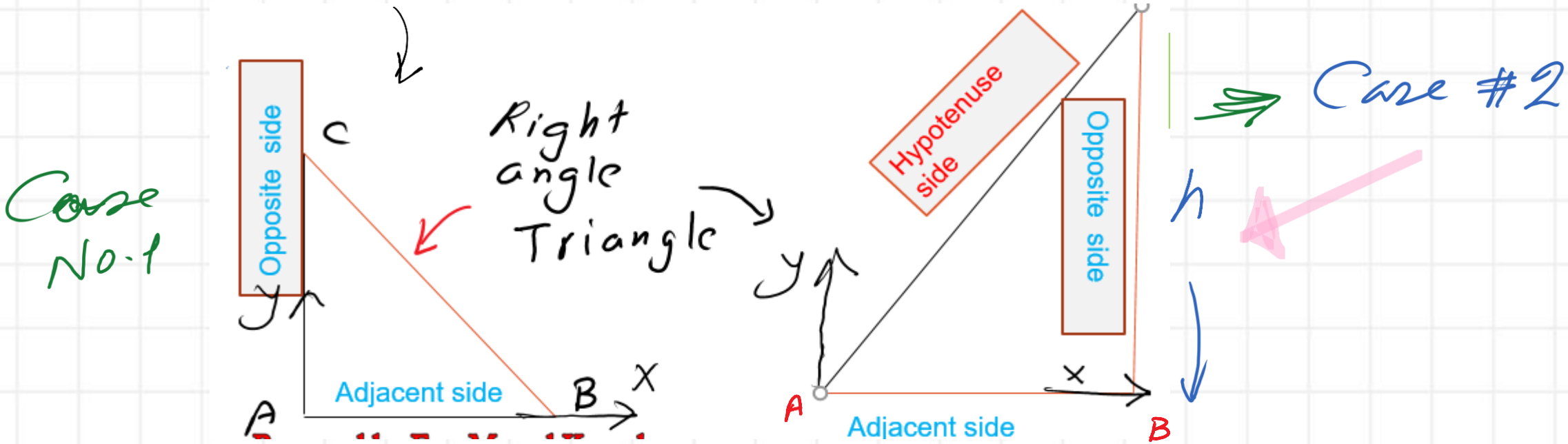
Objective of the lecture

1- How to determine the x bar and y bar for the right- angle triangle by using:

a- Horizontal strip parallel to the external x axis → at the adjacent base

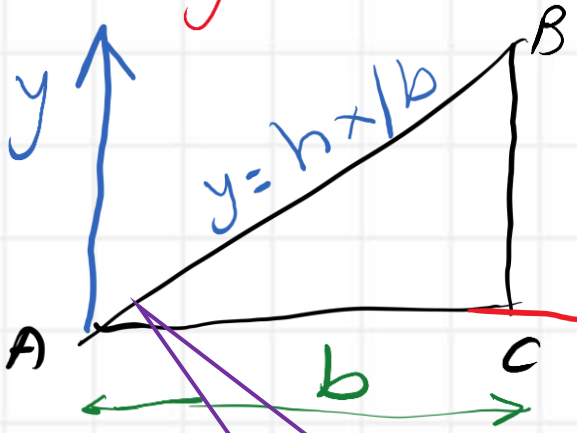
b- Vertical strip perpendicular to the external x axis → at the adjacent base



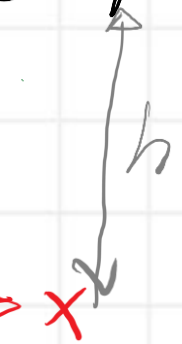
Case # 2

$\bar{y}$ bar

Using a horizontal strip



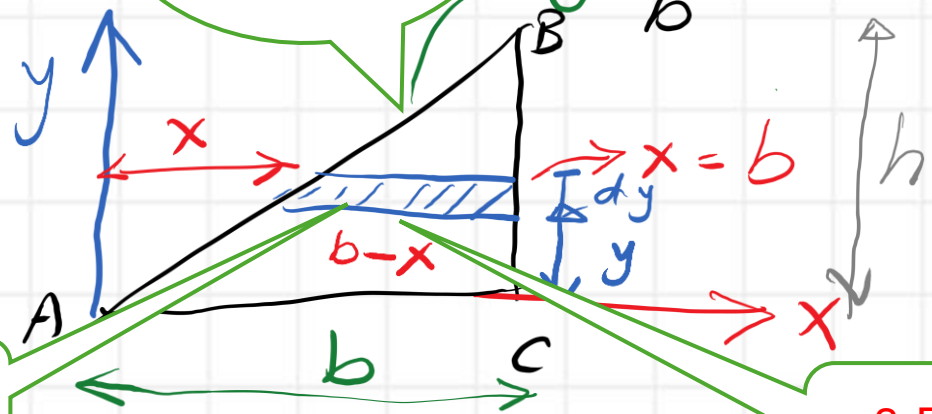
1-Draw two axes x &y.



Area estimation of triangle by integration.

2-Find y value in terms of b & h and x.

$y = \frac{h}{b} x$ → $x = \frac{by}{h}$



4-Integrate.

3-Estimate strip width.

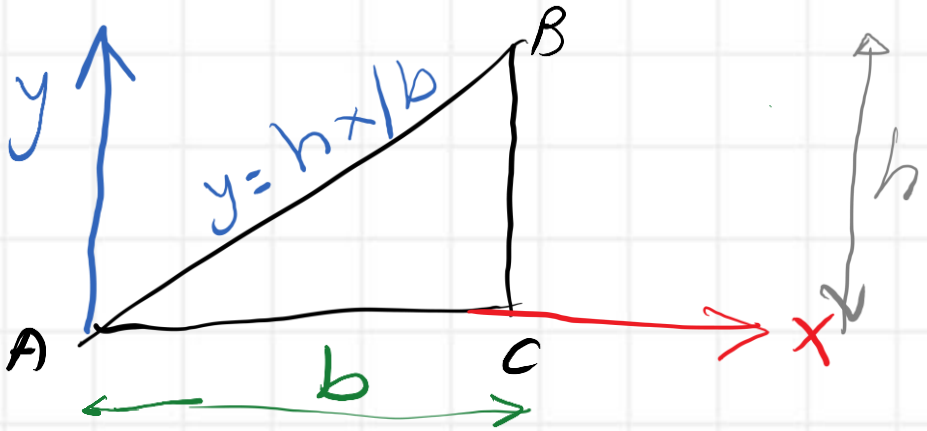
The necessary Four steps are shown in sketch

dA = area of strip $= (b - x)\,dy$, $x = \frac{b}{h}(y)$

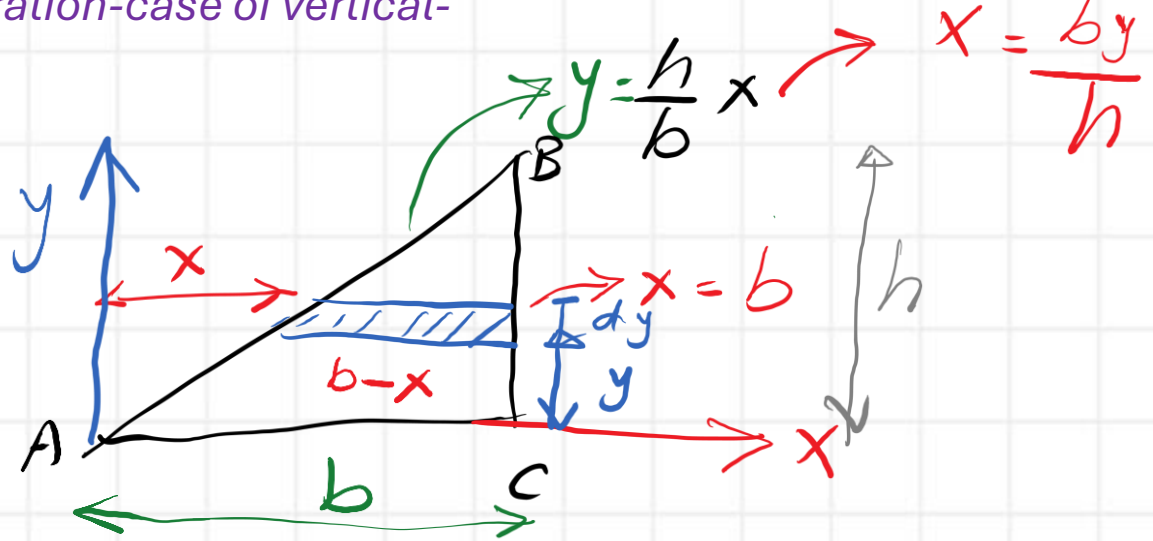
$dA = \left(b - \left(\frac{b}{h}\right)(y)\right) dy = \left(b.dy - b\frac{y}{h}\,dy\right)$.

Area & C.g Determination

Case # 2 → $\bar{y}$ bar



Area estimation of triangle by integration-case of vertical-strip.



$$dA = \left(b \cdot dy - b\frac{y}{h}\,dy\right)$$

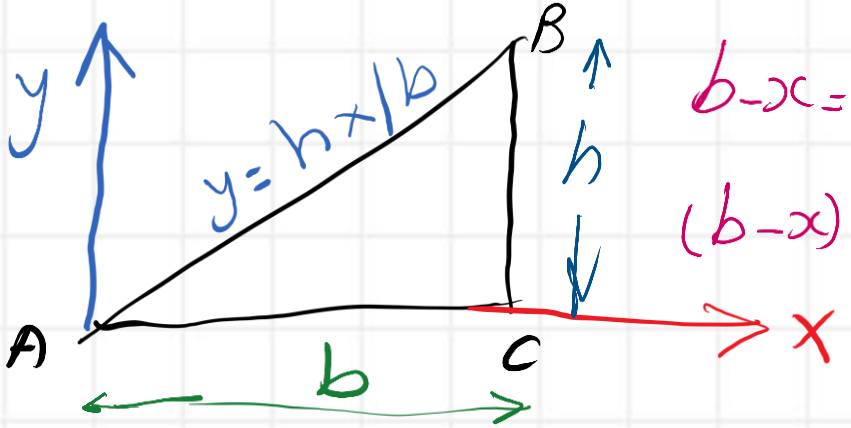
$$\int dA = A = \int b\,dy - \frac{by}{h}\,dy = b \cdot y\Big|_0^h - \frac{b}{h}\,\frac{y^2}{2}\Big|_0^h$$

$$A = \left[bh - \left(\frac{b}{2h}\right)^{0}(h^2)\right] = \frac{1}{2}bh$$

Area & C.g Determination

First moment of area.

Case # 2 → $\bar{y}$ bar

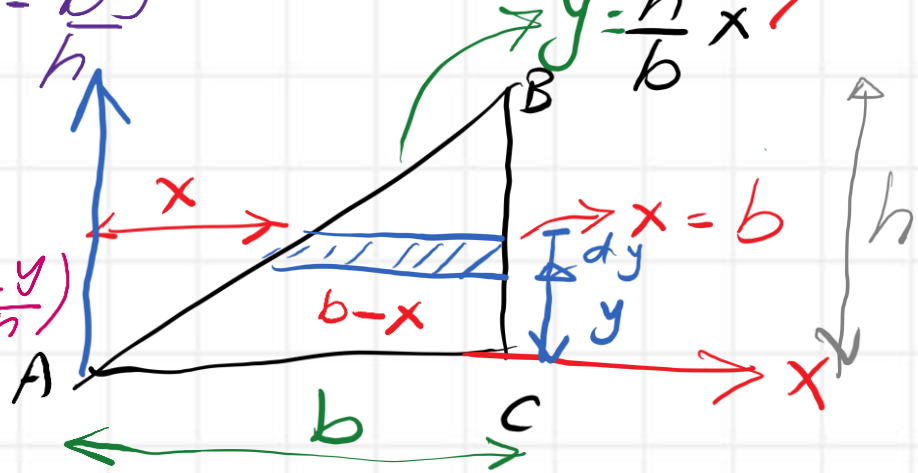


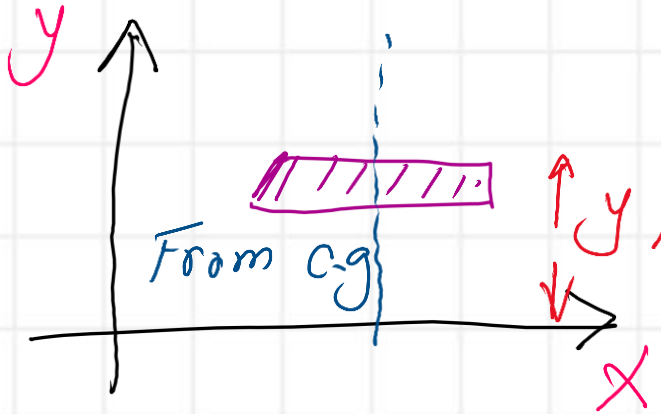
$$b - x = b - \frac{by}{h}$$

$$b - x = b\left(1 - \frac{y}{h}\right)$$

$$(b - x)\,dy = b\,dy\left(1 - \frac{y}{h}\right)$$

$$x = \frac{by}{h}$$





The first moment of area about x-axis = $dA \cdot y$

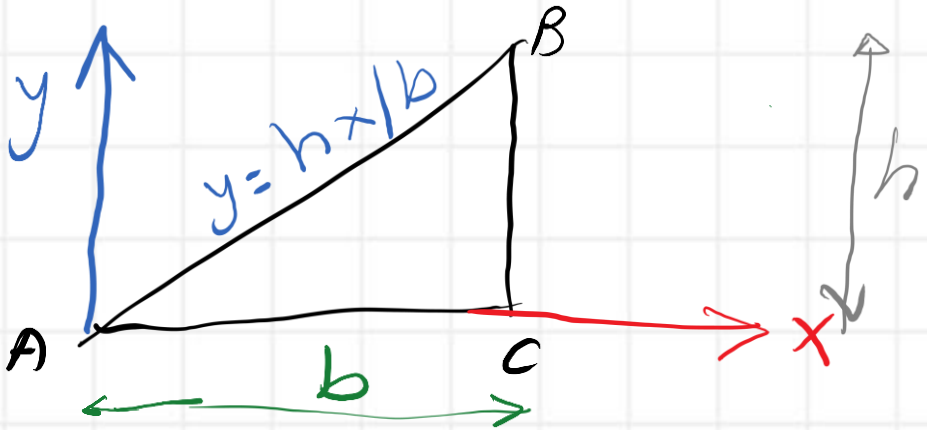
$$dA = (b - x)\,dy \quad \& \quad x = \frac{b}{h}y$$

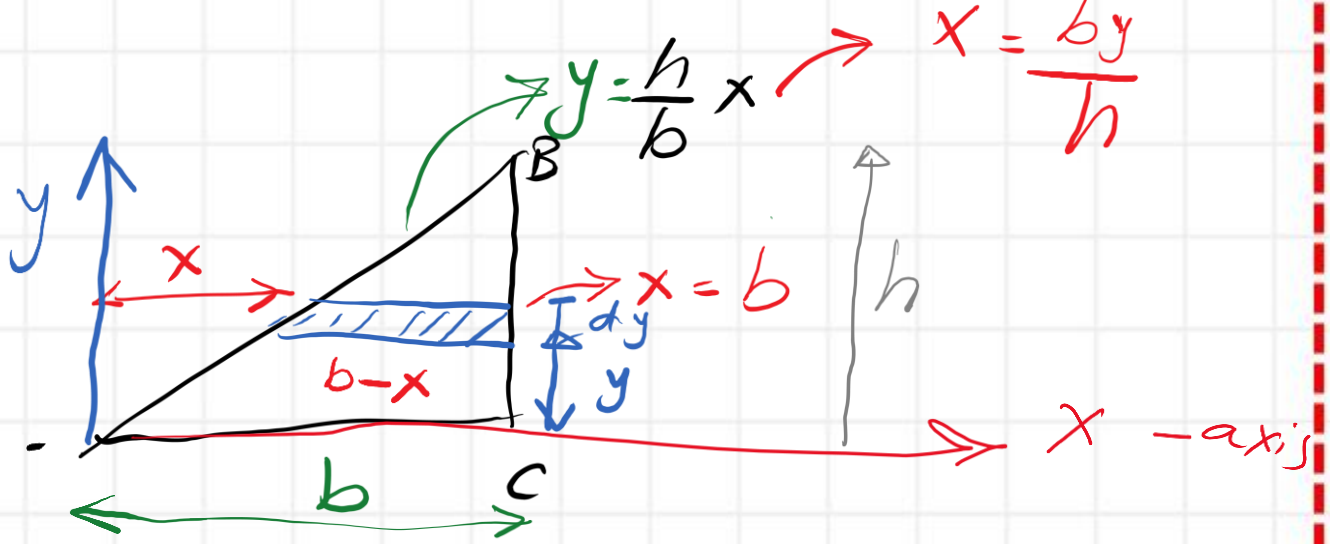
Re-write $dA = \left(b - \frac{b}{h}y\right)dy$

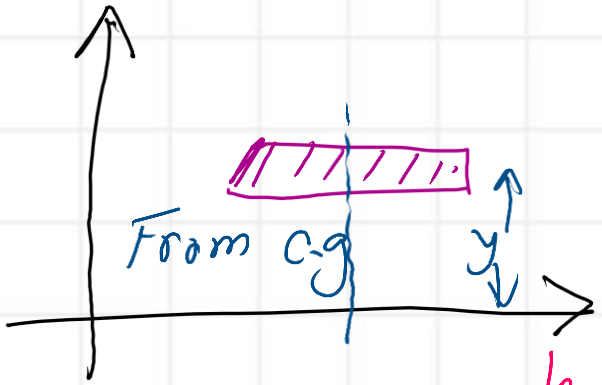
Area & C.g Determination

First moment of area.

Case # 2 → $\bar{y}$ bar







$$dA \cdot \bar{y} = \left(b - \frac{b}{h} y\right) dy \, (y)$$

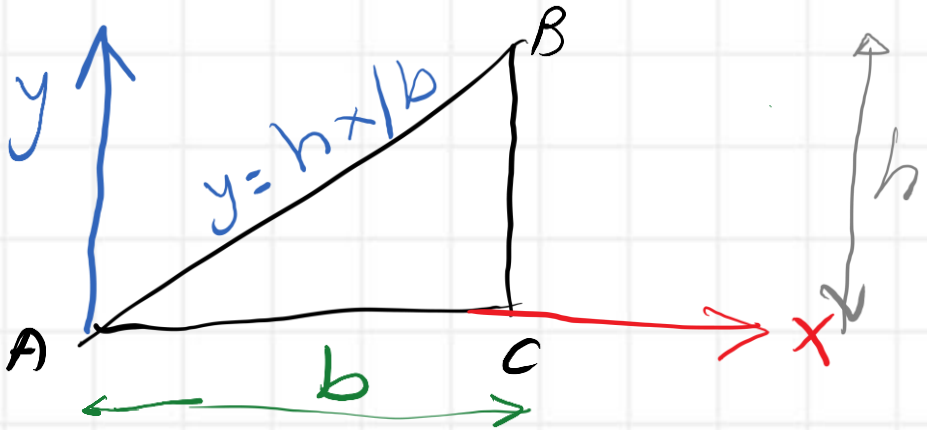
$$\int dA \cdot \bar{y} = \int_0^h b y \, dy - \int_0^h \frac{b}{h} y^2 \, dy$$

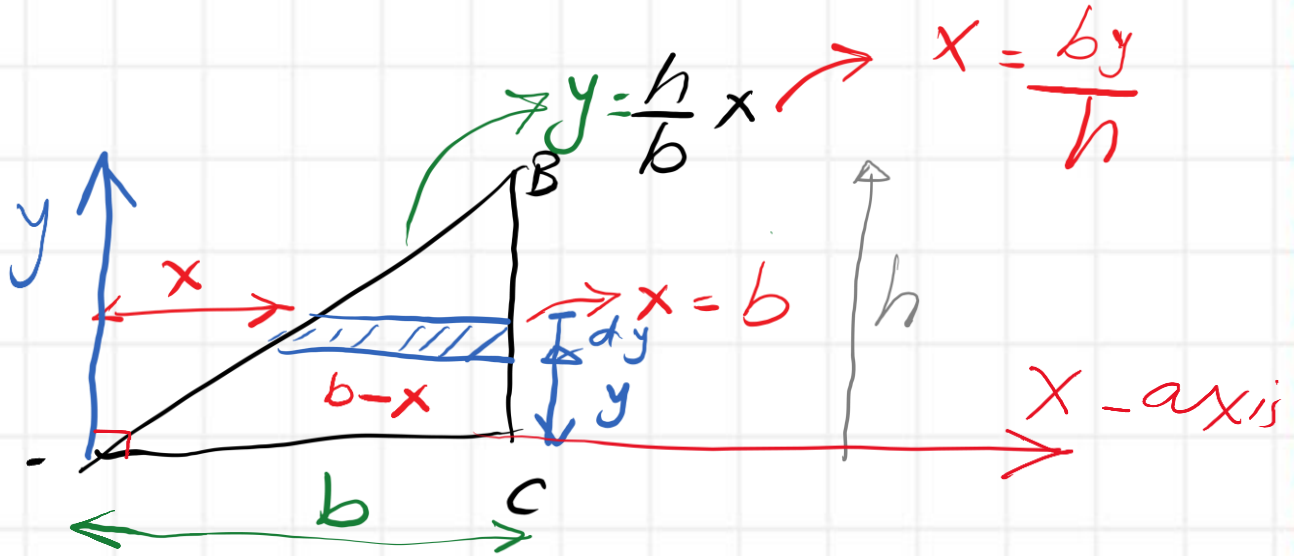
$$A \bar{y} = b \frac{y^2}{2}\Big|_0^h - \frac{b}{h}\left(\frac{y^3}{3}\right)\Big|_0^h = \frac{b}{2} h^2 - \frac{bh^3}{3h} = \frac{b}{2} h^2 - \frac{bh^2}{3} = \frac{bh^2}{6}$$

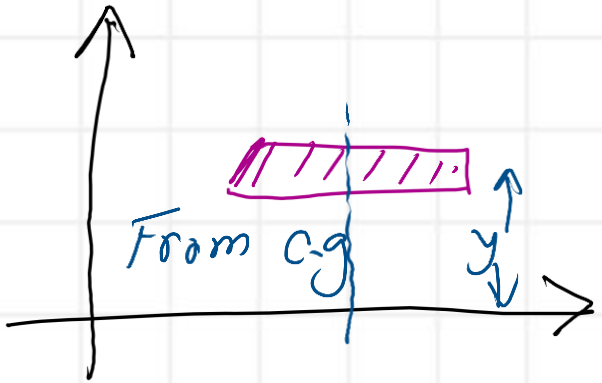
Area & C.g Determination

Case # 2 → $\bar{y}$ bar

First moment of area.







$$A \cdot \bar{y} = bh^2/6$$

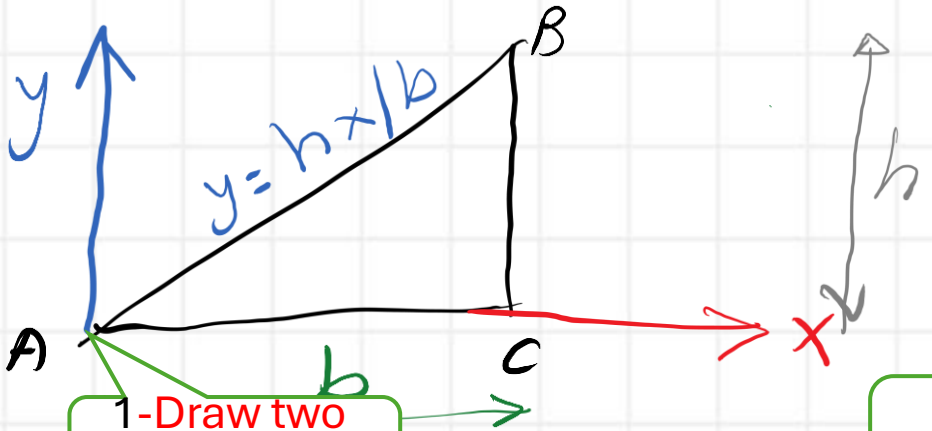
but $A = \frac{1}{2}(b)(h)$

$$\bar{y} = \frac{A \cdot \bar{y}}{A} = \frac{bh^2}{6}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{h}{3}$$

Area & C.g Determination

First moment of area.

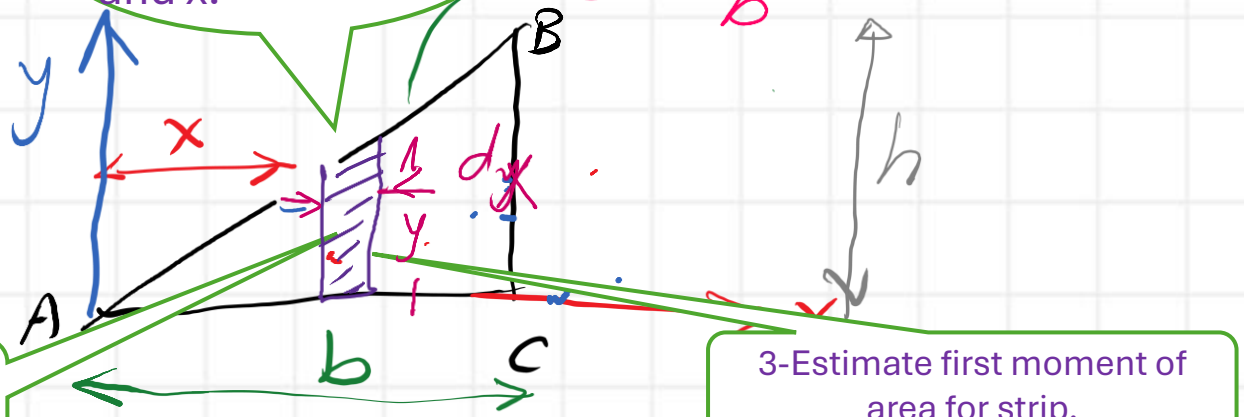
Case # 2 – $\bar{y}$ bar



1-Draw two axes x &y.

2-Find y value in terms of b & h and x.

$y = \frac{h}{b} x$



3-Estimate first moment of area for strip.

4-Integrate.

$\bar{y}$ estimation

strip area $dA = y \cdot dx$

$\bar{y}_{For\ strip} = y/2$

Using Vertical strip

$$dA = \left(\frac{h}{b} x\right)(dx) \rightarrow A = \int_0^b dA = \int_0^b \frac{h}{b} x\, dx = \frac{h}{b} \frac{x^2}{2} \Big|_0^b$$

$$A = \frac{h}{2b}(b^2) = \frac{1}{2} bh$$

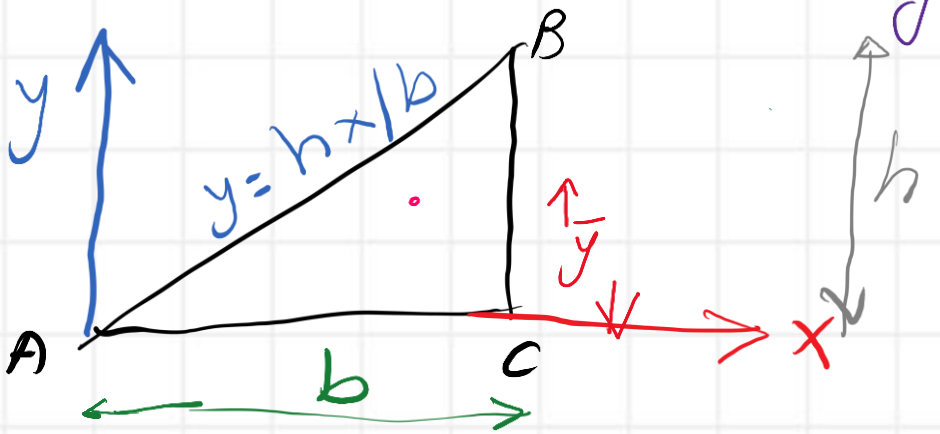
Area & C.g Determination

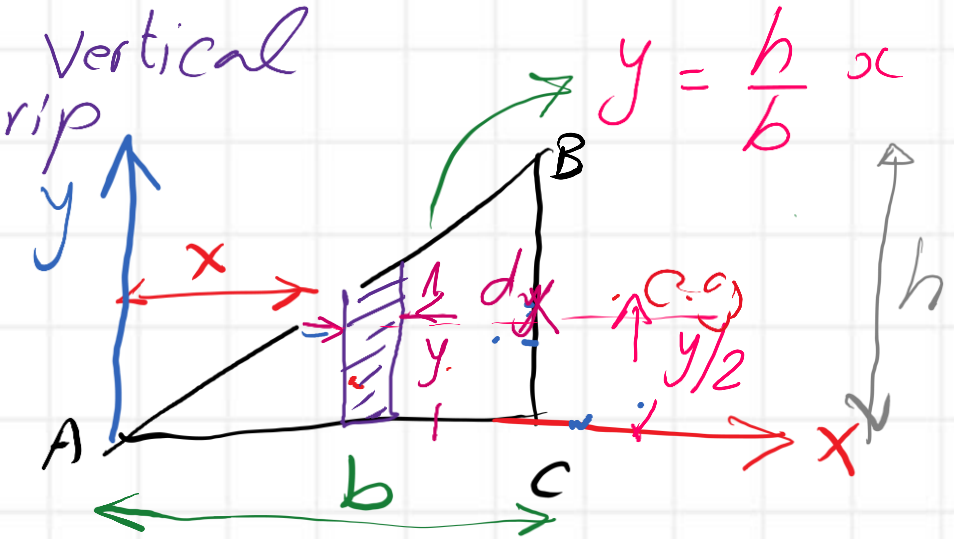
Case # 2

Using a Vertical strip

First moment of area.

$\bar{y}$ estimation





$$dA\,\bar{y}_{str} = (dx)(y)(y/2)$$

$$\frac{y^2}{2} = \frac{1}{2}\left(\frac{h}{b}x\right)^2 = \frac{h^2x^2}{2b^2}$$

$$\int dA\,\bar{y} = \int_0^b \frac{h^2x^2}{2b^2}(dx) = \frac{h^2}{2b^2}\left(\frac{x^3}{3}\right)\Big|_0^b = \frac{h^2}{2b^2}\left(\frac{b^3}{3}\right) = \frac{h^2b}{6}$$

$A \cdot \bar{y} =$

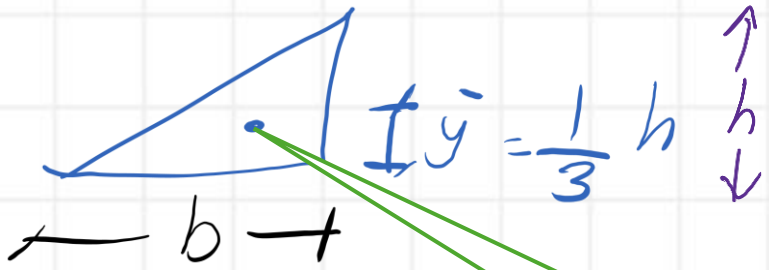
First moment of area value of a triangle.

$\bar{y}$ For right_angle

$$A\bar{y} = \frac{h^2 b}{6}$$

but $A = \frac{1}{2} bh$

$$\bar{y} = (A\bar{y})/A = \frac{h^2 b}{6}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{h}{3}$$



CG height from the base.