Content of Post 6b

(a) How can we express permutation matrix at ij expression?

(b) Practice problems from Handbook → Introduction to Linear Algebra Book Prof. Gilbert strang

Express permutation matrix as I-J.

In Some text books

(1) Introduction to Linear Algebra Prof. Gilbert strang

Use i-j as an Expression (two numbering)

For Example for 3×3 Matrix

where i: old position of row

j: new position of row

$i > j$

For $i < j$ use multipliers

$3! = 3 \times 2 \times 1$
$= 6$

$\boxed{I}$

$1 \to 2 \to 3$

$2 \to 1 \to 3$ $\quad$ $2 \to 3 \to 1$

$1 \to 3 \to 2$

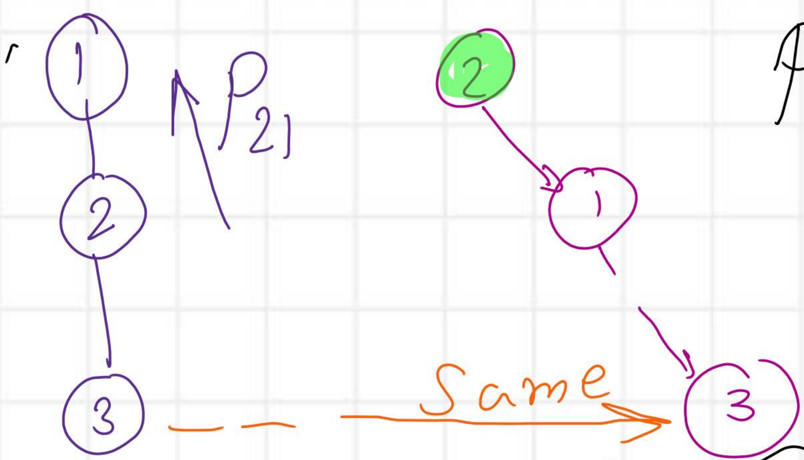
$3 \to 2 \to 1$ $\quad$ $3 \to 1 \to 2$

There are 6 types for permutation matrix as three numbers

$$I = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

C_{213}

$$P_{21} = \begin{bmatrix} & 1 & \\ 1 & & \\ & & 1 \end{bmatrix}$$



$P_{21} \overset{2>1}{=} P_{213}$

Node2

Swap $r_2 \to r_1$ in keep r_3 same position

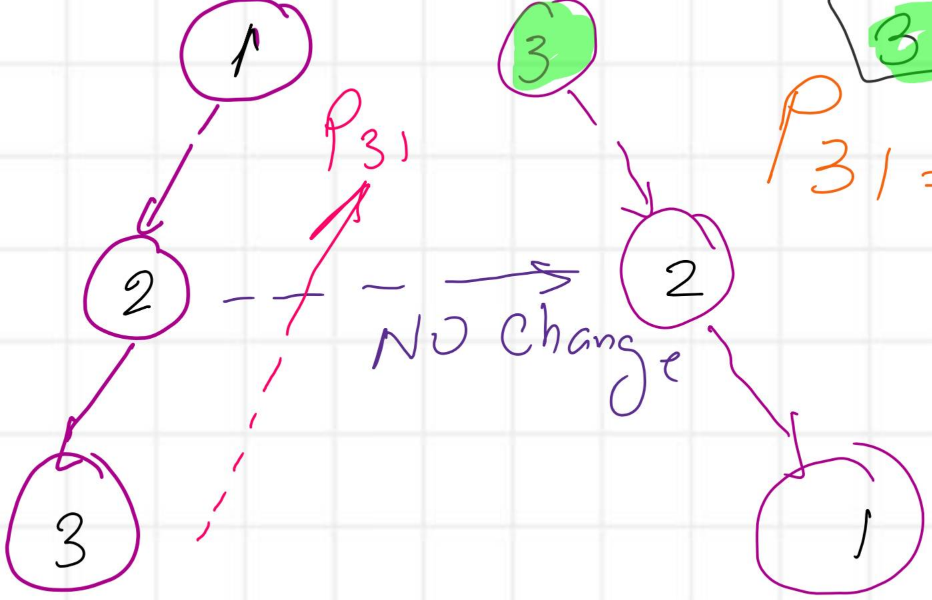
Column: C_{213}

P_{31}

$$= \begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}$$

P_{321}

C_{321} Column



$\boxed{3>1}$

Node 3

$P_{31} = P_{321}$

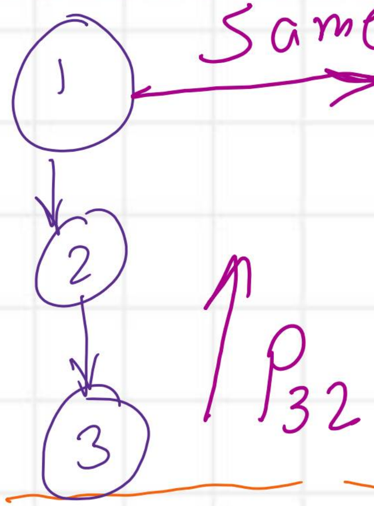
$r_3 \to r_1$ in the r_2 Same position

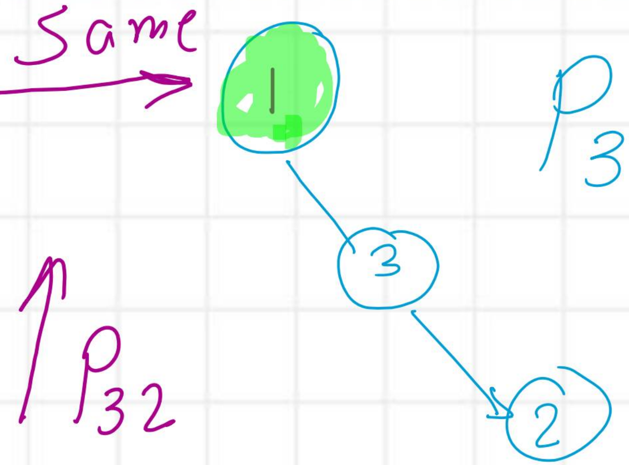
I-J-expression for nodes 2-3
-P21-P31

$$I = \begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix}$$

$$P_{32} = \begin{bmatrix}1&&\\&&1\\&1&\end{bmatrix}$$

C_{132}





$\boxed{3>2}$

$P_{32} = P_{132}$

Node 1

$r_1 \to$ same position

$r_2 \to r_3$

① ✓✓ I, P_{132}

② ✓ $\to P_{231}$

③ ✓

I & P_{21} & P_{31} & P_{32}

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$I \quad P_{213} \quad P_{321} \quad P_{132}$

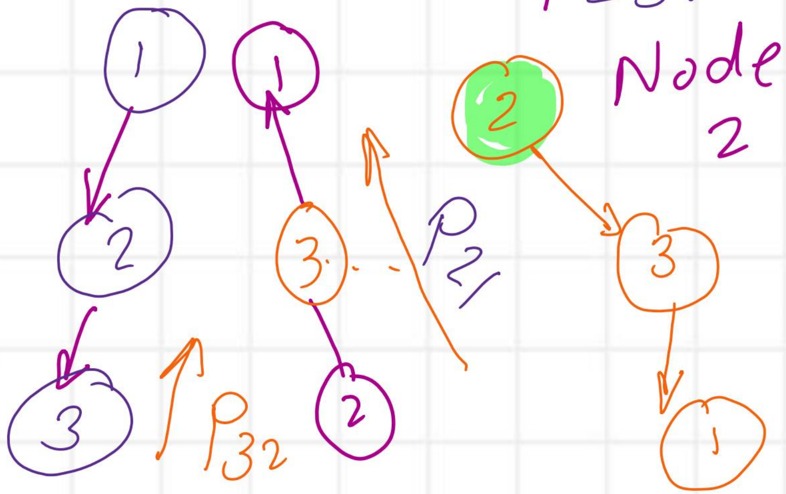
Node wise

$$\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix} \Rightarrow P_{231}$$

remaining

$$P_{231} = P_{132}(P_{213}) = C_{312}$$

$$(P_{32})(P_{21}) \Rightarrow P_{231}$$



No Element in its original position

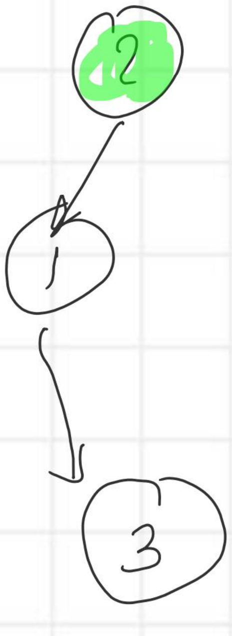
P32-expression

I & P_{21} & P_{31} & P_{32}

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$I \quad P_{213} \quad P_{321} \quad P_{132}$

Node 2



P_{231}

C_{312}

$$P_{231} = \underbrace{P_{132}}_{P_{32}} \underbrace{(P_{213})}_{P_{21}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

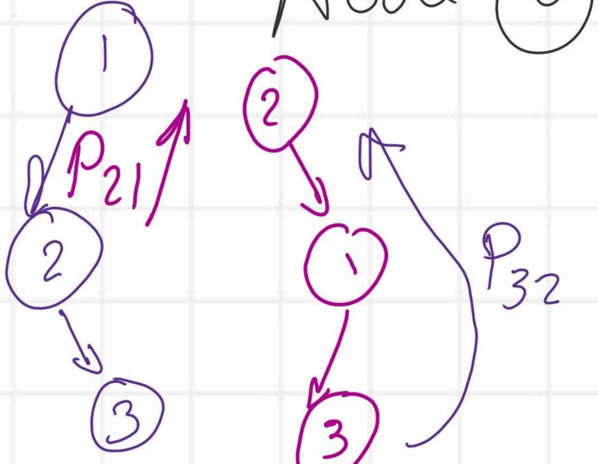
Node 3

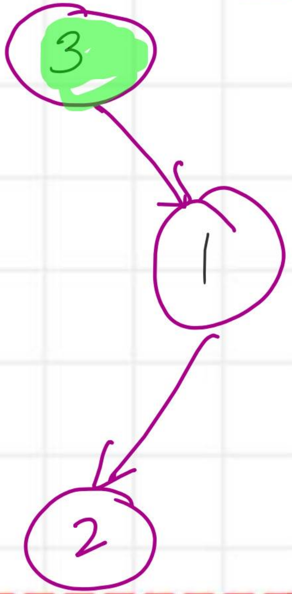
$$P_{312} = P_{213}\, P_{132}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

C_{231}

$$\Rightarrow P_{312} = (P_{21})(P_{32})$$





P231-P312 as a product of two matrices