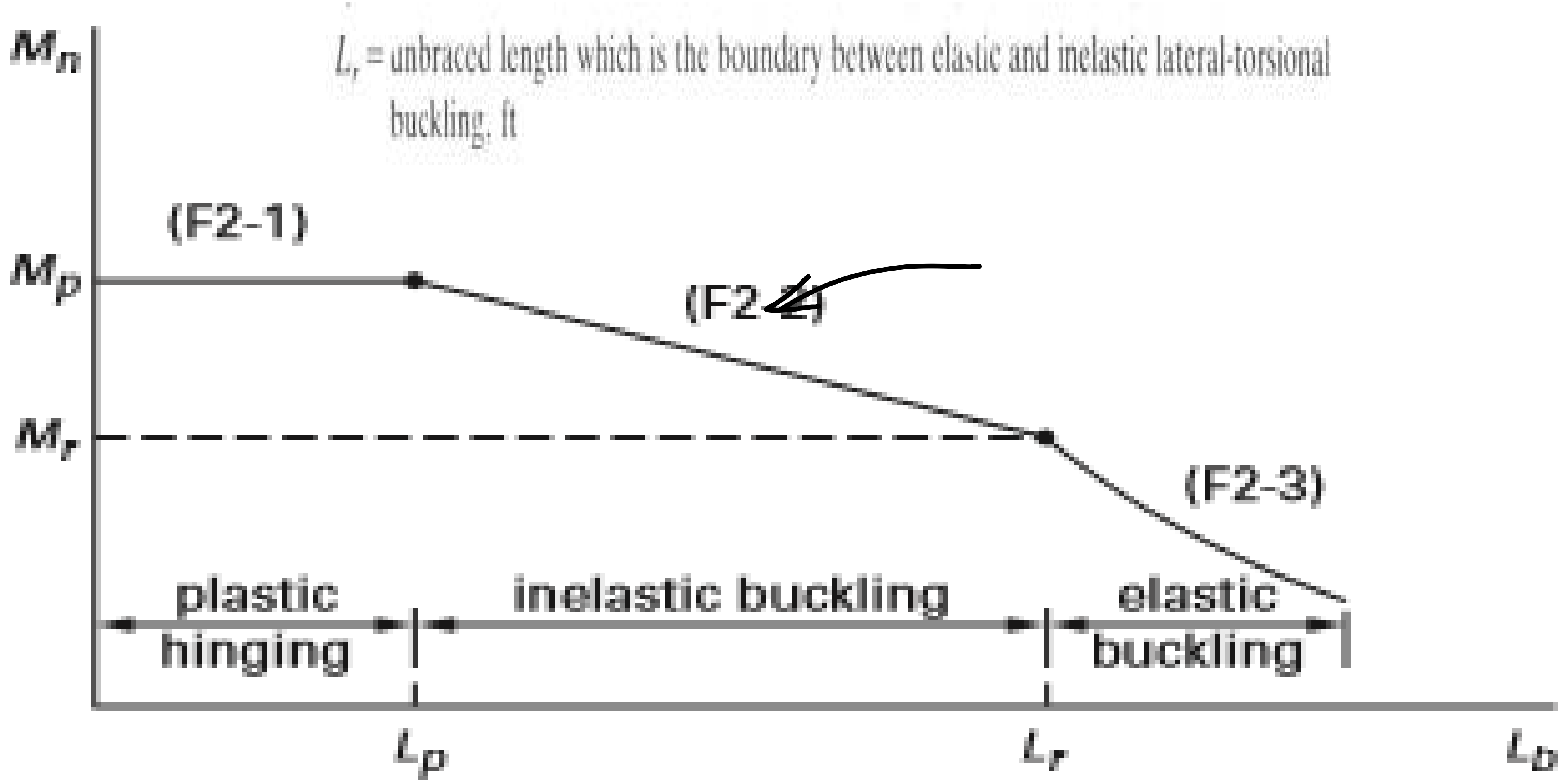

L_r = unbraced length which is the boundary between elastic and inelastic lateral-torsional buckling, ft
M_n
M_p
M_r
(F2-1)
(F2-2)
(F2-3)
plastic hinging
inelastic buckling
elastic buckling
L_p
L_r
L_b

Example 4.6

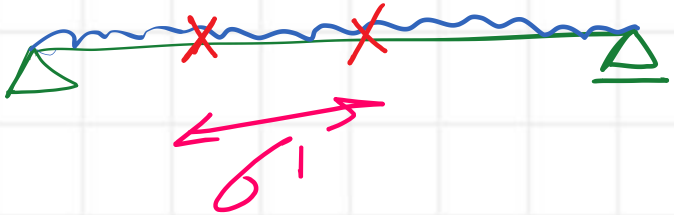
Case of inelastic buckling

when $L_b > L_p$
$< L_r$

A simply supported W16 × 40 beam of grade 50 steel is laterally braced at 6 ft intervals and is subjected to a uniform bending moment with $C_b = 1.0$. Determine the available flexural strength of the beam.

Solution

Analysis of a given steel beam



W16×40

Grade 50 $\Rightarrow$ $F_y = 50$ ksi

$C_b = 1$

Check $L_b > L_p$

$L_p \Rightarrow$ Table 1-1 $\Rightarrow r_y$
└ Table 3-2

$L_r \Rightarrow$ Table 1-1
3-2

or $\geq L_r$

Prepared by Eng.Maged Kamel.

Example 4.6

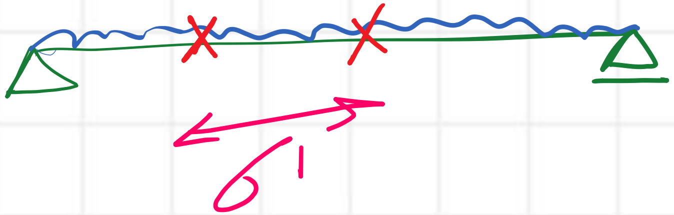
Case of inelastic buckling

when $L_b > L_p$
$< L_r$

A simply supported W16 × 40 beam of grade 50 steel is laterally braced at 6 ft intervals and is subjected to a uniform bending moment with $C_b = 1.0$. Determine the available flexural strength of the beam.

Solution

Analysis of a given steel beam



W16x40

Grade 50 $\Rightarrow F_y = 50$ ksi

$C_b = 1$

$$L_p = \frac{300}{\sqrt{F_y}} \cdot r_y = \frac{300(1.57)}{\sqrt{50}}$$

$$= 66.61'' \Rightarrow 5.55'$$

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W16 X 40

Part-2

Find

Lp, Lr

Table 1-1 (continued)
W-Shapes
Properties

W16-W14

Nom-inal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
50	5.61	37.4	659	81.0	6.68	92.0	37.2	10.5	1.59	16.3	1.89	15.7	1.52	2270
45	6.23	41.1	586	72.7	6.65	82.3	32.8	9.34	1.57	14.5	1.87	15.5	1.11	1990
40	6.93	46.5	518	64.7	6.63	73.0	28.9	8.25	1.57	12.7	1.86	15.5	0.794	1730
36	8.12	48.1	448	56.5	6.51	64.0	24.5	7.00	1.52	10.8	1.83	15.5	0.545	1460

$r_y = 1.57''$

$S_x = 64.70$ inch^3 ; $J = 0.794$ inch^4, $C_w = 1730$ in^6

$Z_x = 73.00$ inch^3 ; $h_o = 15.5''$, $r_{ts} = 1.86''$

L_r, the limiting unbraced length for the limit state of inelastic lateral-torsional buckling, in. (mm), is:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \qquad \text{(F2-6)}$$

where

$$r_{ts}^{\ 2} = \frac{\sqrt{I_y C_w}}{S_x}$$

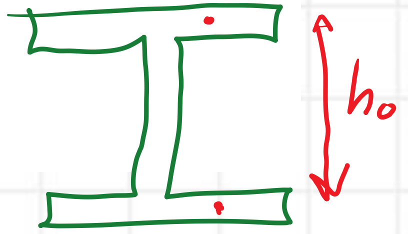
r_y = radius of gyration about y-axis, in. (mm)

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I-shapes

h_o = distance between flange centroids, in (AISC Table 1-1)



or $L_r = \pi r_{ts} \sqrt{\left(\frac{E}{0.70 f_y}\right)}$

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$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \tag{F2-6}$$

where

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \tag{F2-7}$$

and the coefficient c is determined as follows:

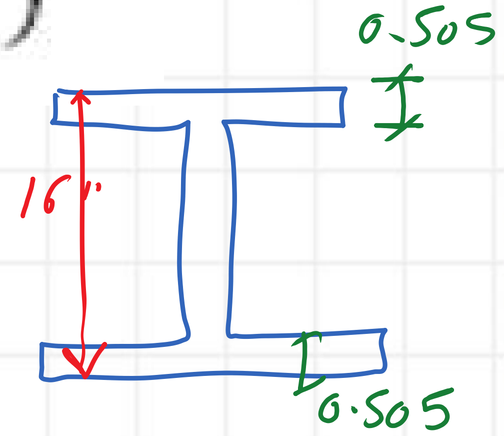
(a) For doubly symmetric I-shapes: $c = 1$ (F2-8a)

(b) For channels: $c = \frac{h_o}{2}\sqrt{\frac{I_y}{C_w}}$ (F2-8b)

W16X40 $\Rightarrow I_y = 28.90 \text{ inch}^4$, $C_w = 1730 \text{ inch}^6$, $S_x = 64.70 \text{ inch}^3$

$C = 1$, $J = 0.794$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$



$$r_{ts} = \left(\sqrt{\frac{28.90(1730)}{64.70}}\right)^{1/2} = 1.86$$

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$

$h_o = 16 - 0.505 = 15.50 \text{ inch}$

$$L_r = 1.95(1.86)\left(\frac{29\times10^3}{0.7)(50}\right)\sqrt{\frac{(0.794)(1)}{64.7(15.50)} + \sqrt{\left(\frac{0.794}{64.7(15.50)}\right)^2 + 6.76\left(\frac{0.7(50)}{29,000}\right)^2}}$$

$$= \sqrt{\frac{6.268}{10^7} + \frac{9.715}{10^6}} = 0.0032$$

$$L_r = 3005.23\sqrt{0.000792 + 0.00321} = 190.12 \text{ inch} = \frac{190.12}{12} = 15.84'$$

$L_r \approx 15.9'$ same as table

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Check Lp, Lr values From Table 3-2

W16x40

$F_y = 50$ ksi

Table 3-2 (continued)
W-Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W16×40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146
W12×50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135
W8×67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154
W14×43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125
W10×54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112

ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

Same results Can be evaluated From table

Lb = 5.55'
Lr = 15.90'

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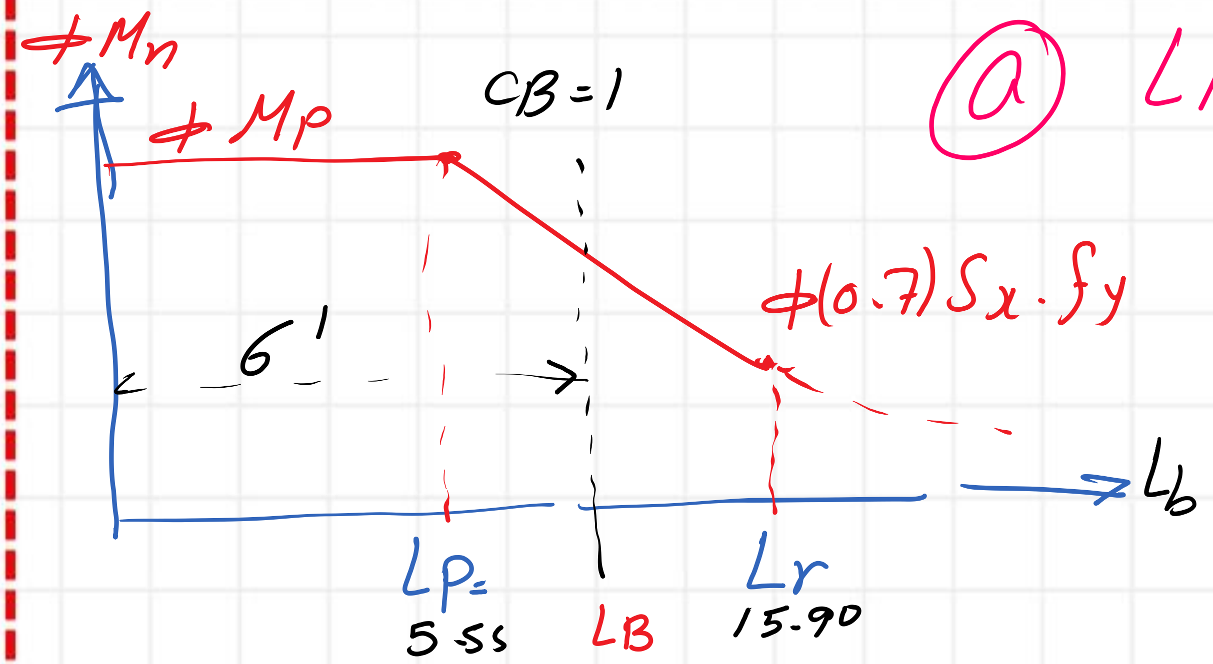

ⓐ LRFD

$Z_x = 73$ inch^3

$S_x = 64.7$ inch^3

$F_y = 50$ ksi

$L_p = 5.55'$ $L_b = 6'$

$L_r = 15.90'$

$C_b = 1, \ \phi_b = 0.90,$

$\phi_b M_p = \phi f_y \cdot Z_x = 0.90(50)(73)\left(\frac{1}{12}\right) = 273.75$ Ft-kips $\simeq 274$ Ft-kips

$\phi_b (0.70) S_x f_y = \phi_b M_{rx} = 0.90(0.70)\frac{64.70}{12}(50) = 169.84 \simeq 170$ Ft-kips

$\phi_b B_F = \frac{\phi M_p - \phi M_{rx}}{L_r - L_p} = \frac{274 - 170}{15.9 - 5.55} = 10.05 \simeq 10.00 \rightarrow$ table

$\phi_b M = 1(274) - 10(6 - 5.55) = 269.50$ Ft-kips =

Prepared by Eng.Maged Kamel.

ⓐ LRFD Design

Table 3-2 (continued)

$F_y = 50$ ksi

W-Shapes

Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W16×40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146
W12×50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135
W8×67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154
W14×43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125
W10×54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112

ASD	LRFD
$\Omega_b = 1.67$	$\phi_b = 0.90$
$\Omega_v = 1.50$	$\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

Same results can be evaluated from table

$\phi M_{px} = 274$ ft.kips

$\phi M_{rx} = 170$ ft.kips

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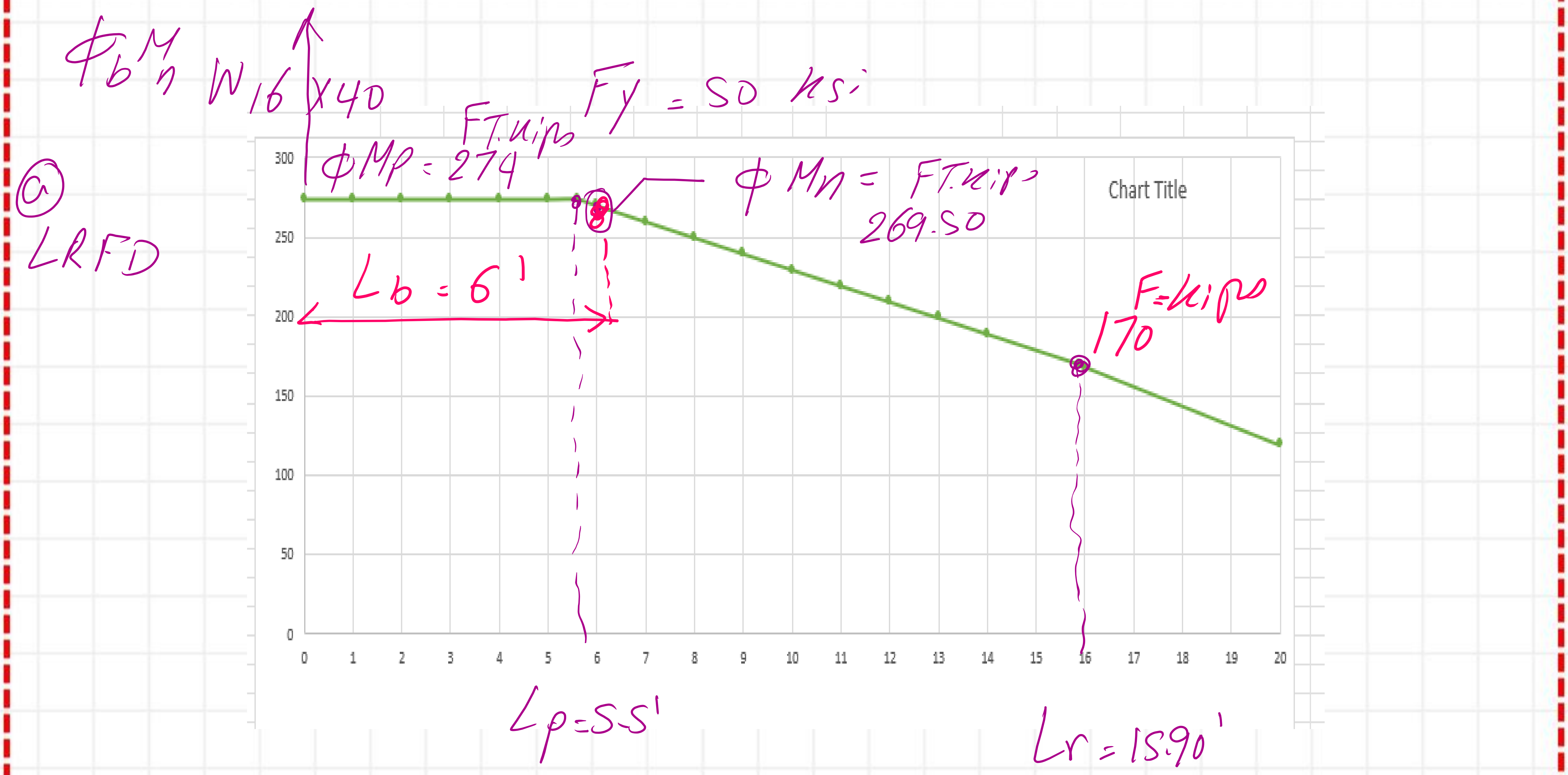

$\phi_b M_n$ W16 X40
Fy = 50 ksi
(a) LRFD
FT.kips
ϕM_p = 274
ϕM_n = FT.kips
269.50
L_b = 6'
F=kips
170
Chart Title
300
250
200
150
100
50
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
L_p = 5.5'
L_r = 15.90'

(b) ASD Design

F_y = 50 ksi

Table 3-2 (continued)
W-Shapes
Selection by Z_x

Z_x

$M_{px}/\Omega_b = 182$ kips.Ft $\Rightarrow$ $\dfrac{M_{rx}}{\Omega_b} = 113$ kips-Ft

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W16×40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146
W12×50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135
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W14×43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125
W10×54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112

ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.

Prepared by Eng.Maged Kamel.

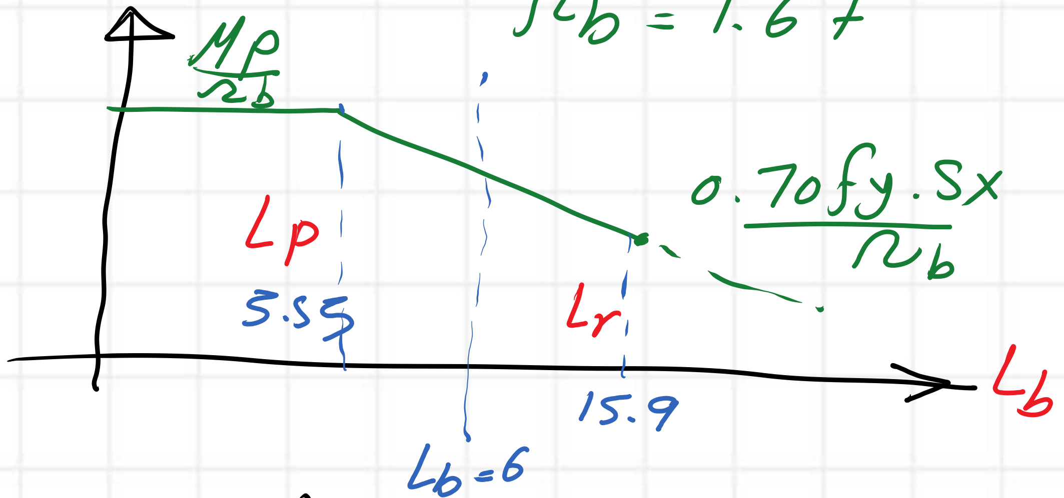
Case of ASD

$M_{Total} = 180$ Ft-kips $\rightarrow$ $Z_x = 73.00 \text{ inch}^3$

$S_x = 64.70 \text{ inch}^3$

$\Omega b = 1.67$

W16 X 40



$Lp = 5.55'$

$Lb = 6'$

$\frac{Mp}{\Omega b} = \frac{fy\, Zx}{\Omega b} = \frac{50(73)}{12(1.67)} = 182.14$ Ft-kips $\simeq 182$ FT.kips

$\frac{M}{\Omega} = \frac{Mp}{\Omega b} - \frac{BF}{\Omega b}(Lb - Lp)$

$\frac{0.70 fy\, Sx}{\Omega b} = \frac{0.7(50)64.70)}{1.67(12)} = 113.00$ Ft-kips

$\frac{BF}{\Omega} = \frac{(182.14 - 113.00}{(15.9) - 5.55)} = 6.68$

table

$\Rightarrow M = 1(182.14) - 6.68(6 - 5.55)$

$= 179.00$ Ft-Kips allowable flextural strength

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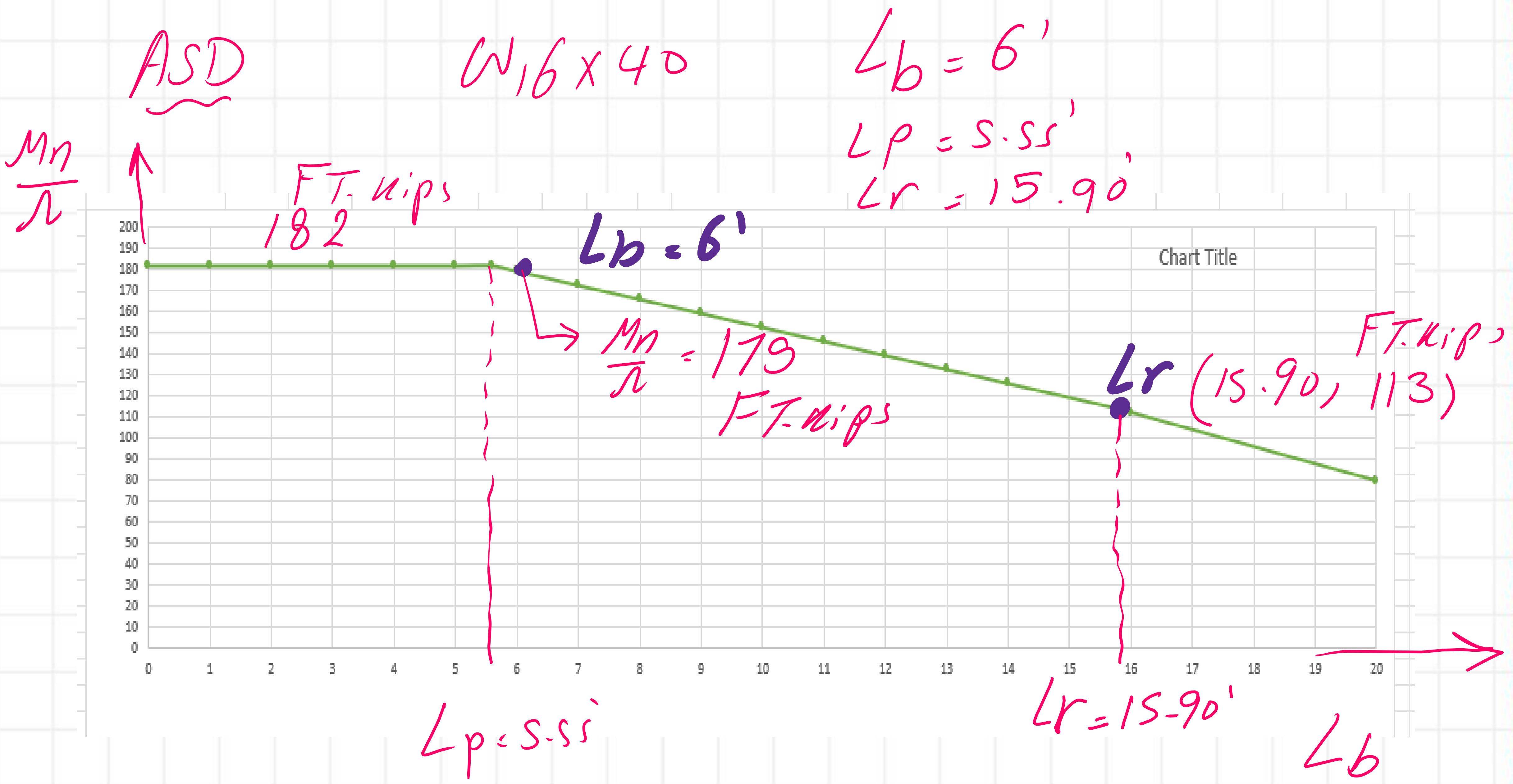

ASD
W16x40
Lb = 6'
Lp = 5.55'
Lr = 15.90'
Mn/Ω
FT. kips
182
Lb = 6'
Mn/Ω = 179 FT-kips
Chart Title
Lr (15.90, 113) FT.kips
Lp = 5.55'
Lr = 15.90'
Lb