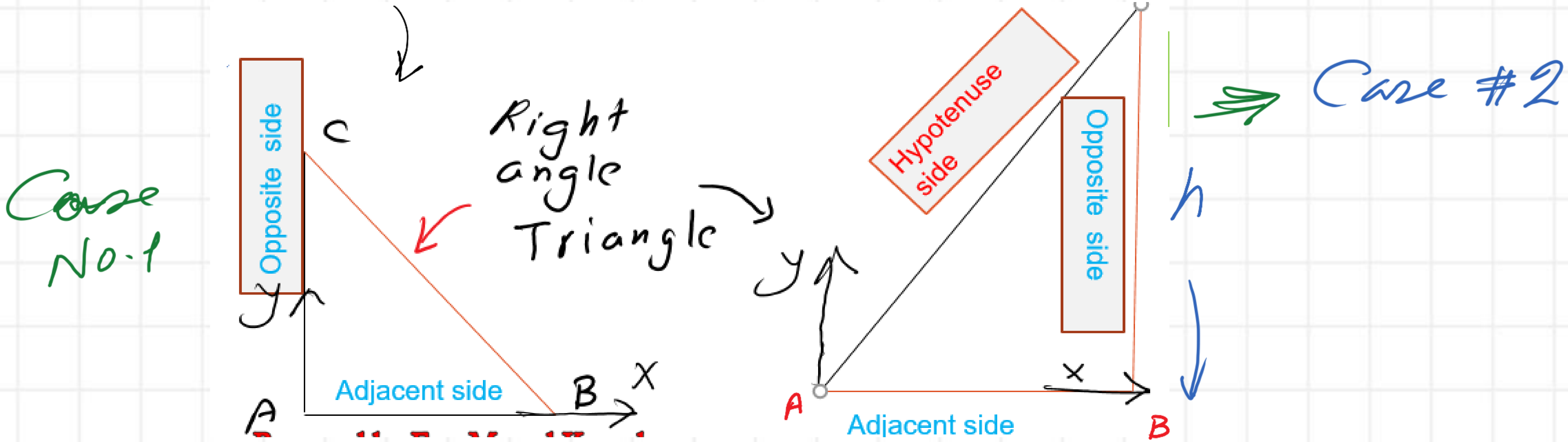
Objective of the lecture

1- How to determine the x bar and y bar for the right angle triangle by using:

a- Horizontal strip parallel to the external x axis → at the adjacent base

b- Vertical strip perpendicular to the external x axis. → at the adjacent base

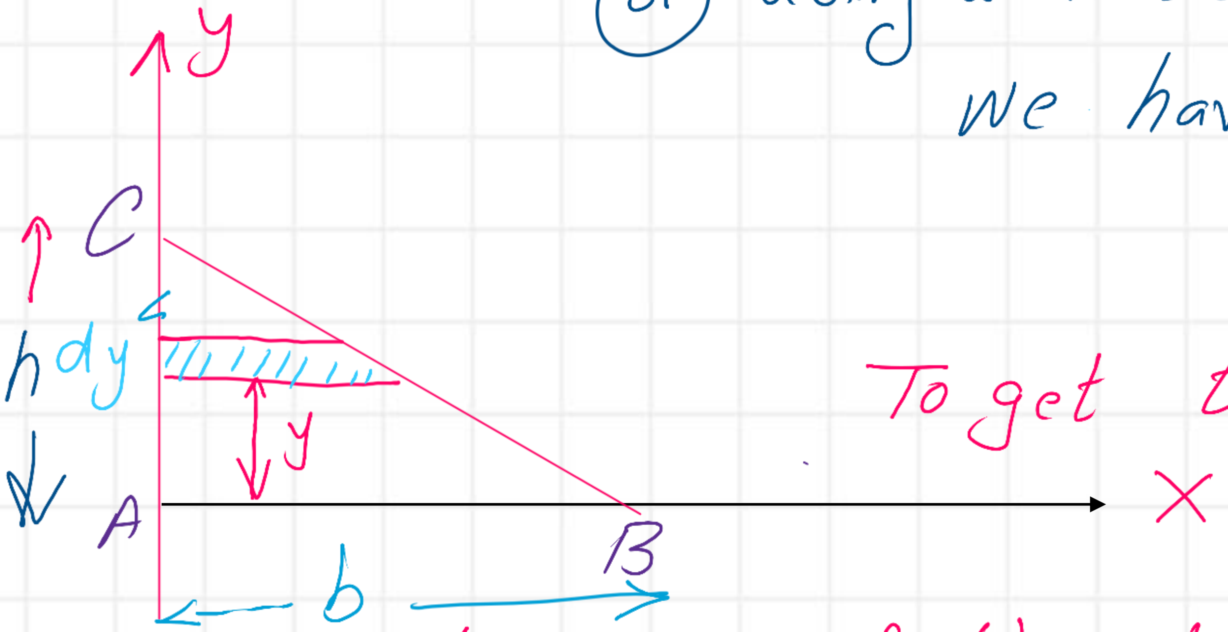


Prepared by Eng.Maged Kamel.

$\bar{x}$ bar for a right angle triangle Case # 1

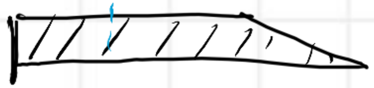
ⓐ using a horizonal strip

we have base = b of the triangle
heigh = h



To get the total area

use a strip



what is the area of the strip ?

if we Consider that strip width = dy

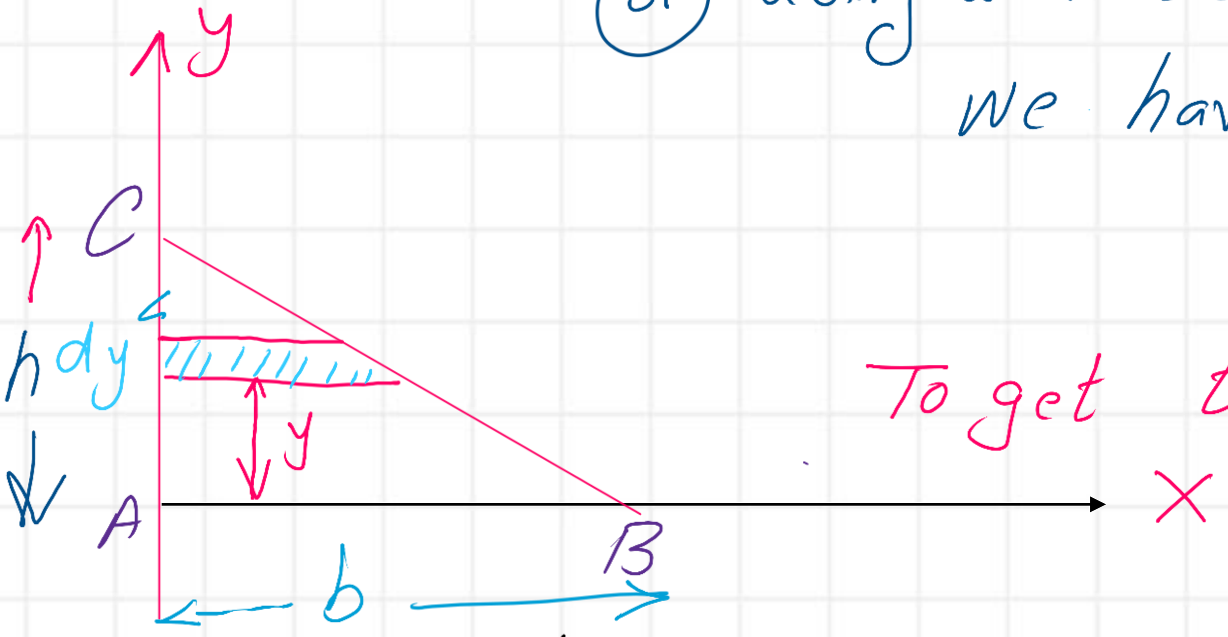
the breadth is = x . $dA \rightarrow$ area of strip

$\bar{x}$ bar for a right angle triangle   Case # 1

(a) using a horizonal strip

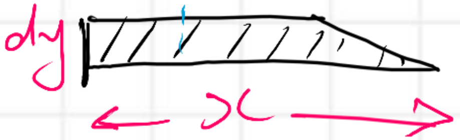
we have base $= b$ of the triangle

heigh $= h$



To get the total area

Use a strip



$$dA = x \cdot dy$$

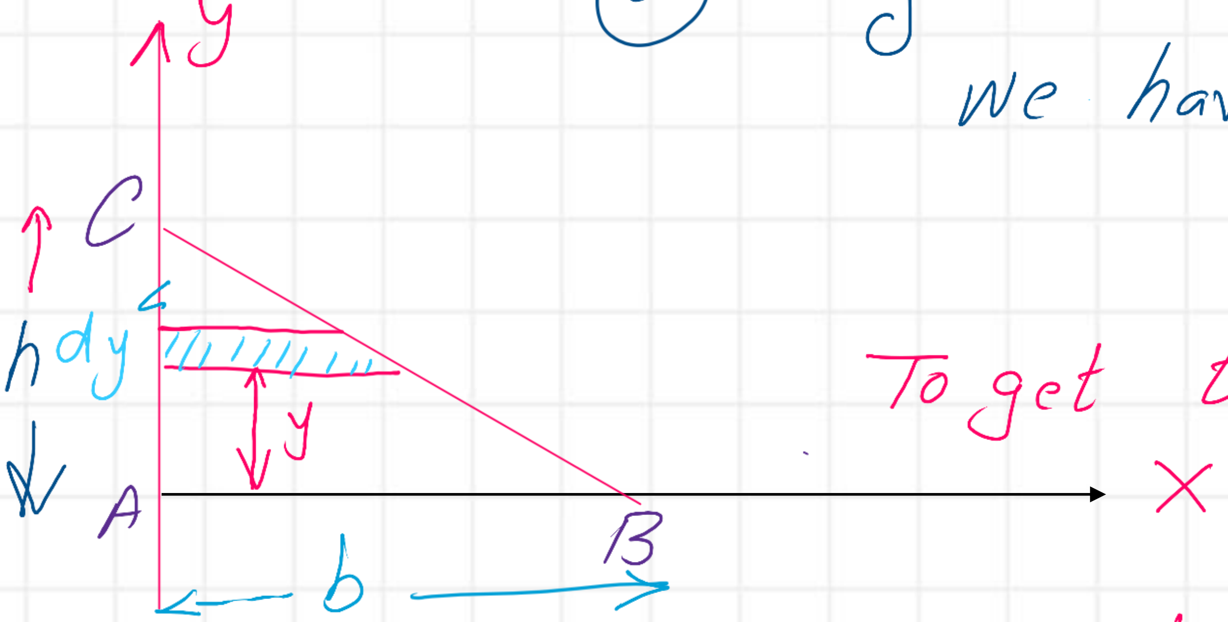
→ From the slope equation of Line BC

$$y = mx + C \qquad \text{where } m = \frac{-h}{b}, \quad C = h$$

$\bar{x}$ bar for a right angle triangle Case # 1

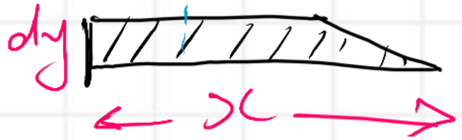
ⓐ using a horizonal strip

we have base $= b$ of the triangle
heigh $= h$



To get the total area

Use a strip



$dA = x \cdot y$

$y = -\frac{h}{b} x + h$

check points C, B

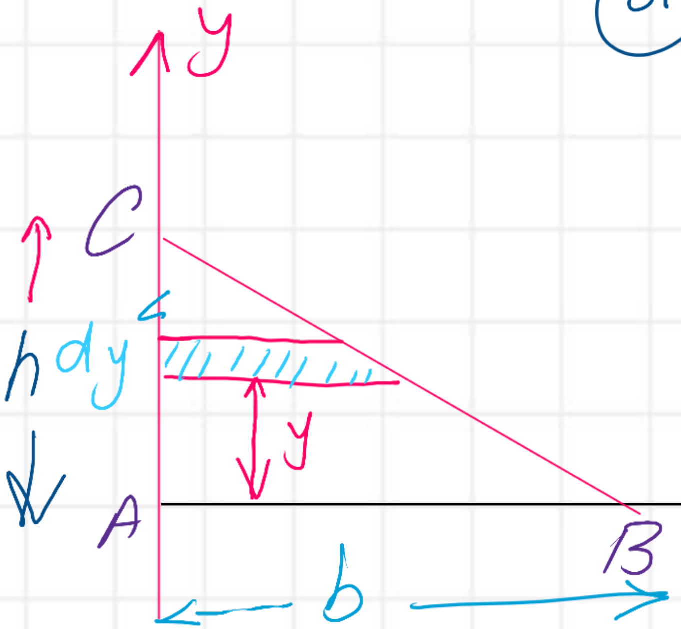
$L.H.S = h$

at $X = 0$, $y = h$ $\rightarrow$ $R.H.S = -\frac{h}{b}(0) + h = +h$

which is Correct

$\bar{x}$ bar for a right angle triangle   Case # 1

(a) using a horizonal strip

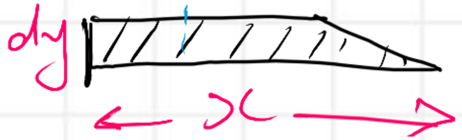


we have base = b of the triangle

heigh = h

To get the total area

Use a strip



$dA = l \cdot y$

$$y = -\frac{h}{b}x + h$$

check Point B

at $\cancel{x} = b$

$y = 0$

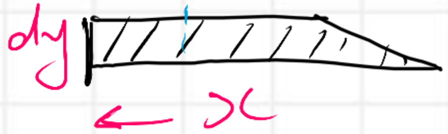
L.H.S $= 0$

$\rightarrow$ R.H.S $= -\frac{h}{b}(b) + h = 0$

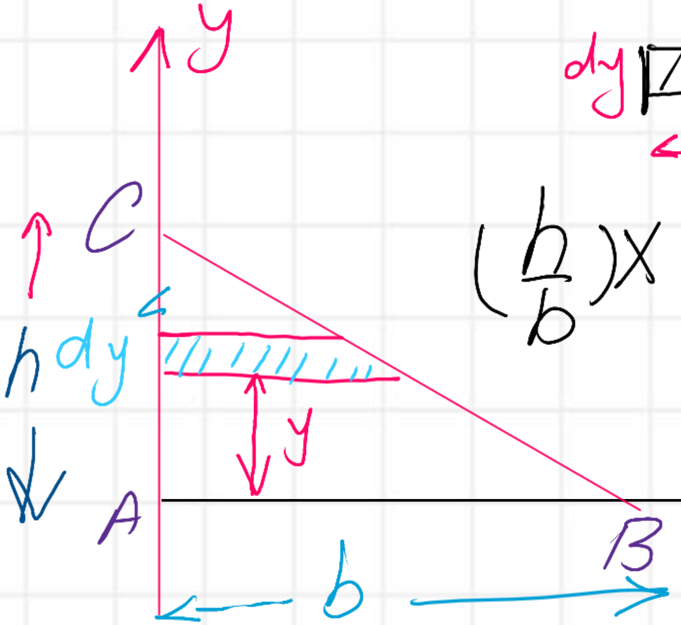
which is Correct

$\bar{x}$ bar for a right angle triangle   Case # 1

Calculate the triangle area by using



$$dA = x(dy)$$



$$\left(\frac{h}{b}\right)x = h - y \Rightarrow x = \left(\frac{h-y}{h}\right)(b)$$

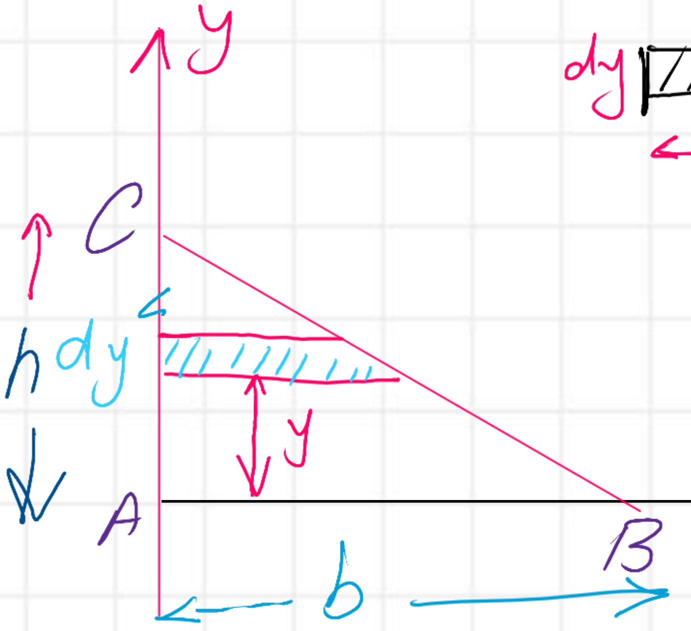
$$dA = \left(\frac{h-y}{h}\right)b\,(dy)$$

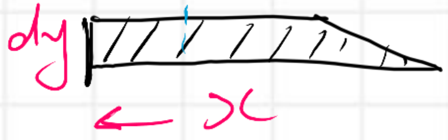
$$A = \int dA = \int_{y=0}^{y=h} \left(1 - \frac{y}{h}\right) b\, dy = b\left[y\Big|_0^h - \frac{y^2}{2h}\Big|_0^h \right]$$

$$A = b(h-0) - \frac{b}{2h}(h^2-0) = bh - \frac{bh}{2} = \frac{1}{2}bh$$

$\bar{x}$ bar for a right angle triangle   Case # 1

Calculate the First moment of Area about y_axis which





$\int dA \cdot \frac{x}{2}$ ⇒ $\frac{x}{2}$ c.g of strip to y_axis

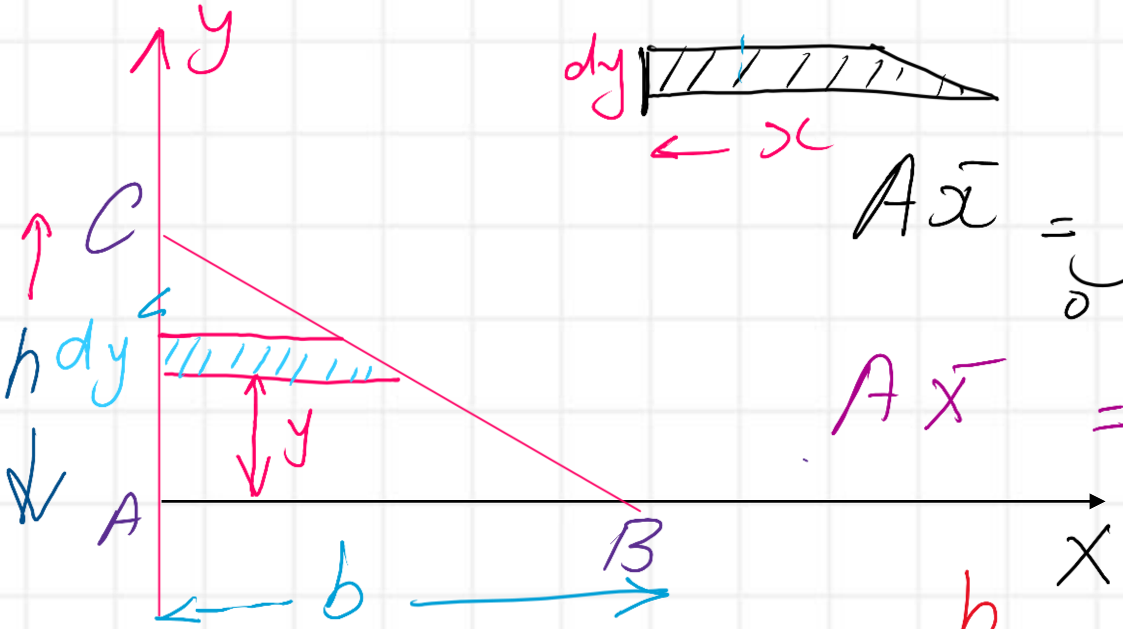
$$x = \frac{(h-y)}{h}(b)$$

$$A\bar{x} = \int dA \cdot \left(\frac{x}{2}\right) = \int_{y=0}^{y=h} \frac{(h-y)}{h}(b)\, dy \left[\frac{(h-y)}{h}\left(\frac{b}{2}\right)\right]$$

$$A\bar{x} = \int_0^h \frac{(h-y)^2}{h^2}\left(\frac{b^2}{2}\right) dy$$

$\bar{x}$ bar for a right angle triangle Case # 1

Calculate the First moment of Area



$$A\bar{x} = \int_0^h \left(\frac{h-y}{h^2}\right)^2 \left(\frac{b^2}{2}\right) dy$$

$$A\bar{x} = \frac{b^2}{2h^2} \int h^2 dy - (2hy + y^2) dy$$

$$A\bar{x} = \frac{b^2}{2(h^2)} \left((h^2 y)\Big|_0^h - \frac{2hy^2}{2}\Big|_0^h + \frac{y^3}{3}\Big|_0^h \right)$$

$$A\bar{x} = \frac{b^2}{2h^2}\left(h^2(h) - h^3 + \frac{h^3}{3}\right) = \frac{b^2 h^3}{2h^2}\left(\frac{1}{3}\right) = \frac{b^2 h}{6}$$

We have $A = \frac{1}{2}(b)(h)$

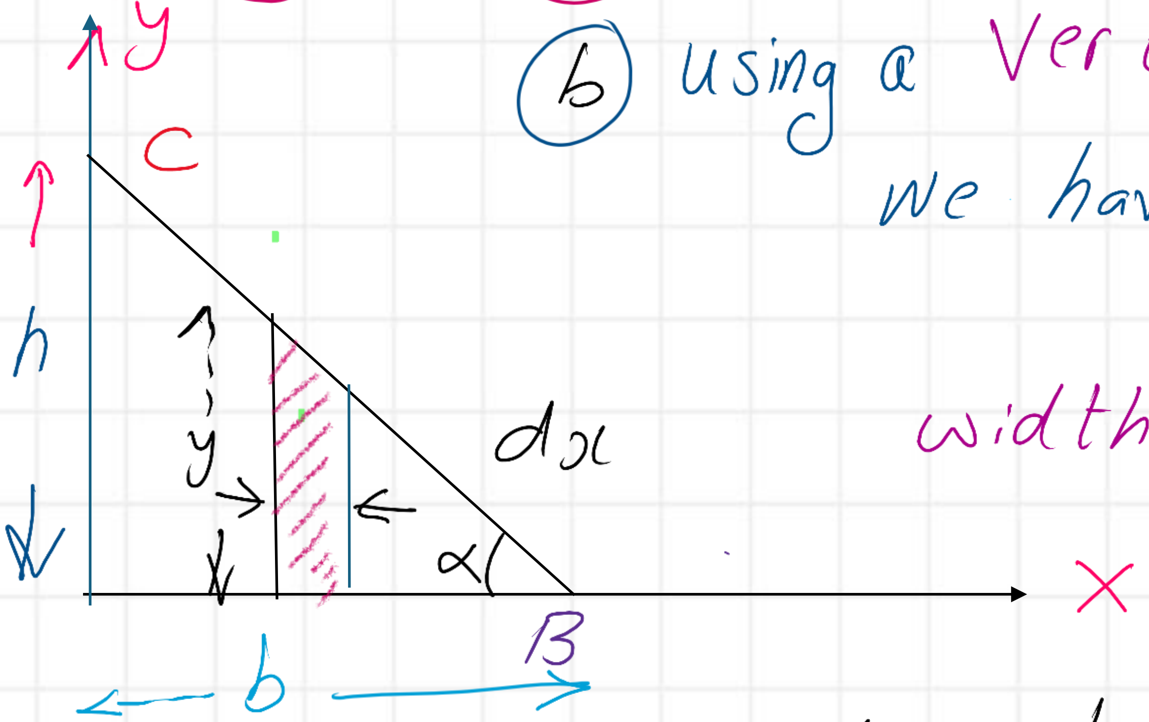
While $A\bar{x} = \frac{b^2 \cdot h}{6}$

Then $\bar{X} = \frac{b^2 h}{6A} = \frac{b^2 h}{6}\left(\frac{1}{\frac{1}{2}(bh)}\right) = \frac{b}{3}$

C.g horizontal distance is apart by one third of the base from the external way axis.

$\bar{x}$ bar for a right angle triangle Case # 1

ⓑ using a Vertical strip



We have base = b of the triangle
heigh = h

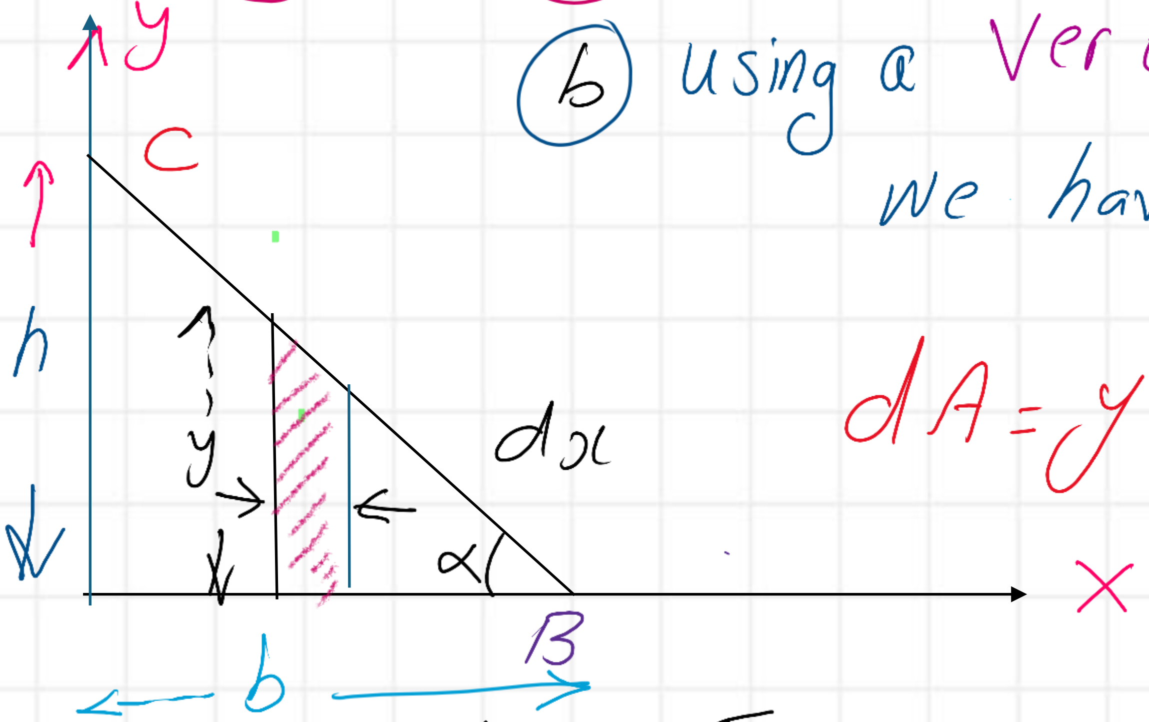
width of strip = dx
heigh = y

We have a relation between x & y as

$y = -\frac{h}{b}(x) + h$ For Line BC

$dA = y \cdot dx$

$\bar{x}$ bar for a right angle triangle — Case # 1



Ⓑ using a Vertical strip

we have base = b of the triangle

heigh = h

$$dA = y \cdot dx = \left(\left(-\frac{h}{b}x\right)+h\right)dx$$

$A = \int dA$ From $x = 0 \rightarrow x = b$

$$A = \int_0^b -\frac{hx}{b}\,dx + h\,dx = -\frac{h}{b}\frac{x^2}{2}\Big|_0^b + h.x\Big|_0^b$$

$$= (h.b) - \frac{h}{2b}(b^2) = h.\frac{b}{2}$$

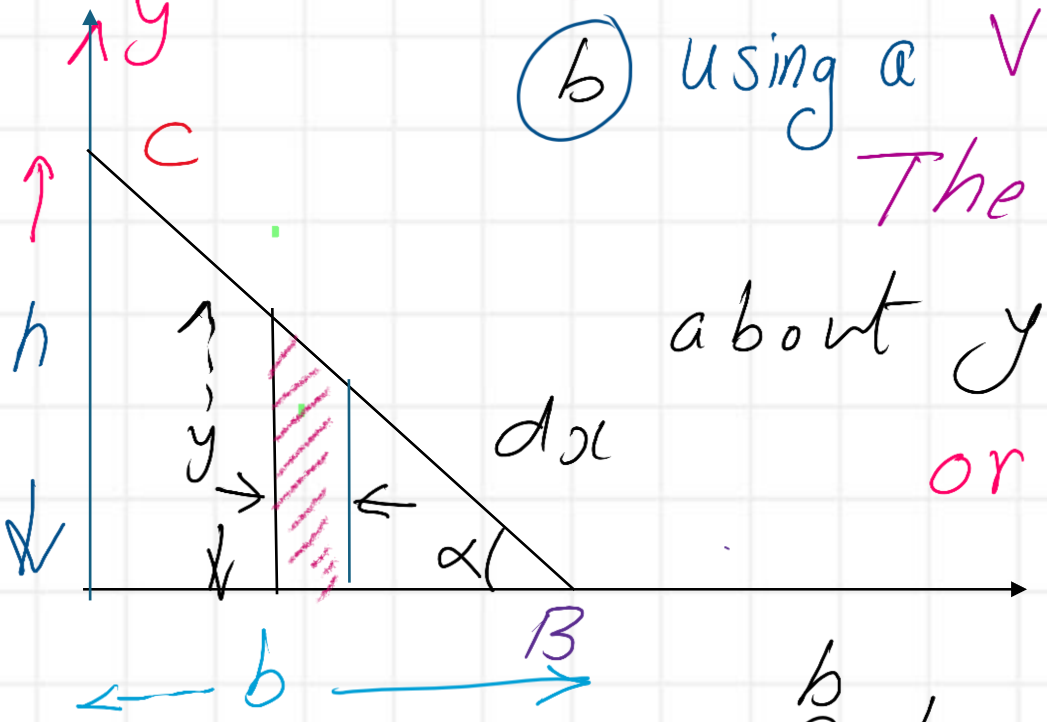
$\bar{x}$ bar for a right angle triangle Case # 1

(b) using a vertical strip

The First moment of area

about y-axis $\rightarrow$ $dA \cdot x = y\, dx \cdot x$

or $= [-\frac{hx}{b} + h)(dx)(x)$



$$A \cdot \bar{x} = \int_0^b h x\, dx - \frac{h}{b} x^2\, dx$$

$$A \cdot \bar{x} = h \frac{x^2}{2} \Big|_0^b - \frac{h}{b} \frac{x^3}{3} \Big|_0^b = \frac{h}{2} \left[b^2 - \frac{2}{3b} (b^3) \right]$$

$$A \cdot \bar{x} = \frac{h}{2} \left(b^2 - \frac{2}{3} b^2 \right) = \frac{hb^2}{6}$$

We have $A = \frac{1}{2}(b)(h)$

While $A\bar{x} = \frac{b^2 \cdot h}{6}$

Then $\bar{X} = \frac{b^2h}{6A} = \frac{b^2h}{6}\left(\frac{1}{\frac{1}{2}(bh)}\right) = \frac{b}{3}$

C.g horizontal distance is apart by one third of the base from the external way axis. {by using a vertical strip}