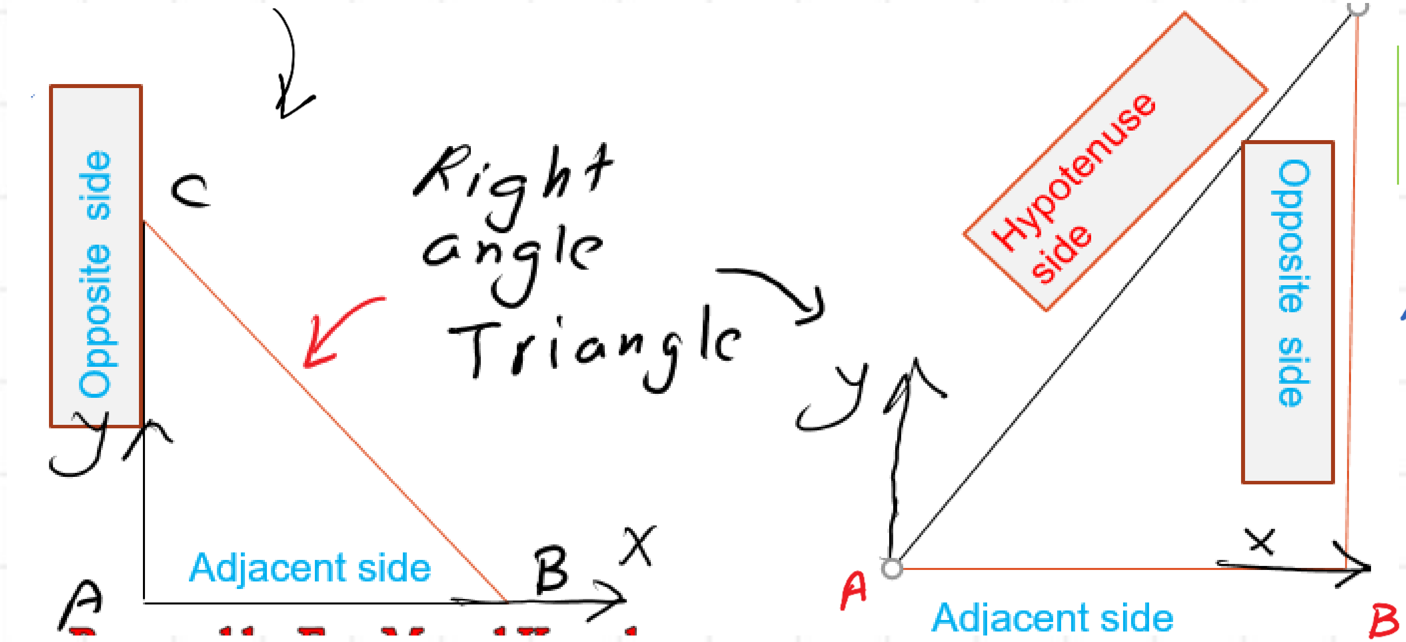
Objective of the lecture

1- How to determine the x bar and y bar for the right- angle triangle by using:

a- Horizontal strip parallel to the external x axis → at the adjacent base

b- Vertical strip perpendicular to the external x axis → at the adjacent base

Case No.1



→ Case #2

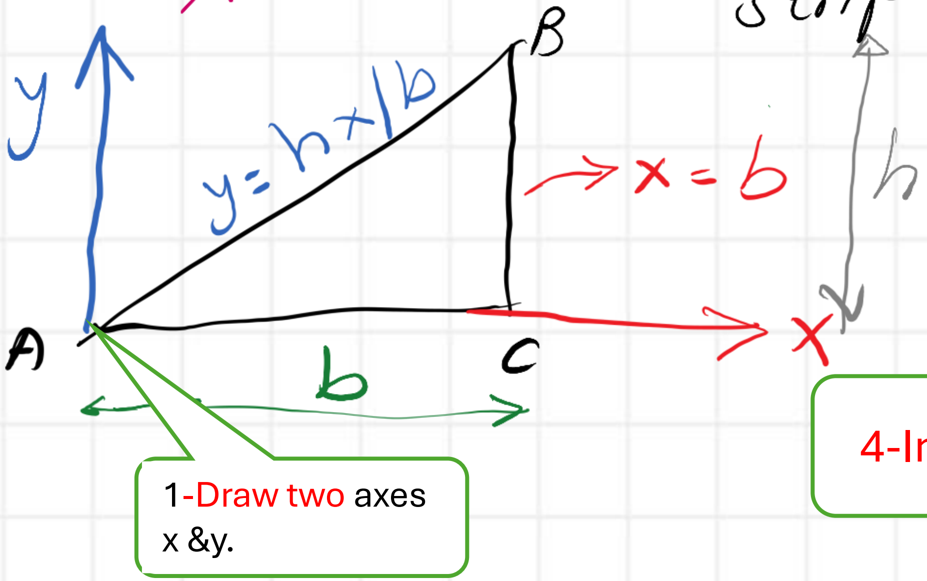
h

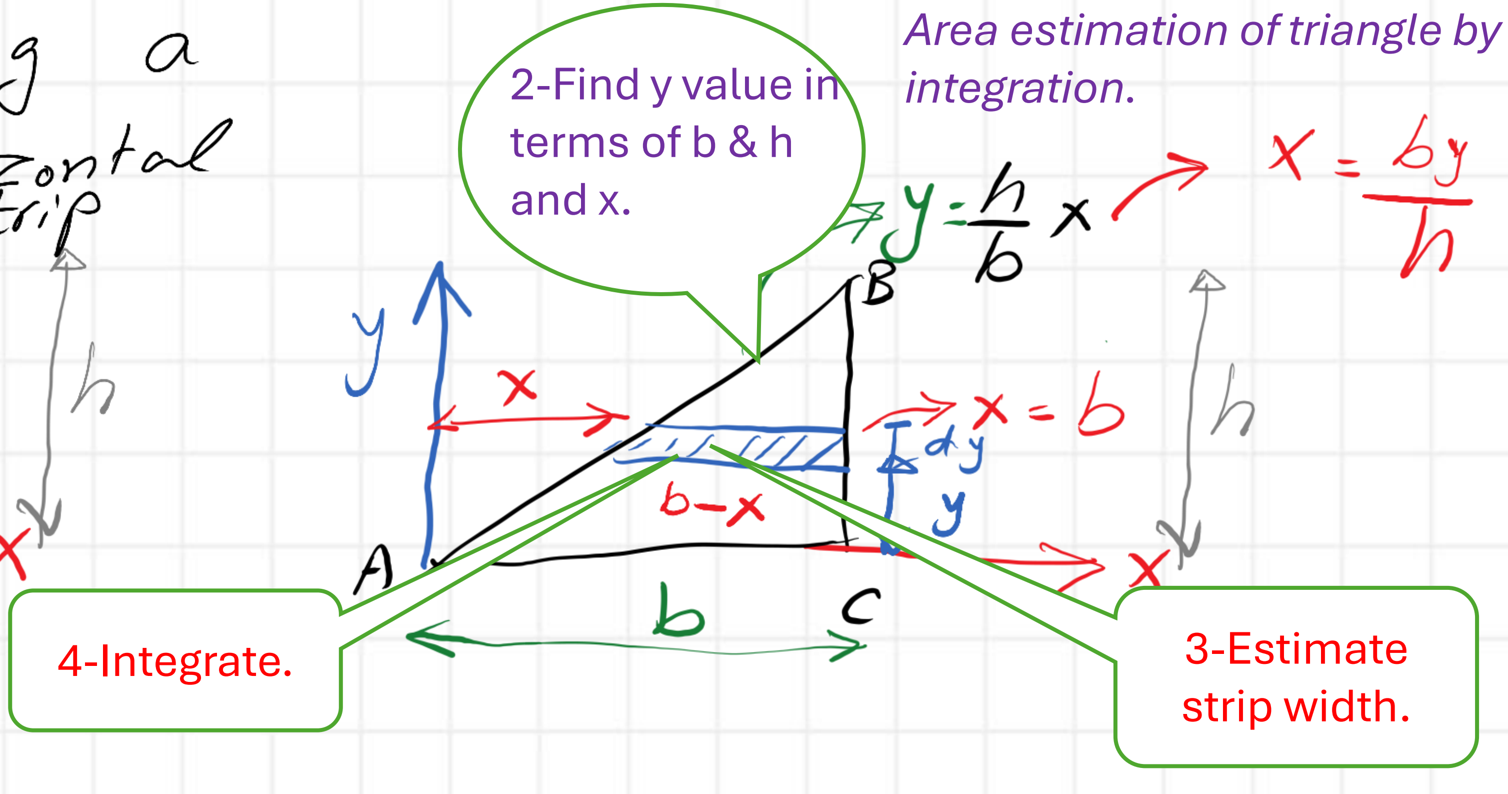
Prepared by Eng.Maged Kamel.

Case # 2 Using a horizontal strip

$\bar{X}$ bar

Area estimation of triangle by integration.





The necessary Four steps are shown in sketch

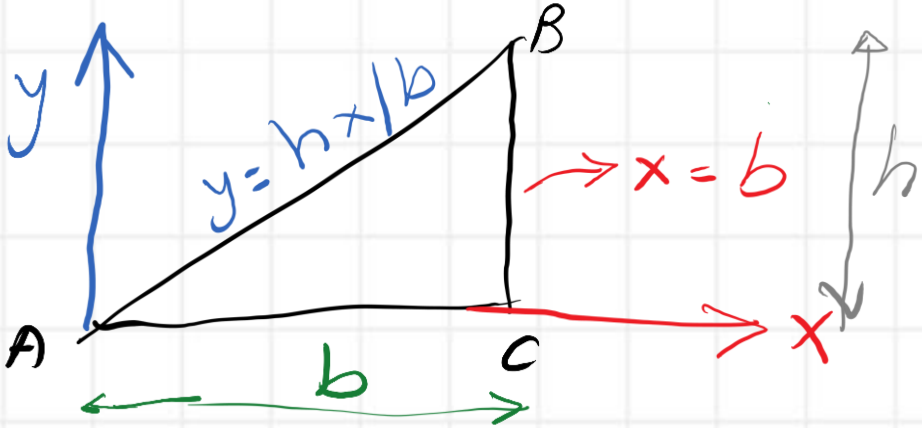
$$dA = \text{area of strip} = (b - x)\,dy, \quad x = \frac{b}{h}(y)$$

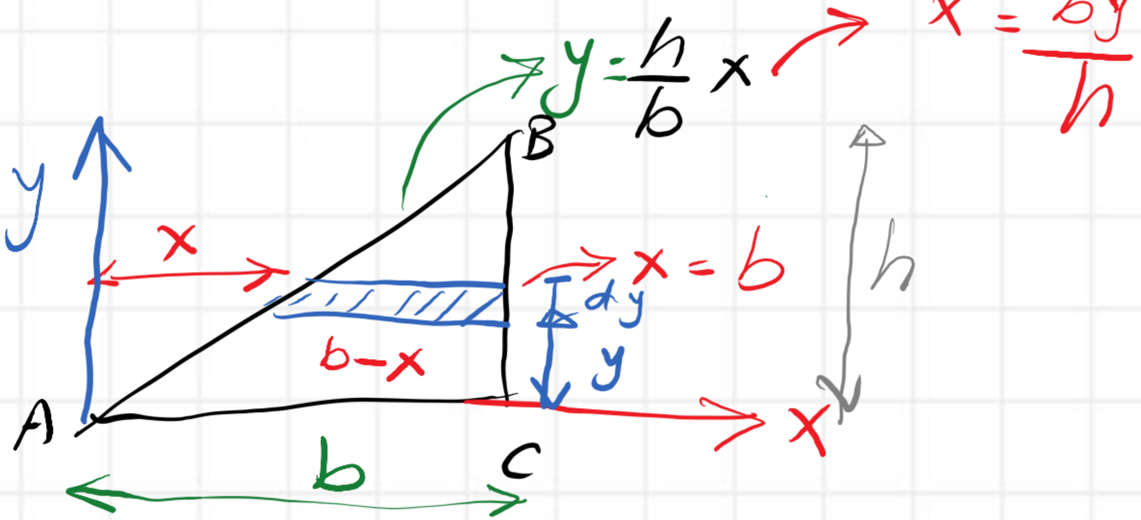
$$dA = \left(b - \left(\frac{b}{h}\right)(y)\right)dy = \left(b.dy - b\frac{y}{h}\,dy\right)$$

Area & C.g Determination

Case # 2 → $\bar{X}$ bar

Area estimation of triangle by integration-case of vertical-strip.





$$dA = \left(b \cdot dy - \frac{by}{h}\, dy\right)$$

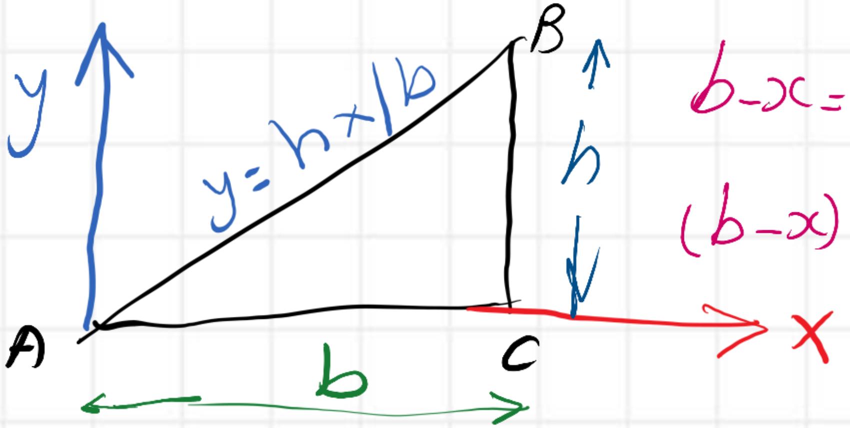
$$\int dA = A = \int b\,dy - \frac{by}{h}\,dy = b \cdot y \Big|_0^h - \frac{b}{h}\frac{y^2}{2}\Big|_0^h$$

$$A = \left[bh - \left(\frac{b}{2h}\right)(h^2)\right] = \frac{1}{2} bh$$

Area & C.g Determination

First moment of area.

Case # 2 → $\bar{X}$ bar

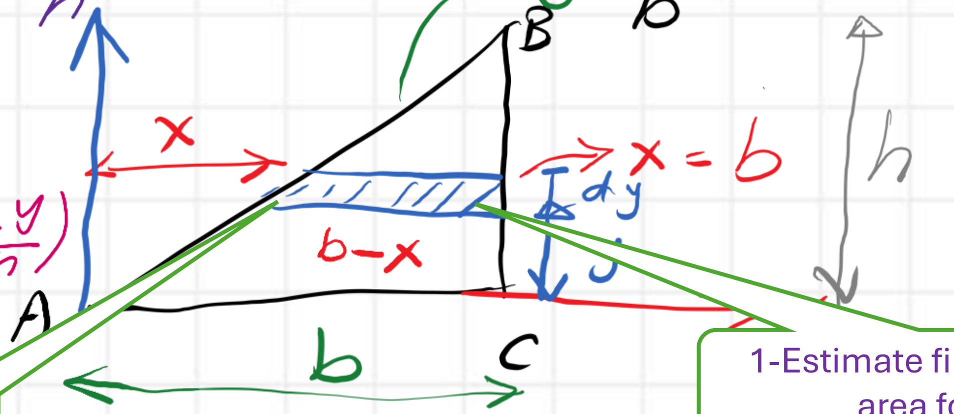


$$b - x = b - \frac{by}{h}$$

$$b - x = b\left(1 - \frac{y}{h}\right)$$

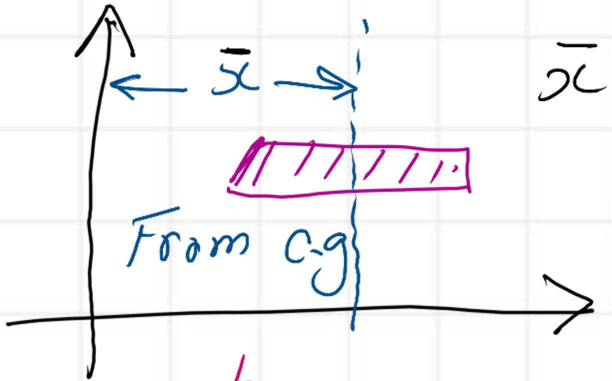
$$(b-x)\,dy = b\,dy\left(1-\frac{y}{h}\right)$$

$$y = \frac{h}{b}x \rightarrow x = \frac{by}{h}$$



1-Estimate first moment of area for strip.

2-Integrate.



$$\bar{x} \text{ for strip} = x + \left(\frac{1}{2}\right)(b-x)$$

$$= \left(\frac{x+b}{2}\right) = \frac{1}{2}\left(\frac{by}{h}+b\right) = \frac{b}{2}\left(\frac{y}{h}+1\right)$$

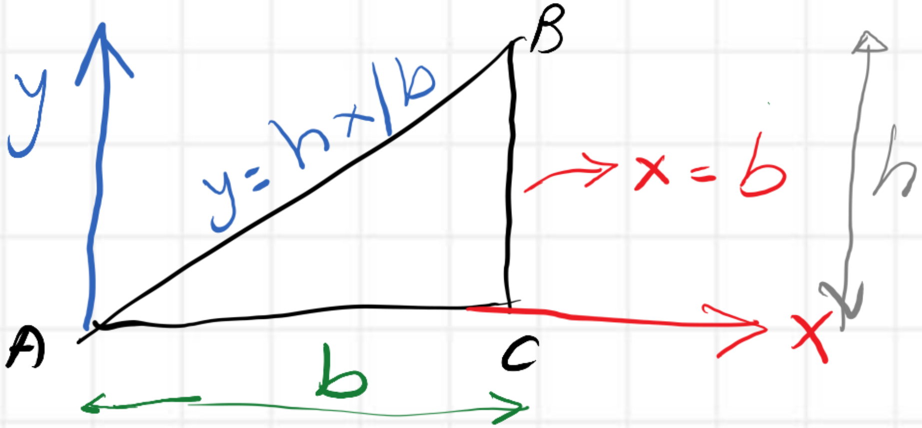
$$dA.\bar{x}_{strip} = \left(b.dy - \frac{by}{h}dy\right)\left(\frac{b}{2}\right)\left(\frac{y}{h}+1\right)$$

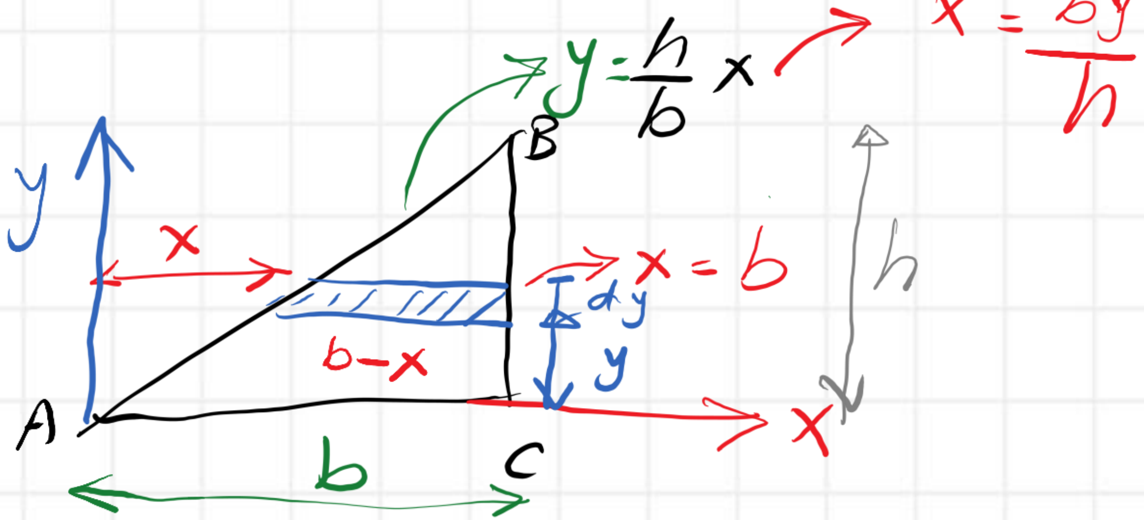
$$A\bar{x} = \int_0^h dA.\bar{x}_{st} = \frac{b^2}{2}\left(\int_0^h \left(\frac{y}{h}+1\right)dy - \int_0^h \frac{y^2\,dy}{h^2} - \int_0^h y\,\frac{dy}{h}\right)$$

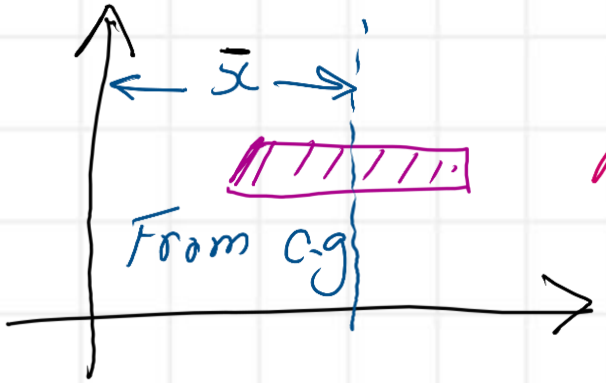
Area & C.g Determination

First moment of area.

Case # 2







$$A\bar{x} = \int_0^h dA\cdot\bar{x}_{st} = \frac{b^2}{2}\int_0^h\left(\left(\frac{y}{h}+1\right)dy - \int_0^h \frac{y^2\,dy}{h^2} - \int_0^h y\,\frac{dy}{h}\right)$$

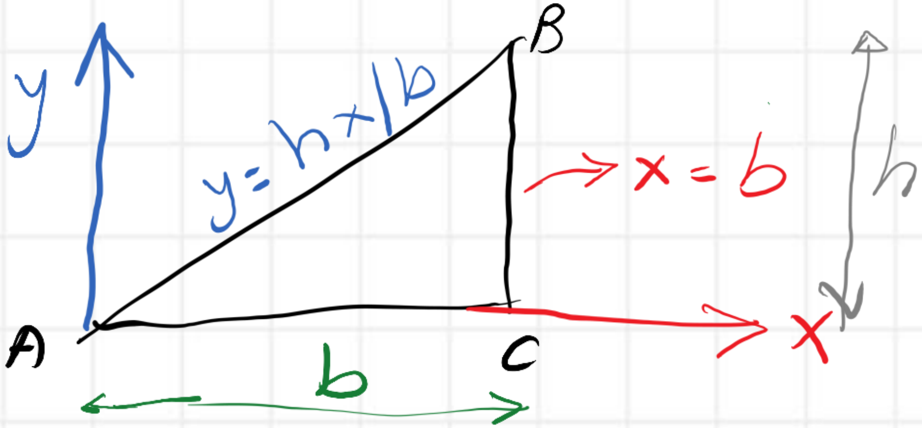
$$\frac{b^2}{2}\left(\frac{y^2}{2h}\Big|_0^h + y\Big|_0^h - \frac{y^3}{3h^2}\Big|_0^h - \frac{y^2}{2h}\Big|_0^h\right)$$

$$A.\bar{x} = \frac{b^2}{2}\left(\frac{h^2}{2h} + h - \frac{h^3}{3h^2} - \frac{h^2}{2h}\right) = \frac{b^2}{2}\left(\frac{h}{2} + h - \frac{h}{3} - \frac{h}{2}\right)$$

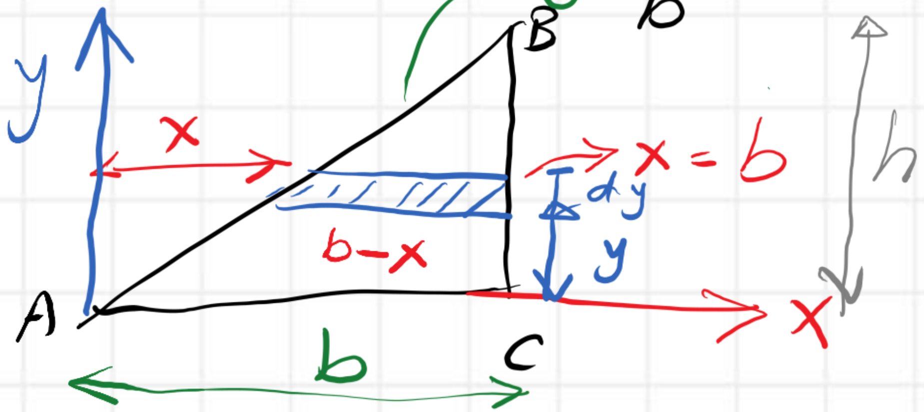
Area & C.g Determination

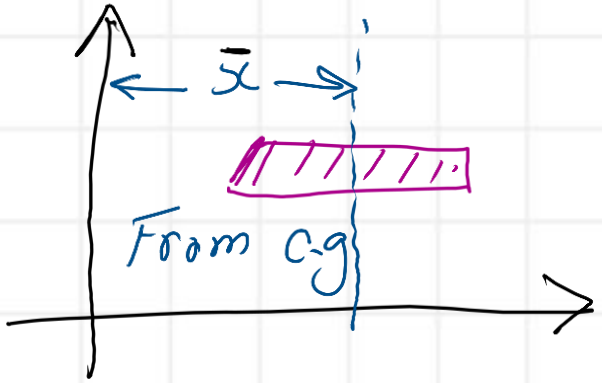
First moment of area.

Case # 2



$$y = \frac{h}{b}x \rightarrow x = \frac{by}{h}$$





$$A \cdot \bar{x} = \frac{b^2}{2}\left(\frac{h}{2} + h - \frac{h}{3} - \frac{h}{2}\right)$$

$$A\bar{x} = \frac{b^2}{2}\left(\frac{3h}{2} - \frac{5h}{6}\right) = \frac{1}{2}b^2\left(\frac{9-5}{6}\right)h$$

$$= \frac{1}{2}b^2\left(\frac{2}{3}h\right)$$

$$A = \frac{bh}{2}$$

$$A\bar{x} = \frac{1}{3}b^2h$$

$$\bar{x} = \frac{A \cdot \bar{x}}{A} = \frac{\frac{1}{3}b^2h}{\left(\frac{1}{2}bh\right)} \Rightarrow \left(\bar{x} = \frac{2}{3}b\right)$$

Area & C.g Determination

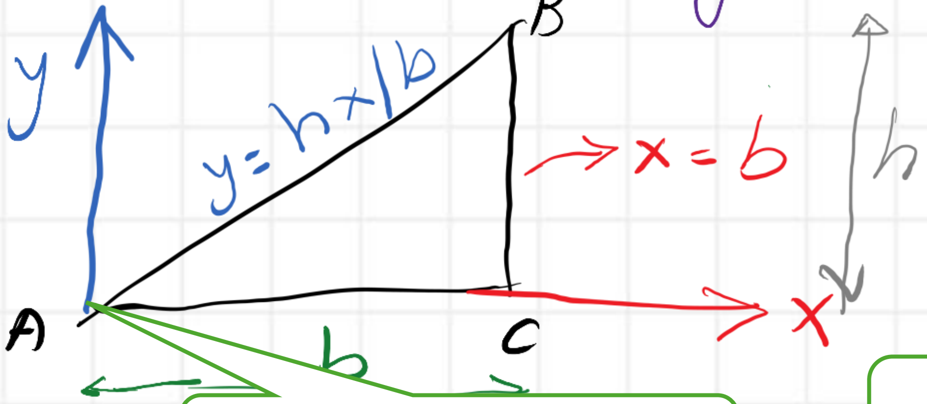
$\bar{X}$ estimation

First moment of area.

Case # 2

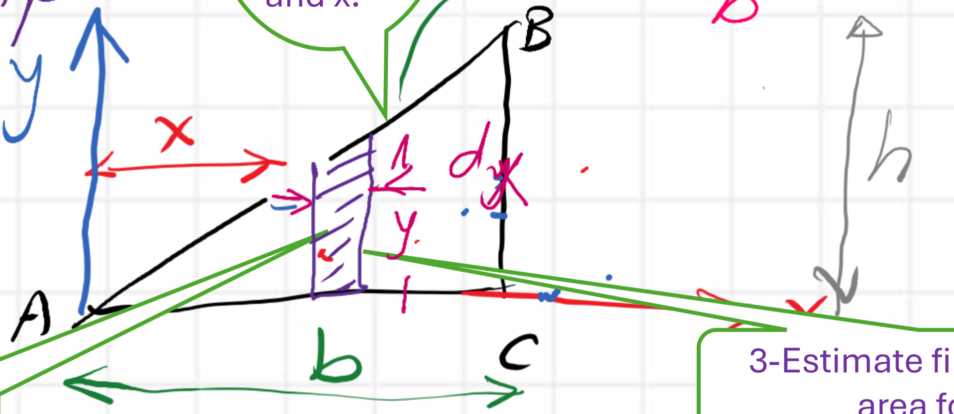
Using Vertical strip

$$y = \frac{h}{b} x$$



1-Draw two axes x &y.

2-Find y value in terms of b & h and x.



4-Integrate.

3-Estimate first moment of area for strip.

strip area $dA = y \cdot dx$

$\bar{x}$ bar $= x$

$$dA = \left(\frac{h}{b}x\right)(dx) \rightarrow A = \int_0^b dA = \int_0^b \frac{h}{b} x \, dx = \frac{h}{b} \frac{x^2}{2} \Big|_0^b$$

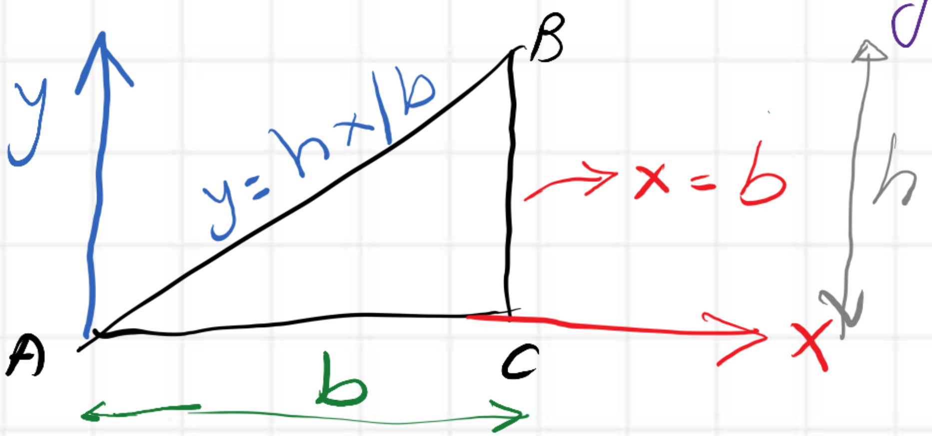
$$A = \frac{h}{2b}(b^2) = \frac{1}{2} hb$$

Area & C.g Determination

First moment of area.

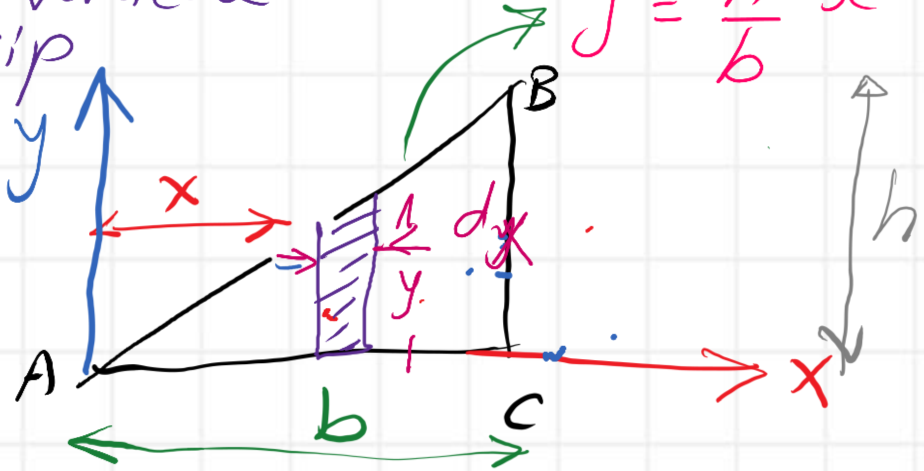
$\bar{X}$ estimation

Case # 2



Using a vertical strip

$$y = \frac{h}{b}x$$



$$dA.x = h\frac{x}{b}(dx)(x)$$

$$\int dA.x = \int_0^b h\left(\frac{x^2}{b}\right)dx = \frac{h}{b}\,\frac{x^3}{3}\Big|_0^b = \frac{h}{3b}(b^3) = \frac{hb^2}{3}$$

$$\bar{x} = \frac{A\bar{x}}{A} \rightarrow = \frac{hb^2}{3}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{2}{3}b \rightarrow$$

Same result obtained by using a hL strip