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First Solved problem

Find L & U values using option - 2

Turn A → upper matrix - U

[Use row operation

[Use Elementary matrices

Find inverse of L and check

$LU = A$ & $L^{-1}L = I$

First Example

Write the lower and upper matrix for $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ Crout's method

Option - 2

Solution: Convert A → an upper matrix

$\begin{bmatrix} 1 & ? & ? \\ & 1 & ? \\ & & 1 \end{bmatrix}$

$|A| = +10$ Why?

Check if matrix is invertible, estimate the determinant value.

$$|A| = +1\begin{vmatrix} 3 & -1 \\ 5 & 3 \end{vmatrix} - 1\begin{vmatrix} 4 & -1 \\ 3 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix}$$

$$= +1(9+5) - 1(12+3) + (20-9)$$

$$= +14 - 15 + 11$$

$$= +10$$

$$A = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \cdot \begin{bmatrix} +1 & U_{12} & U_{13} \\ 0 & +1 & U_{23} \\ 0 & 0 & +1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{22}+L_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

Compare the first column of A and LU

$L_{11} = a_{11} = 1$

$L_{21} = a_{21} = 4$

$L_{31} = a_{31} = 3$

Compare the first row of A and LU

$$L_{11} = 1$$

$$L_{11} U_{12} = 1 \Rightarrow L_{11} = U_{12} = \frac{1}{1} = +1$$

$$L_{11} U_{13} \Rightarrow U_{13} = +1$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{22}+L_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

By definition

$$U_{11} = U_{22} = U_{33} = 1$$

$$\left\{ \begin{array}{l} L_{11} = a_{11} = 1 \\ L_{21} = a_{21} = 4 \\ L_{31} = a_{31} = 3 \end{array} \right\}$$

Check what do we have ?

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & ? & 0 \\ 3 & ? & ? \end{bmatrix} \quad L_{22} ? \;\&\; L_{32} = ? \quad L_{33} ?$$

$$\begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & ? \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow U_{23} \; ??$$

Find L22 and L32

Elementary matrix E_1

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow$$

Use a_{11} as a pivot

$-\frac{a_{21}}{a_{11}} = \frac{-4}{1} = -4 \rightarrow$ multiply by $\frac{R_1+R_2}{R_2}$

$-\frac{a_{31}}{a_{11}} = \frac{-3}{1} = -3$ multiply by $\frac{R_1+R_2}{R_2}$

A

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & +2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & L_{32} & L_{32}U_{23}+L_{33} \end{bmatrix}$$

A_1

same

$L_{22} = -1$

$L_{32} = +2$

Find U23

$$\Rightarrow \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

Elementary matrix E2

A_1

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & +2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & L_{32} & L_{32}U_{23}+L_{33} \end{bmatrix}$$

R_2

same

$L_{22} = -1$
$L_{32} = +2$

$$\left(\frac{a_{23}}{a_{22}}\right)_{A_1} = \frac{L_{22}U_{23}}{L_{22}} = U_{23} = -5/-1 = +5$$

U matrix is Complete $\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Find L33

$\Rightarrow$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

$L_{11} = 1$ & $L_{31} = 3$

$L_{22} = 1$

A_1

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & +2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & L_{32} & L_{32}U_{23}+L_{33} \end{bmatrix}$$

R_2

same

Use L_{22} as a pivot $\rightarrow \frac{L_{32}}{L_{22}}(-1) = \left(\frac{2}{-1}\right)(-1) = +2$

$-2R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & -10+0 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & 0 & L_{33} \end{bmatrix}$$

To get U matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & -10 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ 0 & L_{22} & L_{22}U_{23} \\ 0 & 0 & L_{33} \end{bmatrix}$$

$1/L_{11} \Rightarrow [1/1]$

$1/L_{22} \Rightarrow [1/-1]$

$1/L_{33} \hookrightarrow [1/-10]$

For Elementary operation E_3

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & +1 & +5 \\ 0 & 0 & +1 \end{bmatrix} \Rightarrow \text{U matrix}$$

$L_{33} = -10 \Rightarrow$ L matrix is Complete

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Check $L\,U \Rightarrow A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow \text{Same given } A$$