Elementary matrices: Convert A to U

Pivot

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Zeros

E_1: multiplier $-\frac{a_{21}}{a_{11}} R_1 + R_2$

multiplier $= -\frac{a_{31}}{a_{11}} R_2 + R_3$

$-\frac{a_{21}}{a_{11}} R_1 = -4(1) + 0 = -4$

$-\frac{a_{31}}{a_{11}} R_1 + R_3 = (-3)(1) + 0 = -3$

$$\underset{E_1}{\begin{bmatrix} 1 & 0 & 0 \\ (-4) & 1 & 0 \\ (-3) & 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}} = \underset{E_1 A}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & \boxed{-1} & -5 \\ 0 & 2 & 0 \end{bmatrix}}$$

$$\Rightarrow (E_3 E_2 E_1) A \Rightarrow U$$

$$E_1 A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 2 & 0 \end{bmatrix}$$

Use a_{32} as a pivot

$$E_2 \Rightarrow -\frac{a_{32}}{a_{21}} \frac{R_2 + R_3}{R_3} = \frac{-2}{-1} = +2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \xrightarrow{R_2} \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2(0) & +2(1)+0 & +2(0)+1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \Rightarrow E2$$

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$$E_2 \cdot (E_1 A)$$

still not U

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 2 & 0 \end{bmatrix}}_{E_1 A} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & \boxed{-1} & -5 \\ 0 & 0 & \boxed{-10} \end{bmatrix}}_{E_2 E_1 A}$$

$L_{33} = -10$

$$\underbrace{\begin{bmatrix} (1) & 0 & 0 \\ 0 & (-1) & 0 \\ 0 & 0 & (-\frac{1}{10}) \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & -10 \end{bmatrix}}_{E_2 E_1 A} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}}_{U \text{ Final}}$$

$$E_3\,E_2\,E_1\,A = U$$

$$(E_3\,E_2\,E_1)\,L\,U = U$$

How can we get L^{-1}?

$$E_3\,E_2\,E_1\,L = I$$

Then

$$E_3\,E_2\,E_1 = L^{-1}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\frac{1}{10} \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}}_{E_1}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\frac{1}{10} \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}}_{E_1} \quad L^{-1}$$

$$E_3 \cdot E_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -\frac{2}{10} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ -3 & -\frac{2}{10} & -\frac{1}{10} \end{bmatrix}$$

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Check $L L^{-1} = I$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ -3 & -\frac{2}{10} & -\frac{1}{10} \end{pmatrix}}_{L^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$$