e = 2.71828$; in fact, the approximate value to 20 decimal places is

$$e = 2.71828182845904523536$$

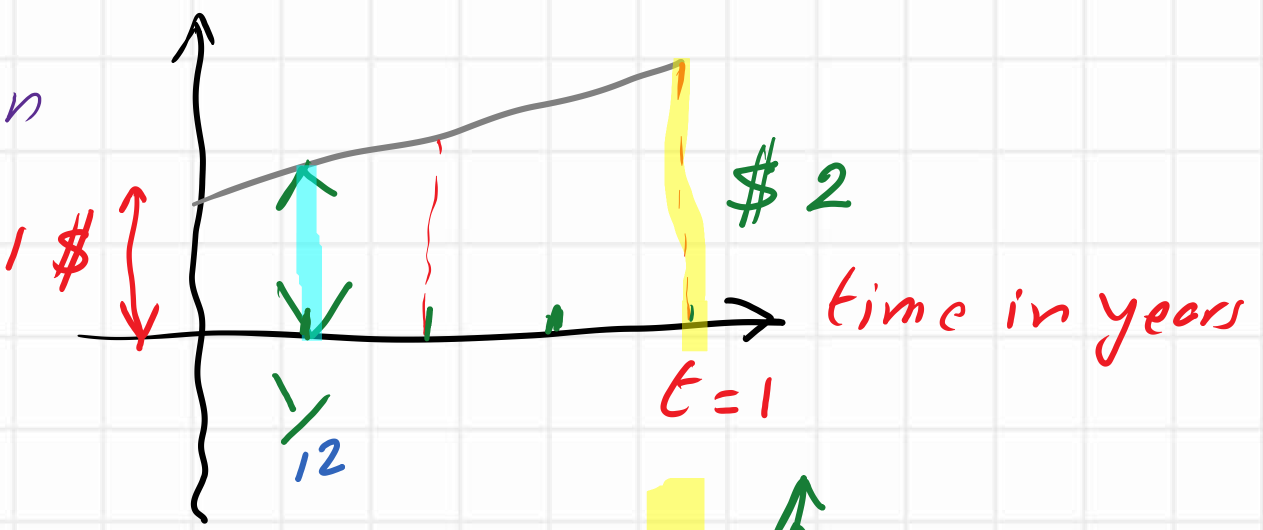
which is the same value as the table

$$FV = \left(1 + \frac{i}{n}\right)^{nt} = \left(1 + \frac{(100/100)}{4}\right)^{4(1)} \Rightarrow \$2.4414$$

Fv Value at one month.

The value can be shown as

$$F_v(n=1\ \text{year}) = P_0 + in(P_0)$$



$P_0 = \$1$

$n = 1$ year

$i = 100\%$

$$Fv(n=1) = 1 + \left(\frac{100}{100}\right)(1)(1) = \$2$$

To change the Frequency to monthly

get the value at $t = \frac{1}{12}$ year = one month

$$Fv(n=1\ \text{month}) = P_0 + inP_0 = 1 + \left(\frac{100}{100}\right)\left(\frac{1}{12}\right)(1) = \$1.08333$$

Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

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1 (annually)	\$2.00
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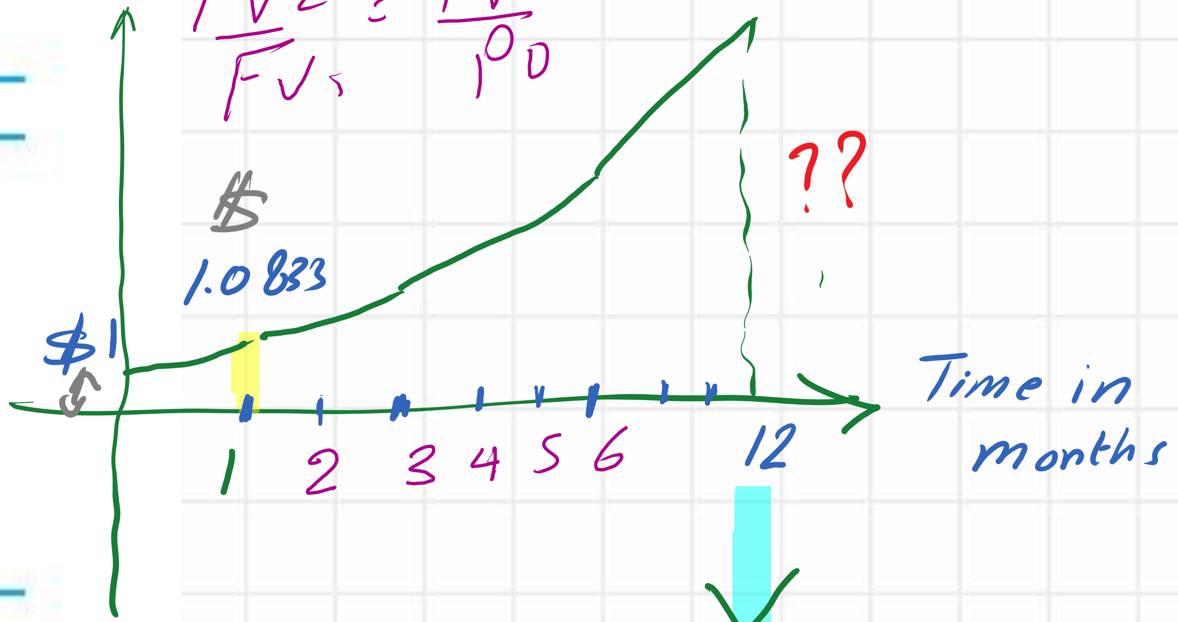
12 (monthly)	\$2.613035
365 (daily)	\$2.714567
8760 (hourly)	\$2.718127
525,600 (each minute)	\$2.718279
31,536,000 (each second)	\$2.718282

$$\frac{FV_2}{FV_1} = \frac{FV_1}{100}$$



one year

$FV(n = 1 \text{ month}) = (1.0833/1)$

$FV(n = 2 \text{ month}) = (1.0833)^2 = 1.1735$

$FV(n = 3 \text{ month}) = (1.0833)^3 = 1.2714$

$FV(n = 12 \text{ month}) = (1.0833)^{12}$ \$ $= 2.613035$

It appears that, correct to five decimal places, $e \approx 2.71828$; in fact, the approximate value to 20 decimal places is

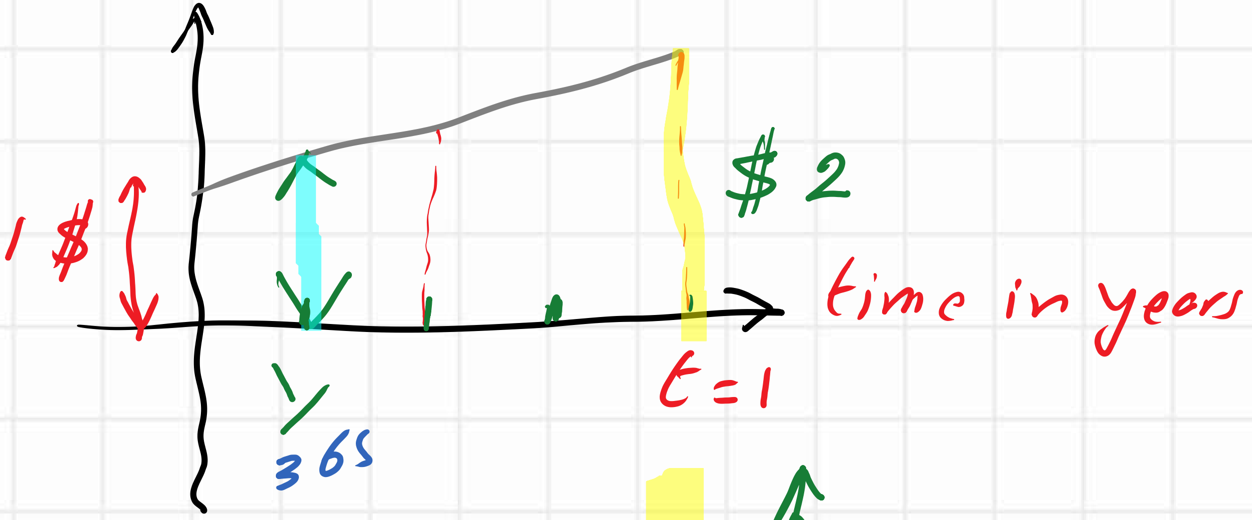
$$e \approx 2.71828182845904523536$$

which is the same value as the table

$$FV = \left(1 + \frac{i}{n}\right)^{nt} = \left(1 + \frac{(100/100)}{12}\right)^{(12)(1)} = \$\,2.61305$$

F_V Value at one day

The value can be shown as



$F_V(n=1\ year) = P_0 + in(P_0)$

$P_0 = \$1$
$n = 1\ year$
$i = 100\%$

$Fv(n=1) = 1 + \left(\frac{100}{100}\right)(1)(1) = \$\ 2$

To change the frequency to get the value at $t = \frac{1}{365}$ year $=$ 1 - daily

$Fv(n=1\text{-daily}) = P_0 + in P_0 = 1 + \left(\frac{100}{100}\right)\left(\frac{1}{365}\right)(1) = \$\ 1.0027397$

Table 4.13 *Balance after 1 year, 100% nominal interest, various compounding frequencies*

Frequency	Approximate balance
1 (annually)	\$2.00
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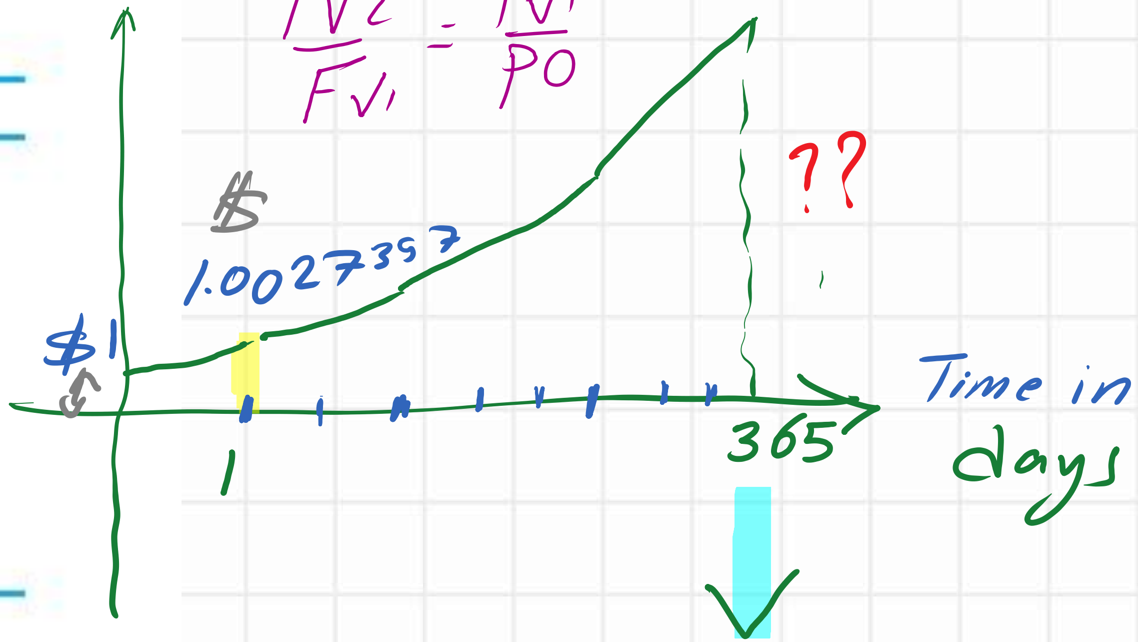
31,536,000 (each second)	\$2.718282

Compounded daily

$$\frac{Fv_2}{Fv_1} = \frac{Fv_1}{P_0}$$



$$Fv(n = 1\ \text{day}) = 1.002739$$

$$Fv(n = 2\ \text{days}) = (1.002739)^2 \Rightarrow Fv(n = 365\ \text{days}) = (1.002739)^{365}$$

$$= 2.714567$$

It appears that, correct to five decimal places, $e \approx 2.71828$; in fact, the approximate value to 20 decimal places is

$$e \approx 2.71828182845904523536$$

which is the same value as the table

$$Fv = \left(1 + \frac{i}{n}\right)^{nt} = \left(1 + \frac{(100/100)}{365}\right)^{(365)(1)} = 2.714567$$

Types of Compound Interest

Annual Compound yearly = 6%

Compounded Quarterly $\Rightarrow \frac{6}{100} \times \frac{1}{4} = 1.5\%$

Compounded monthly $\Rightarrow \frac{6}{100} \times \frac{1}{12} = 0.5\%$

Compounded daily $\Rightarrow \frac{6}{100}\left(\frac{1}{365}\right) = \frac{6}{365}\%$

Compounded Continuously $= e^{rt}$

$FV_1 = P_1(1+i)^n$ $\quad i = 6\%$ $\quad n = 1$ $\quad$ Yearly

$FV_2 = P_2(1+i_2)^4$ $\quad i_2 = 1.5\%$ $\quad n_2 = 4$ $\quad$ Quarterly

$F_2 = P_2(1+0.015)^4$

$F_3 = P_3(F/P, i, n)$

$F_3 = P_3[1 + i_3)^{n_3}$

Monthly

$i_3 = \frac{6}{12} = \frac{1}{2}\%$ & $n_3 = 12$

$F_3 = P_3(1 + \frac{1}{2}\%)^{12}$ $\Rightarrow$ Compounded monthly

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where P = principal
r = annual interest rate expressed as a decimal
n = number of interest periods per year
t = number of years P is invested
A = amount after t years.

EXAMPLE 3–7

Suppose the bank changed the interest policy in Example 3-5 to "6% interest, compounded quarterly." For this situation, how much money would be in the account at the end of 3 years, assuming a $500 deposit now?

Example-3-5 ⇒ 6% Compounded yearly

$P = \$500$

$i = 6\%$ Compounded quarterly

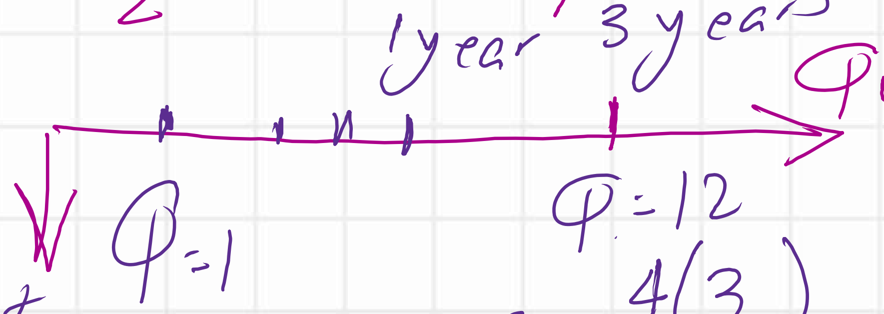
Solution

Quarters

$n = 4$

$t = 3$ year

$500



$$F = P\left(1 + \frac{i}{n}\right)^{nt} = 500\left(1 + \frac{6}{100}\left(\frac{1}{4}\right)\right)^{4(3)}$$

$$= 500[1 + 0.015]^{12}$$

$$= 597.809 \ \$$$

We Can use Excel Built in Function

Syntax FV(rate, nper, , pv, [type])

For Future Value

rate Quarterly $= \frac{6}{100}\left(\frac{1}{4}\right) = 0.015\%$

nper $= 12$ Quarters

PV $= -500$

FV(0.015, 12, , -500,)

Future value		£597.81