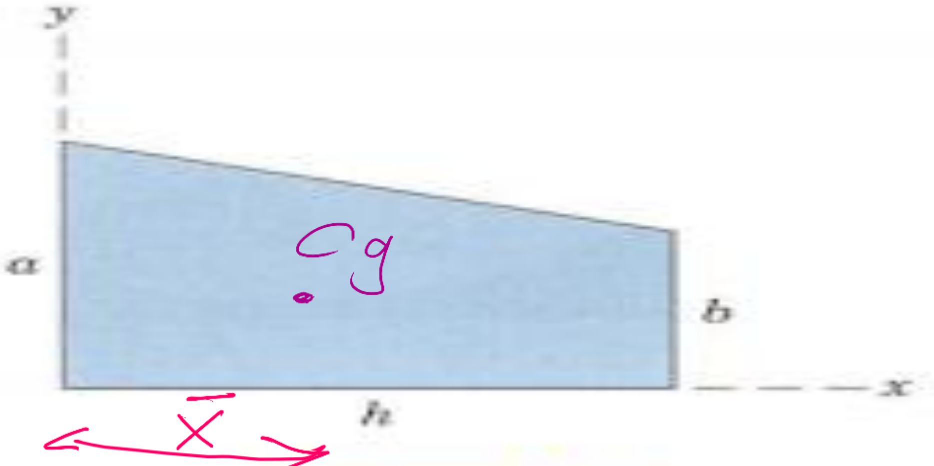
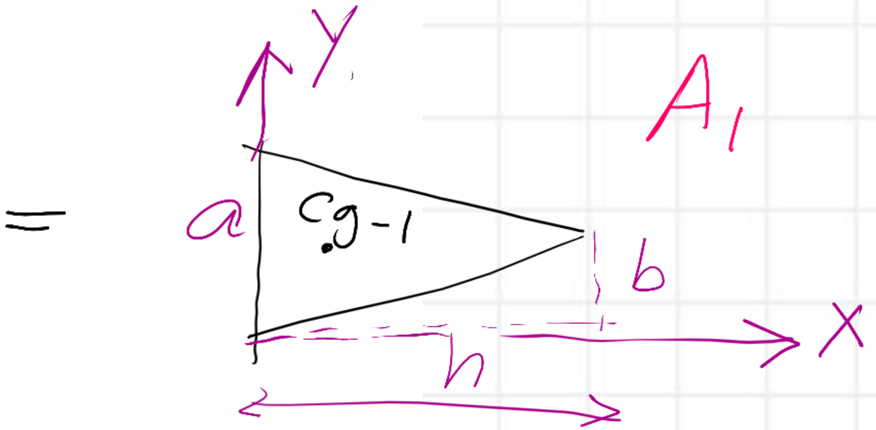
William Kraige

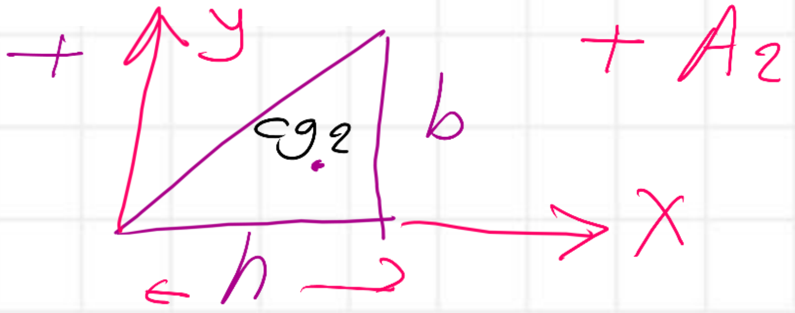
**5/7** Determine the $x$- and $y$-coordinates of the centroid of the trapezoidal area.

$$Ans.\ \bar{x} = \frac{h(a + 2b)}{3(a + b)}, \bar{y} = \frac{a^2 + ab + b^2}{3(a + b)}$$




**Problem 5/7**




Solution : Consider that Trapezoidal area Consists of two areas $A_1 + A_2$

$(x_{c.g})_1 = \frac{h}{3}$ $\qquad A_1 = \frac{1}{2} a h$

$(x_{c.g})_2 = \frac{2}{3} h$ $\qquad A_2 = \frac{1}{2} h b$

**Prepared by Eng.Maged Kamel.**

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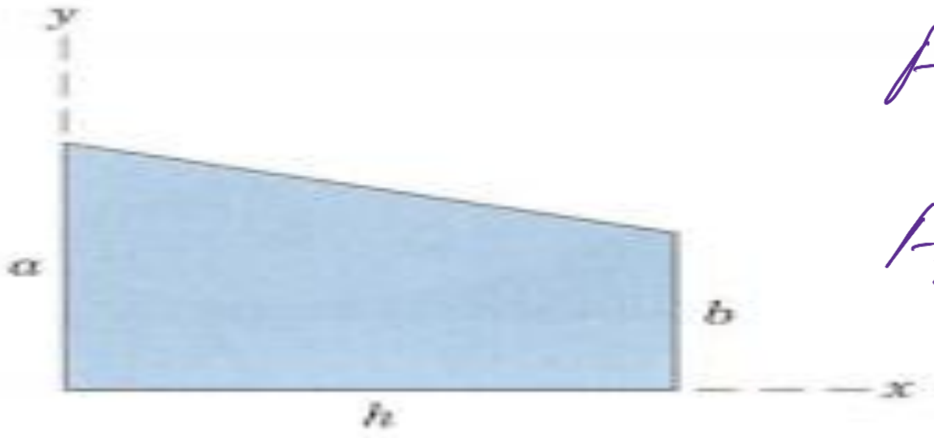


**Problem 5/7**

$A_1 = \frac{1}{2} a h \ , (x_{cg})_1 = \frac{h}{3}$

$A_2 = \frac{1}{2} b h \ , (x_{cg})_2 = \frac{2}{3} h$

But $A = A_1 + A_2 = \frac{1}{2} a h + \frac{1}{2} b h = \frac{h}{2}(a + b)$

Use $x_{c.g} = A_1 (x_{cg})_1 + A_2 (x_{cg})_2 / A_1 + A_2$

$A_1 x_{c.g_1} + A_2 x_{cg_2} = \frac{1}{2} a h (h/3) + \frac{bh}{2}\left(\frac{2}{3} h\right)$

$= \frac{ah^2}{6} + \frac{bh^2}{3} = \frac{h^2}{6}(a + 2b)$

Prepared by Eng.Maged Kamel.

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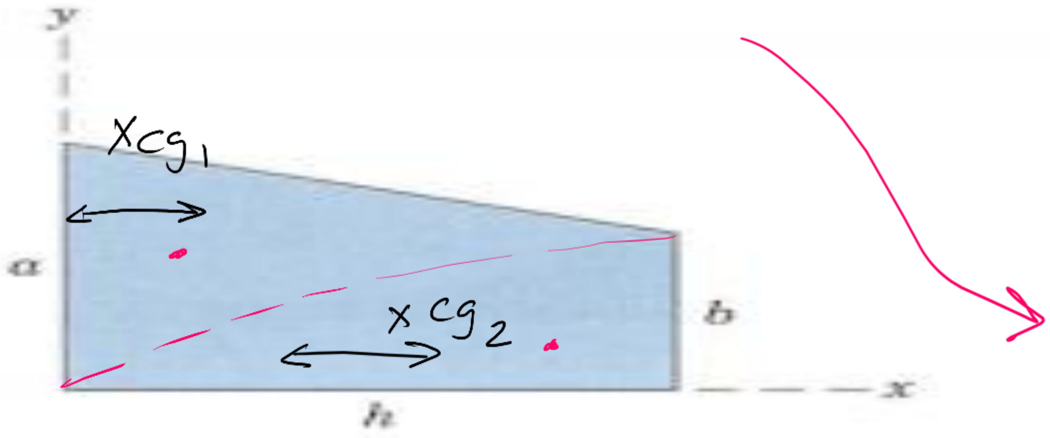


**Problem 5/7**

$$\bar{X} = \frac{h^2(a+2b)/6}{\frac{h}{2}(a+b)} = \frac{h(a+2b)2}{6(a+b)} = \frac{h(a+2b)}{3(a+b)}$$

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William Kraige

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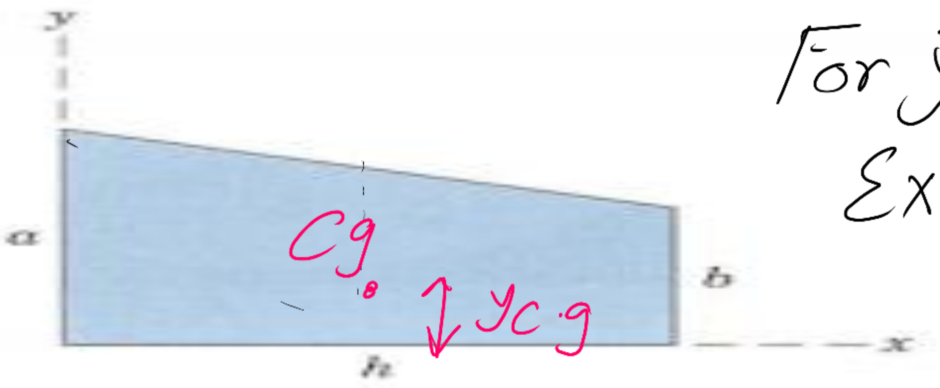
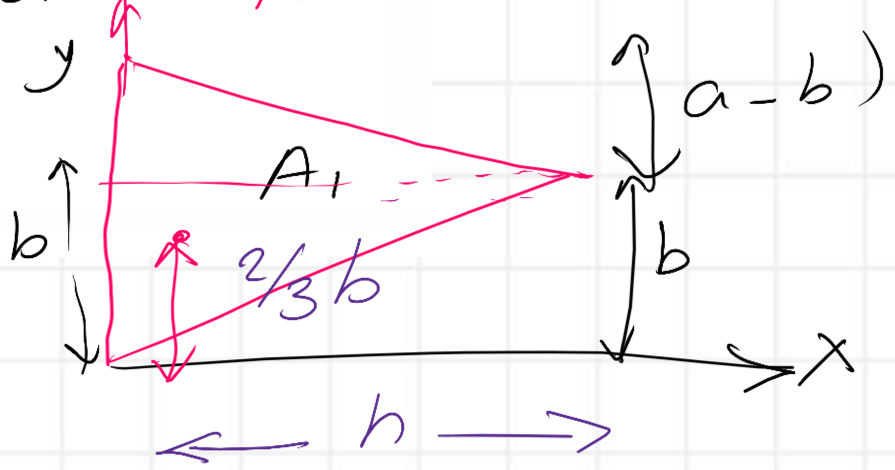
$$Ans.\ \bar{x} = \frac{h(a+2b)}{3(a+b)},\ \bar{y} = \frac{a^2+ab+b^2}{3(a+b)}$$




**Problem 5/7**

For $\bar{y}$ part

Examine First area $A_1$




Solution : $A_1 = \frac{1}{2}bh + \frac{1}{2}(a-b)h$

$A_1 = \frac{1}{2}ah$

$y_{cg_1} = \frac{2}{3}b,\quad y_{c.g_2} = b + \frac{1}{3}(a-b) = \frac{2}{3}b + \frac{a}{3} = \frac{2b+a}{3}$

$\Sigma Ay = \frac{1}{2}bh\left(\frac{2}{3}b\right) + \frac{1}{2}(a-b)h\cdot\left(\frac{2b+a}{3}\right)$

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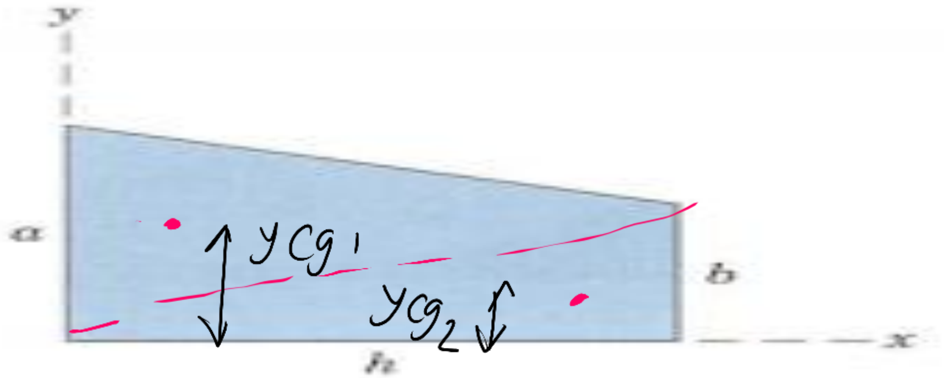
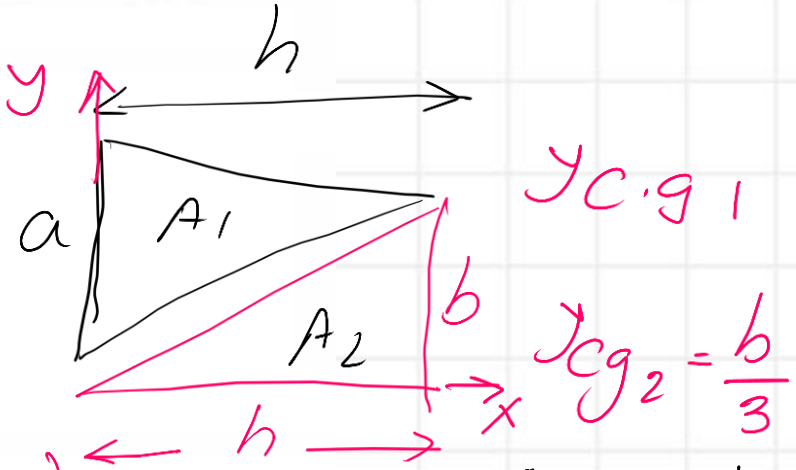



**Problem 5/7**




$$A_2 = \frac{1}{2}hb$$

$$(\sum Ay)_{A_1} = \frac{1}{2}bh\left(\frac{2}{3}b\right) + \frac{1}{2}(a-b)h.\left(\frac{2b+a}{3}\right)$$

$$= \frac{2hb^2}{6} + \frac{h}{6}(a-b)(2b+a)$$

$$= \frac{h}{6}\left(2b^2 + 2ab + a^2 - 2b^2 - ba\right)$$

$$(\sum Ay)_{A_1} = \frac{h}{6}\left(a^2 + ab\right)$$

**Prepared by Eng.Maged Kamel.**

**5/7** Determine the $x$- and $y$-coordinates of the centroid of the trapezoidal area.

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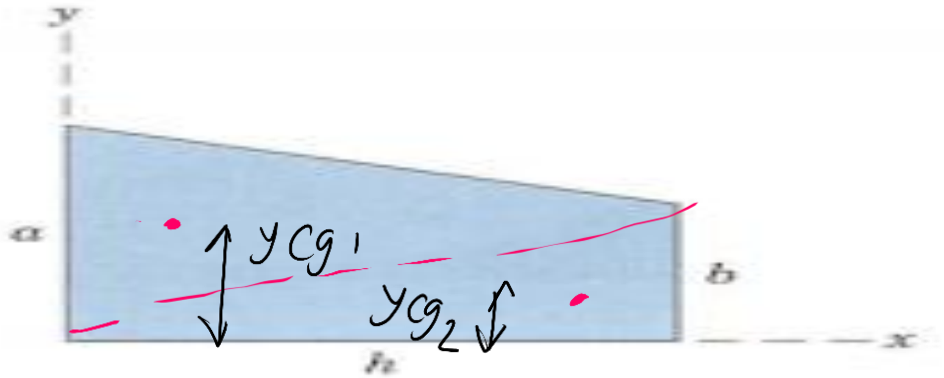
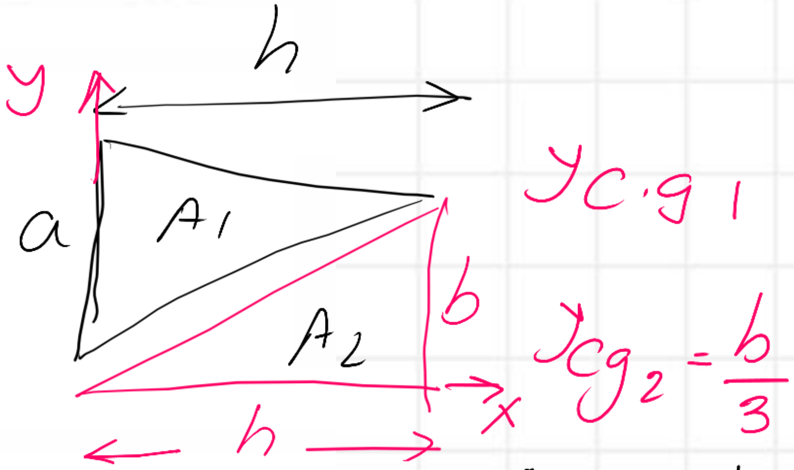



**Problem 5/7**




$$y_{cg_2} = \frac{b}{3}$$

$$A_2 = \frac{1}{2} h b$$

$$\Sigma(Ay)_{A_1} = \frac{h}{6}(a^2 + ab)$$

$$\Sigma(Ay)_{A_2} = \frac{1}{2} h b\left(\frac{b}{3}\right) = \frac{h}{6}(b^2)$$

$$y_{c.g} = \frac{h}{6}(a^2 + ab + b^2) \Big/ \frac{1}{2}(ah + bh)$$

$$y_{cg} = \frac{a^2 + ab + b^2}{3(a+b)}$$

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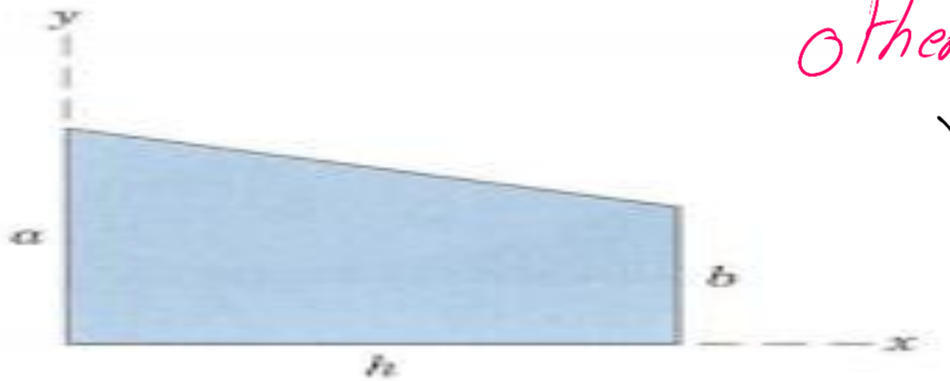



**Problem 5/7**

$Y_{cg}$

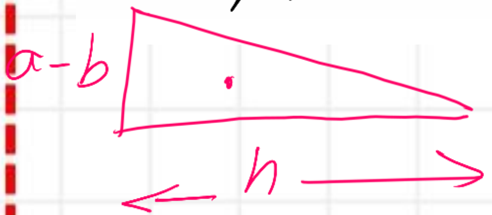
other alternative

$\gg$ $A_1$: Triangle

$A_2$ rectangle

$y_{cg_1} = \frac{a-b}{3}$

$A_1$

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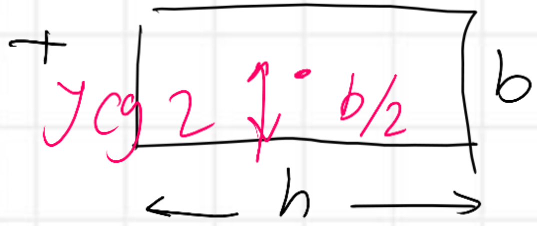


$$\Sigma A y = \frac{1}{2}(a-b)h\left(b+\frac{1}{3}(a-b)\right) + bh\left(\frac{b}{2}\right)$$

$$= \frac{h}{2}(a-b)\left[b+\frac{a}{3}-\frac{b}{3}\right] + \frac{h}{2}b^2$$

$$= \frac{h}{2}(a-b)\left(\frac{3b+a-b}{3}\right) + \frac{h}{2}b^2$$

$$= \frac{h}{2}\frac{(a-b)(2b+a)}{3} + \frac{h}{2}b^2$$

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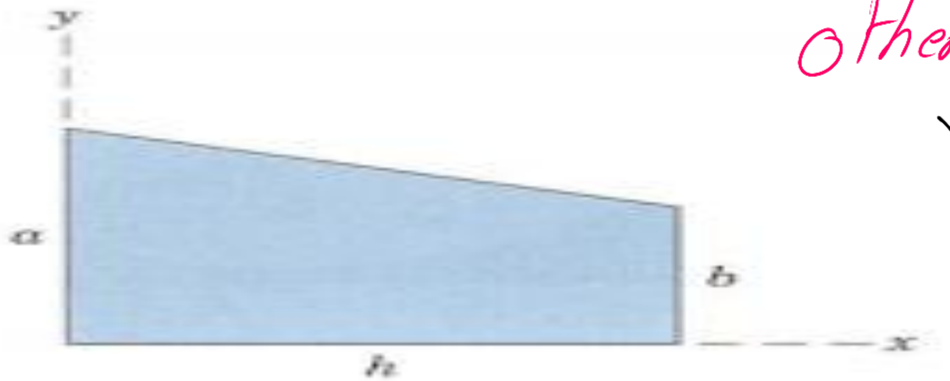
Ycg

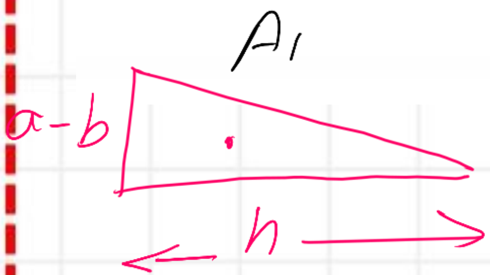



**Problem 5/7**

other alternative

⋙ $A_1$ : Triangle

$A_2$ rectangle

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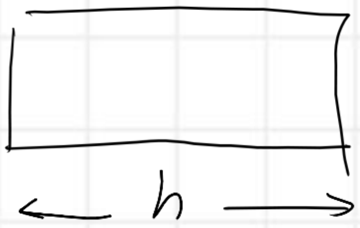


$$\sum Ay = \frac{h}{2}\frac{(a-b)}{3}(2b+a) + \frac{h}{2}b^2\left(\frac{3}{3}\right)$$

$$= \frac{h}{6}(2ab+a^2-2b^2-ba) + \frac{h}{6}(3b^2)$$

$$= \frac{h}{6}(a^2+ba-2b^2+3b^2) = \frac{h}{6}(a^2+ab+b^2)$$

$$\sum A = \frac{(a-b)}{2}h + bh = \frac{h}{2}(a+b)$$

$$Ycg = (a^2+ab+b^2)/3(a+b)$$

**Prepared by Eng.Maged Kamel.**