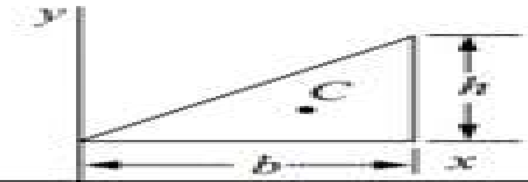
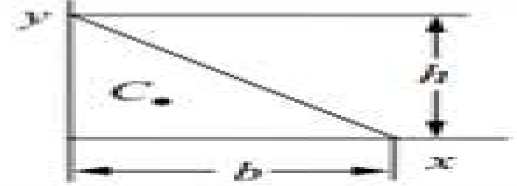
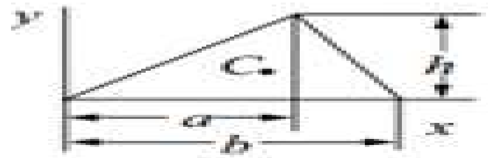
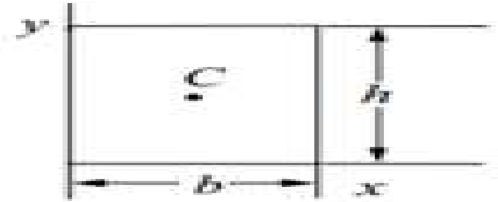
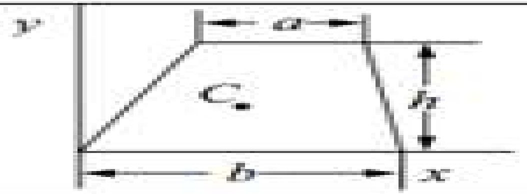
FE Reference Handbook 10.0

Figure	Area & Centroid
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$
	$A = bh/2$ $x_c = (a + b)/3$ $y_c = h/3$
	$A = bh$ $x_c = b/2$ $y_c = h/2$
	$A = h(a + b)/2$ $y_c = \dfrac{h(2a + b)}{3(a + b)}$

Area and C.g of Trapezium

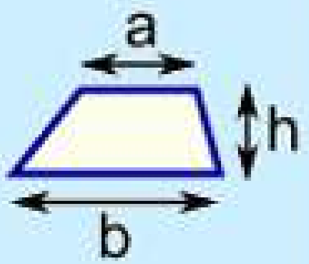
$\bar{x}$ and area.
$\bar{y}$

← Trapezium

$$y_c = \frac{h}{3}\frac{(2a+b)}{(a+b)}$$

Trapezium

Area and c.g.



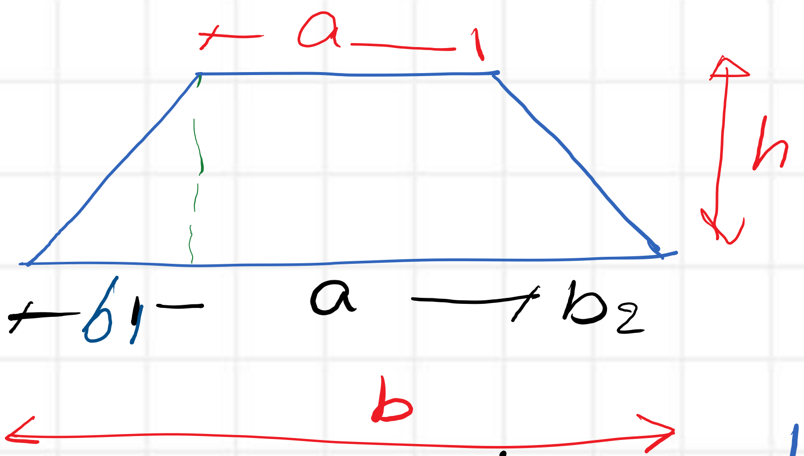
Trapezoid (US)

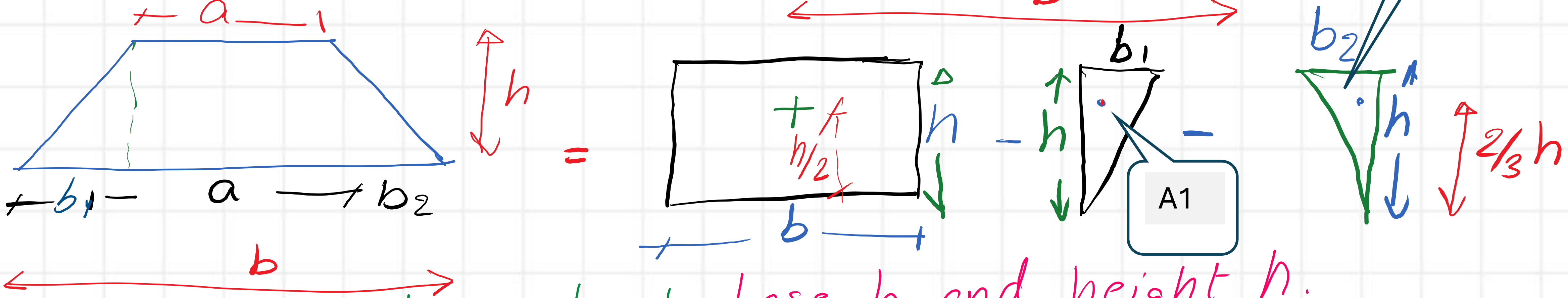
Trapezium (UK)

Area = ½(a+b) × h

h = vertical height

How to estimate the area?

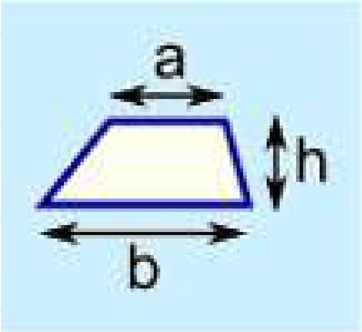




Trapezium as a big rectangle base b and height h. then deduct two triangles of A_1 and A_2

Trapezium x_bar

How to estimate the X bar?

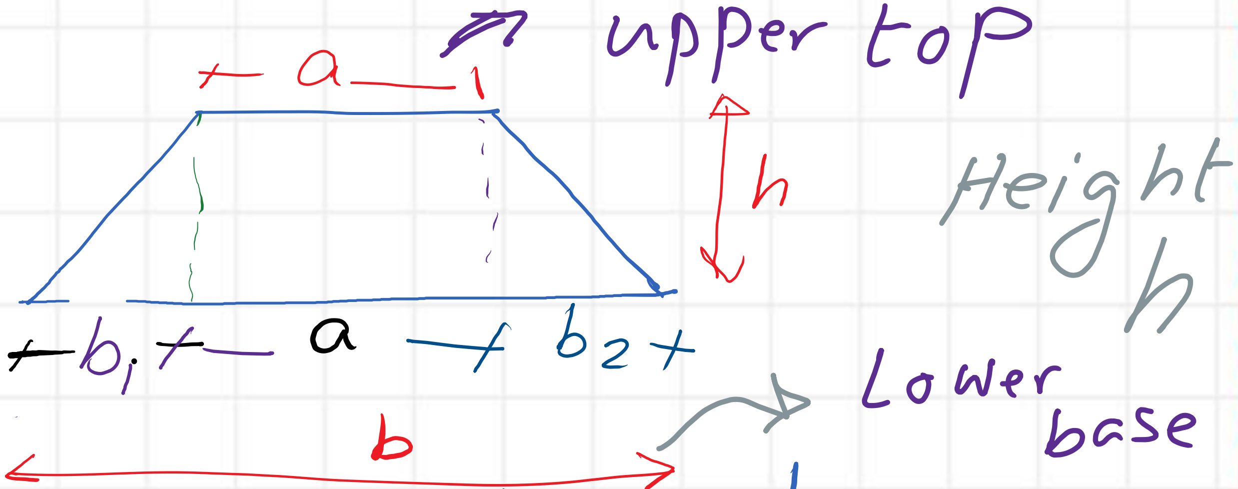


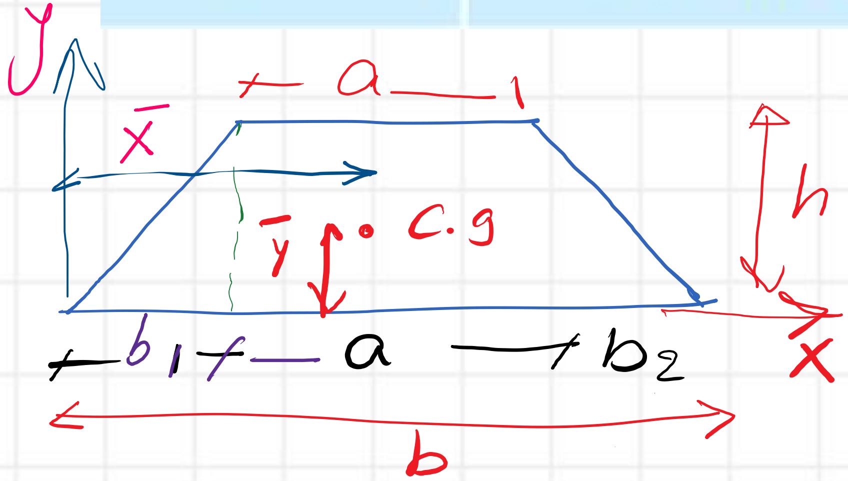
Trapezoid (US)

Trapezium (UK)

Area = ½(a+b) × h

h = vertical height





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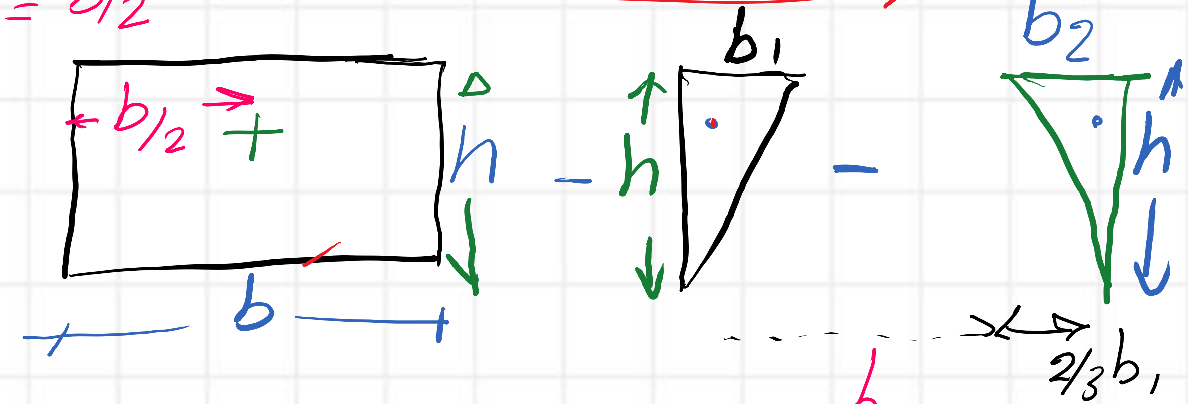
$x_1 = b/2$



$x_2 = \frac{b_1}{3}$

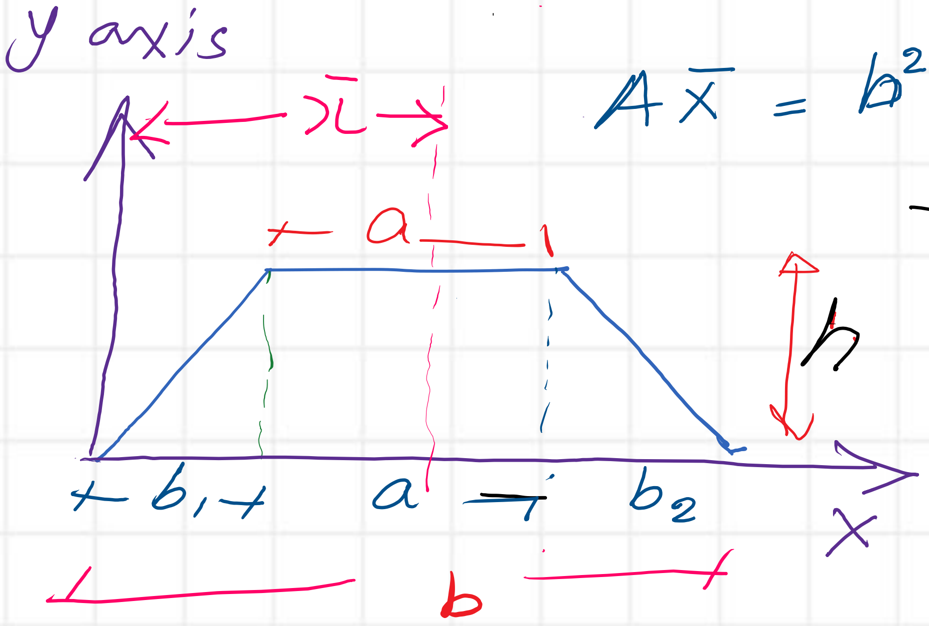
$x_3 = b_1 + a + \frac{b_2}{3}(2)$

$A \cdot x_{bar} =$

rectangle	First triangle	Second triangle
A*x bar = A1*x1	- A2*x2	-A3*x3

First moment about y-axis

$$A\bar{x} = b\cdot\left(\frac{b}{2}\right)h - \frac{1}{2}(b_1 h)\left(\frac{1}{3}b_1\right) - \frac{1}{2}(b_2 h)\left(a + b_1 + b_2\left(\frac{2}{3}\right)\right)$$



$$A\bar{x} = b^2\left(\frac{h}{2}\right) - \frac{b_1^2 h}{6} - \frac{1}{2}b_2 h a - \frac{1}{2}b_1 b_2 h - \frac{1}{3}(b_2^2)\cdot h =$$

$$A\cdot x_{bar} = \frac{h}{6}\left(3b^2 - b_1^2 - 3a(b_2) - 3(b_1)(b_2) - 2b_2^2\right)$$

$$A\bar{x} = \frac{h}{6}\left(3b^2 - b_1^2 - b_2(3a + 3b_1 - 2b_2)\right)$$

$$b_2 = b - a - b_1$$

$$A\bar{x} = \frac{h}{6}\left(3b^2 - b_1^2 - 3ab_2 - 3b_1b_2 + 2b_2^2\right)$$

$$A = \frac{1}{2}[a+b]h \Rightarrow A\bar{x} = \frac{h}{6}\left(3b^2 - b_1^2 - b_2(3a + 3b_1 + 2b - 2a - 2b_1)\right)$$

$$A\bar{x} = \frac{h}{6}\left(3b^2 - b_1^2 - b_2(a + b_1 + 2b)\right)$$

$$-b_2(a+b_1+2b) \quad \Rightarrow b_2 = b - a - b_1$$

$$-(b-a-b_1)(a+b_1+2b)$$

$$= -(\boxed{ba} + \cancel{bb_1} + 2b^2 - a^2 - ab_1 - \boxed{2ab} - b_1a - b_1^2 - \cancel{2bb_1})$$

$$= -(-ba - bb_1 + 2b^2 - a^2 - 2ab_1 - b_1^2)$$

$$A\bar{x} = \frac{h}{6}(3\cancel{b^2} - \cancel{b_1^2} + ba + bb_1 - \cancel{2b^2} + a^2 + 2ab_1 + \cancel{b_1^2})$$

$$= \frac{h}{6}(b^2 + ba + bb_1 + a^2 + 2ab_1)$$

$$A = \frac{1}{2}(a+b)h \Rightarrow \text{Area of Trapezium}$$

Simplify Expresion

$$\bar{X} = \frac{h}{6}\left(b^2 + ba + bb_1 + a^2 + 2ab_1\right) \Big/ \left(\frac{1}{2}\right)(a+b)h$$

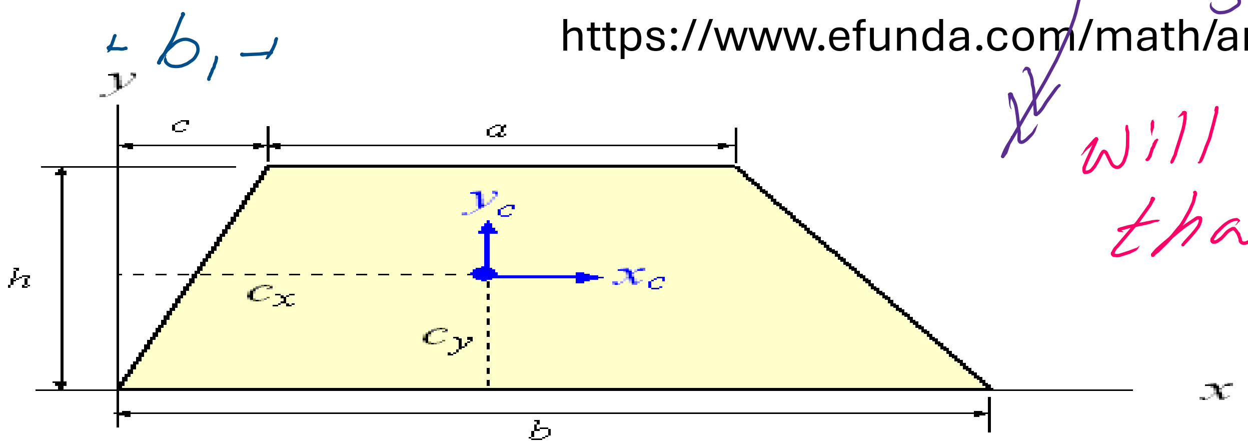
$$\bar{X} = \frac{(b^2 + ba + bb_1 + a^2 + 2ab_1)}{3(a+b)} \rightarrow \bar{X} = \frac{b^2 + ba + bc + a^2 + 2ac)}{3(a+b)}$$

if we Put $b_1 = c$

$\bar{X} = C_x$ $\rightarrow$

will match that expression C_x

https://www.efunda.com/math/areas/trapezoid.cfm



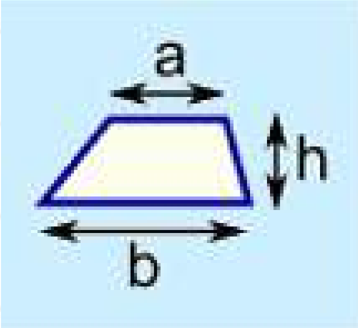
C_x	C_y	Area
$\frac{2ac + a^2 + cb + ab + b^2}{3(a+b)}$	$\frac{h(2a+b)}{3(a+b)}$	$\frac{h(a+b)}{2}$

Area and Cg.

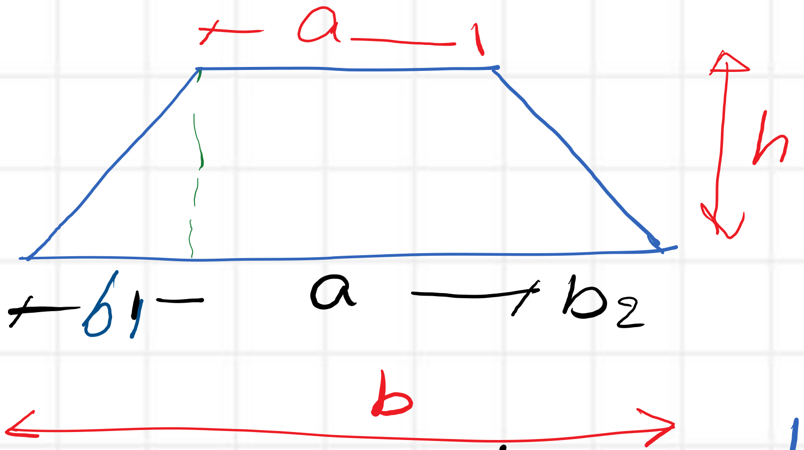
Case # 3 Trapezium.

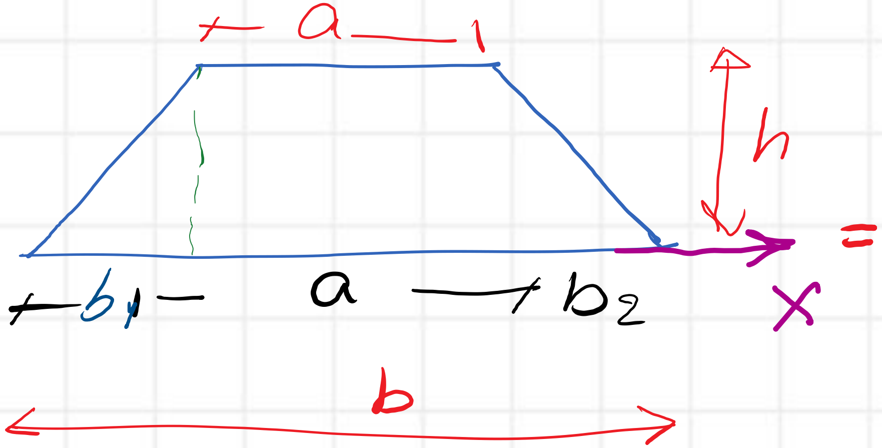
First moment of area about x–x

How to estimate the area?



[Trapezoid (US)](#)
[Trapezium (UK)](#)
Area = ½(a+b) × h
h = vertical height





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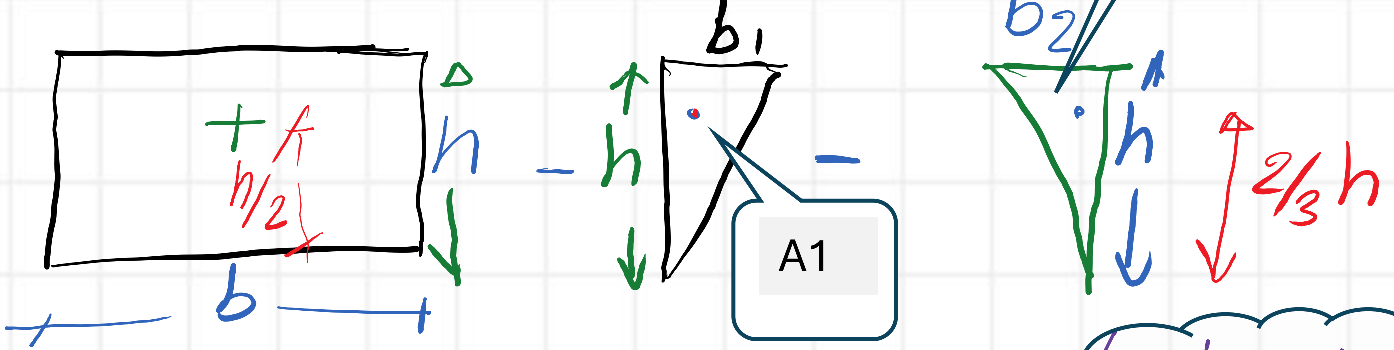
A2



+ b
b

b − b1 − b2 = a

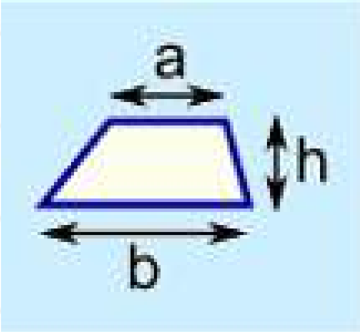
$$A_{net} = A_{Rec} - A_{1_{Tr}} - A_{2_{Tr}} = bh - \frac{1}{2} b_1 h - \frac{1}{2} b_2 h$$

$$= h\left(b - \frac{b_1}{2} - \frac{b_2}{2}\right] = \frac{h}{2}\left(2b - b_1 - b_2\right] = \frac{h}{2}\left[b + a\right]$$

Area & C.g Determination

Case #3 Trapezium

How to estimate the area?



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Trapezium (UK)
Area = ½(a+b) × h
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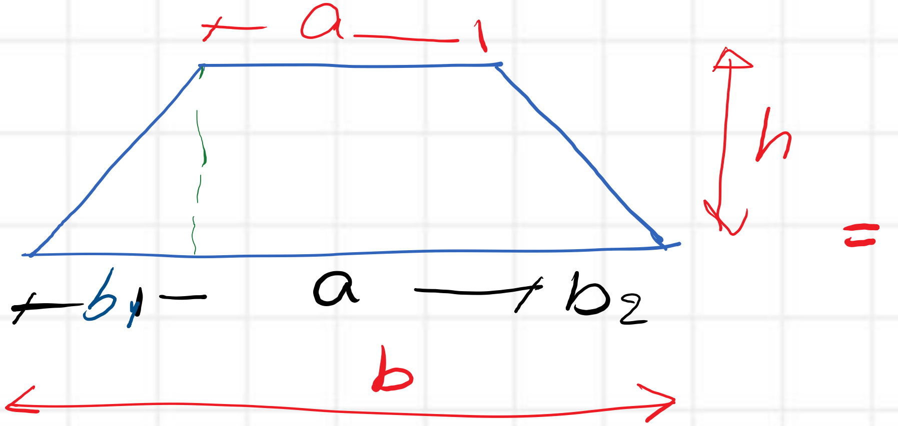
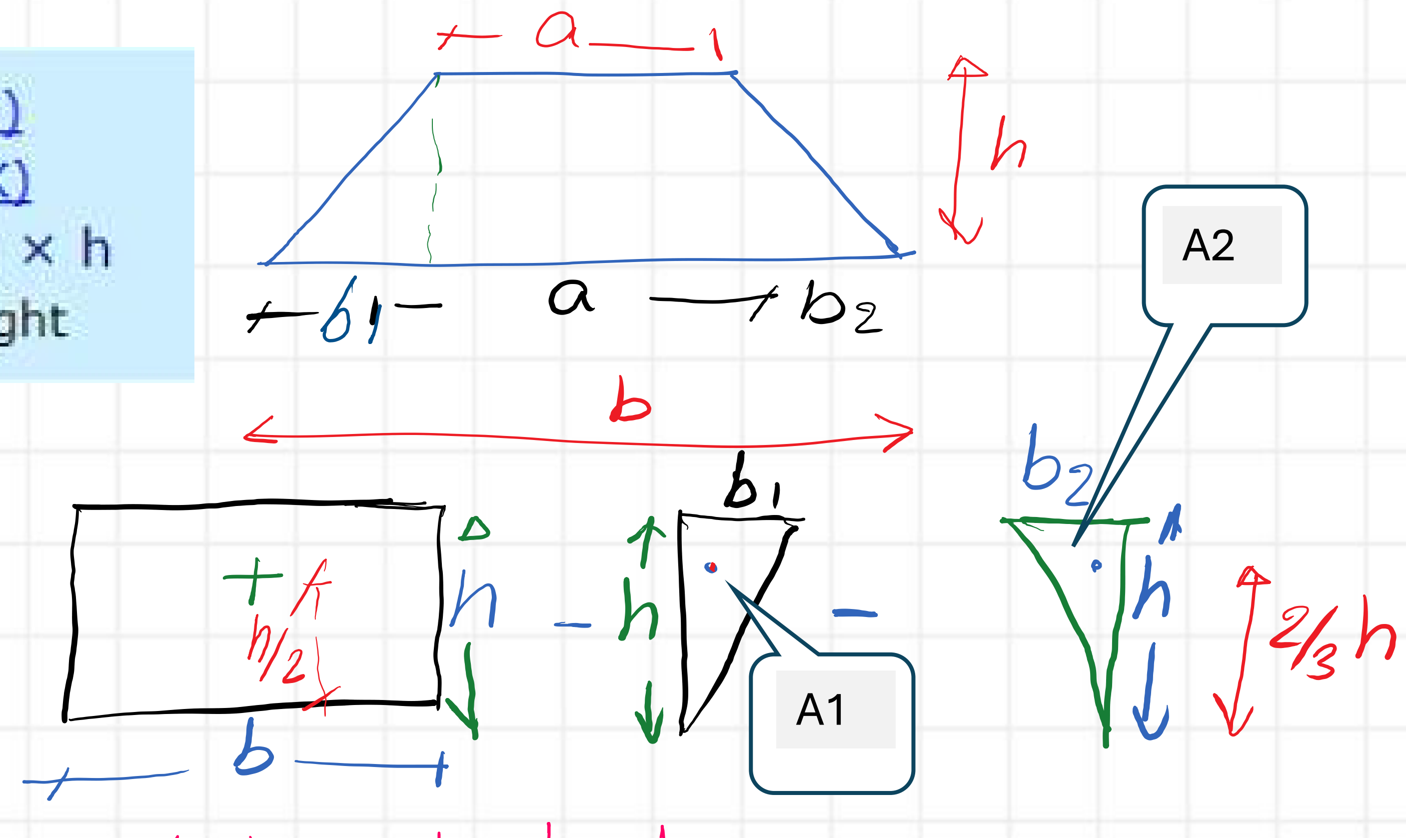
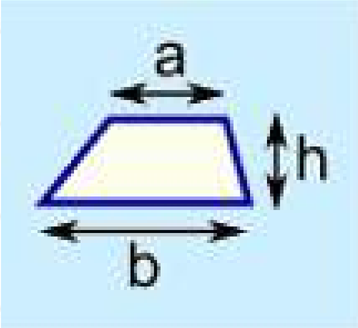




$$A_{net} = A - A_1 - A_2 = bh - \frac{1}{2} b_1 h - \frac{1}{2} b_2 h$$

$$A_{net} = h\left(b - \frac{1}{2} b_1 - \frac{1}{2} b_2\right)$$

Y- bar Estimation For Trapezoid.



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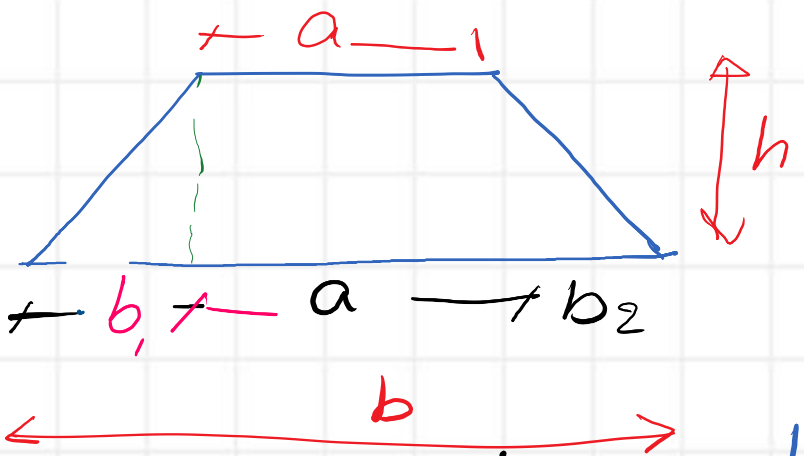
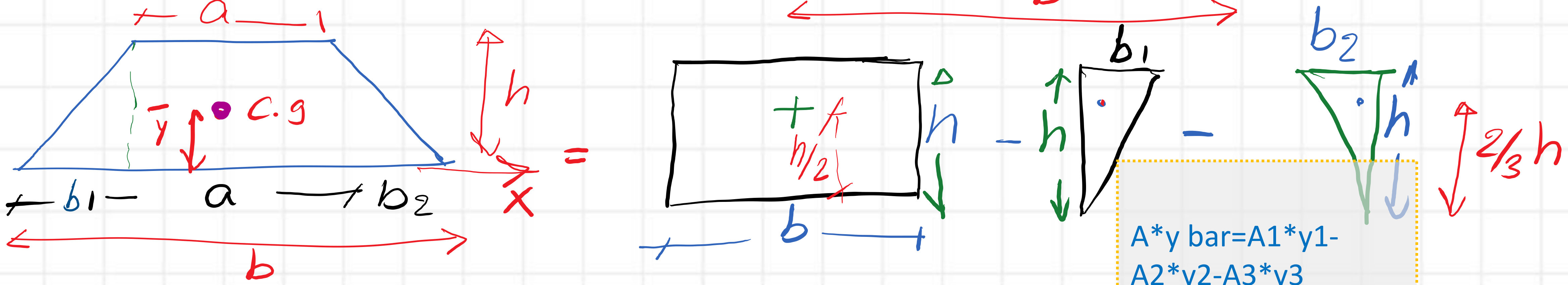




A*y bar=A1*y1-A2*y2-A3*y3

rectangle First triangle Second triangle

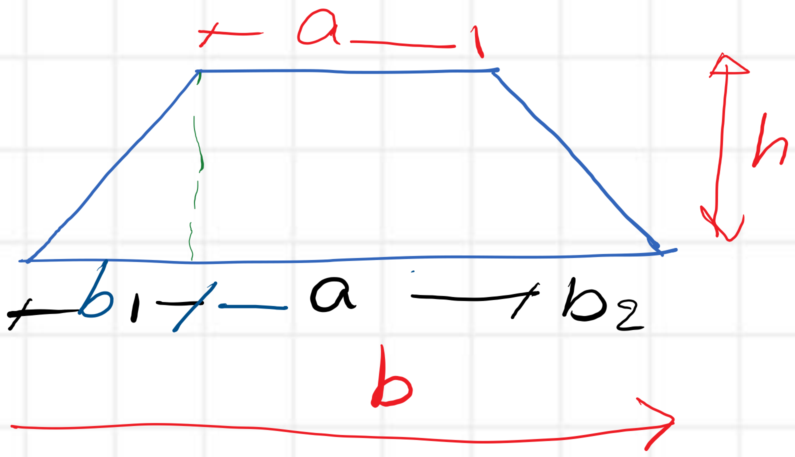
$$A\bar{y} = bh\cdot\left(\frac{h}{2}\right) - \frac{1}{2}b_1 h\left(\frac{2}{3}h\right) - \frac{1}{2}b_2 h\left(\frac{2}{3}h\right)$$

$$= \frac{1}{2}bh^2 - \frac{1}{3}b_1 h^2 - \frac{1}{3}b_2 h^2 = \frac{h^2}{2}\left[b - \frac{2}{3}b_1 - \frac{2}{3}b_2\right]$$

$$A\bar{y} = bh\cdot\left(\frac{h}{2}\right) - \frac{1}{2}b_1 h\left(\frac{2}{3}h\right) - \frac{1}{2}b_2 h\left(\frac{2}{3}h\right)$$

First moment of area about x-axis

$$= \frac{1}{2}bh^2 - \frac{1}{3}b_1 h^2 - \frac{1}{3}b_2 h^2 = \frac{h^2}{2}\left[b - \frac{2}{3}b_1 - \frac{2}{3}b_2\right]$$



$$A\bar{y} = \frac{h^2}{2}\left[\frac{3b - 2b_1 - 2b_2}{3}\right]$$

$$[b = b_1 - a - b_2] \Rightarrow 3b = 3b_1 - 3a - 3b_1$$

$$A\bar{y} = \frac{h^2}{6}\left[(3b_1 + 3a + 3b_2) - 2b_1 - 2b_2\right]$$

$$A\bar{y} = \frac{h^2}{6}\left[b_1 + 3a + b_2\right] = \frac{h^2}{6}\left[b_1 + a + 2a + b_2\right] = \frac{h^2}{6}(2a+b)$$

$= b$

$$A = \frac{1}{2}[a+b]h$$

$$\frac{A\bar{y}}{A} = \frac{h^2}{6}[2a+b]\left[\frac{2}{(a+b)\cancel{h}}\right] = \frac{1}{3}h(2a+b)/a+b$$

$\bar{y}$ bar

Final expression