

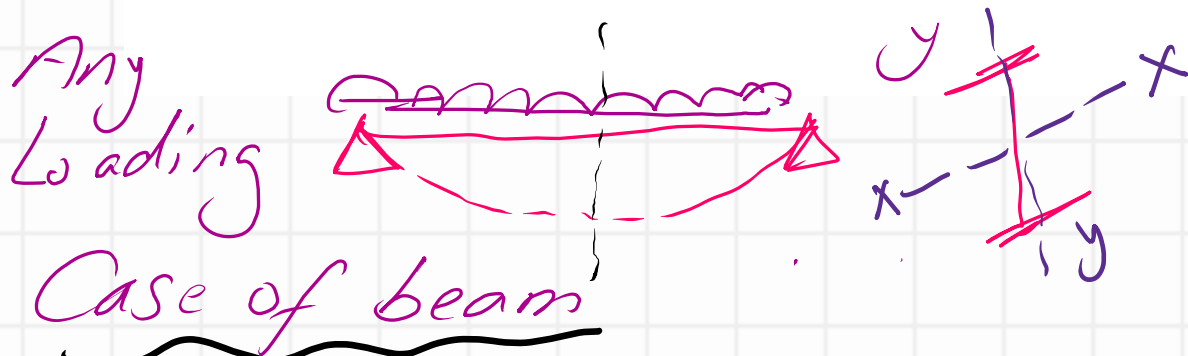
## Summary

- ① Difference between beam buckling and a column buckling
- ② What is the difference between local and general buckling
- ③ Table for  $K$  values based on Euler's formula for different end conditions  
Table A7.1 and Commentary C A7.1
- ④ Practice problem for column strength

Summary of the content of posts 2-2a

### 3.2 COLUMN BUCKLING

- Consider a long slender compression member. If an axial load  $P$  is applied and increased slowly, it will ultimately reach a value  $P_{cr}$  that will cause buckling of the column.  $P_{cr}$  is called the critical buckling load of the column.



Due to Loading (any type)  
There will be a moment  
about major axis  $x-x$   
and deflection

$x-x$  is the major axis  
 $y-y$  is the minor axis  
Deflection is about  
major axis  
movement in  $y-y$   
direction

Prepared by Eng. Maged Kamel.

Difference between beam buckling and column buckling

### 3.2 COLUMN BUCKLING

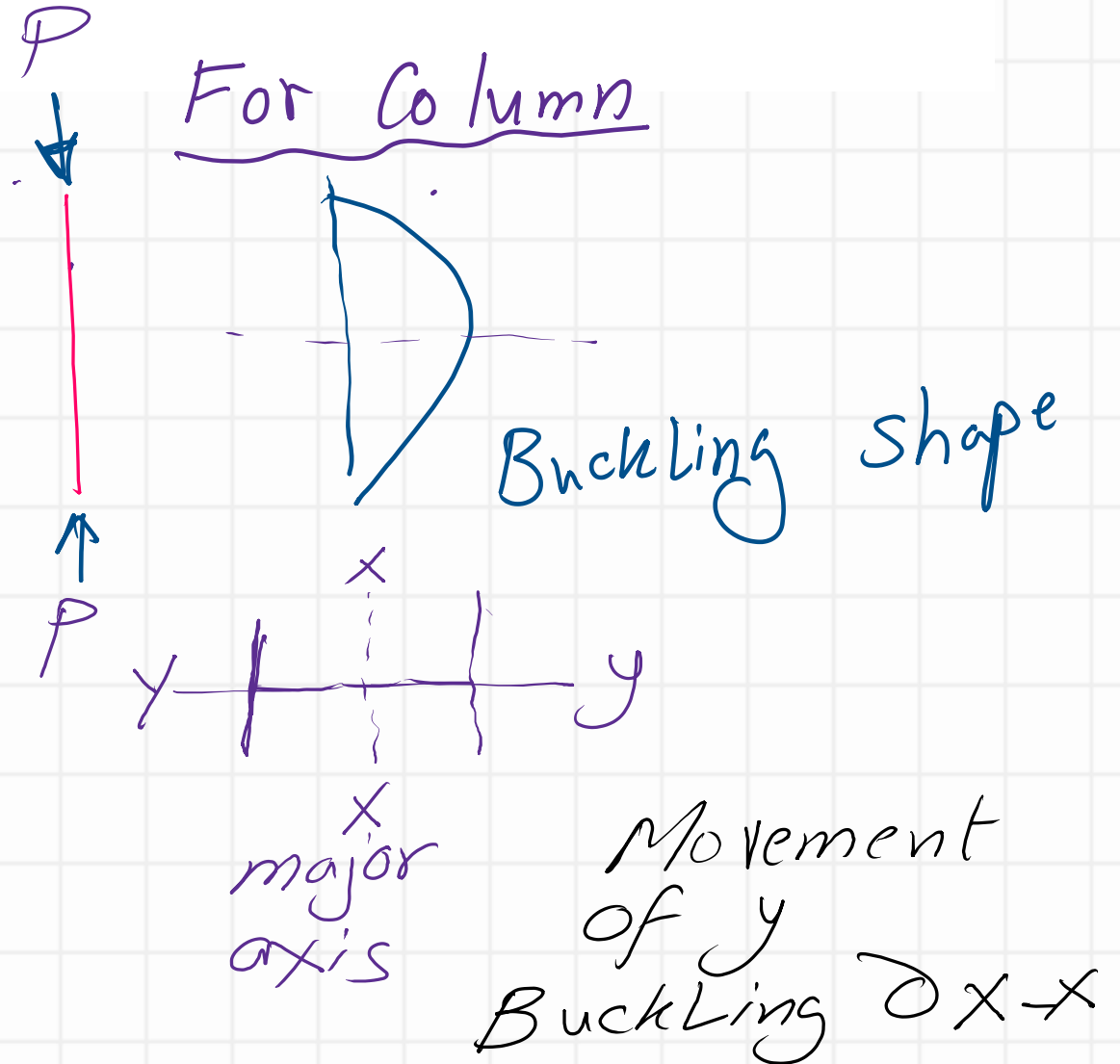
- Consider a long slender compression member. If an axial load  $P$  is applied and increased slowly, it will ultimately reach a value  $P_{cr}$  that will cause buckling of the column.  $P_{cr}$  is called the critical buckling load of the column.

Difference between buckling about strong/weak axis

Case of Column

Buckling occurs when a straight column under axial compression under goes bending

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## APPENDIX 7


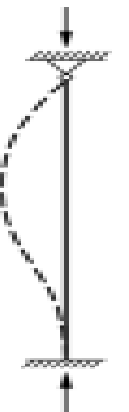
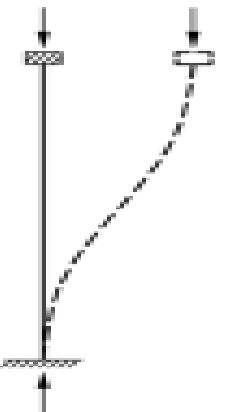
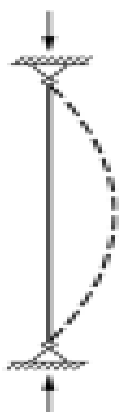
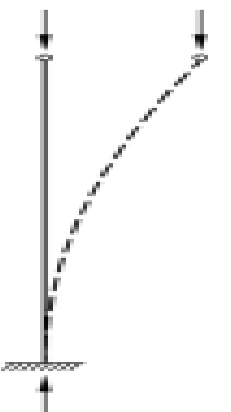
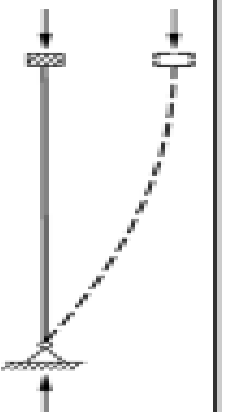
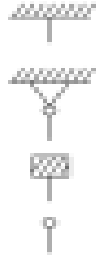




### ALTERNATIVE METHODS OF DESIGN FOR STABILITY

The effective length method and first-order analysis method are addressed in this Appendix as alternatives to the direct analysis method, which is presented in Chapter C. These alternative methods of design for stability can be used when the limits on their use as defined in Appendix 7, Sections 7.2.1 and 7.3.1, respectively, are satisfied.

The effective length,  $L_c = KL$ , for column buckling based upon elastic (or inelastic) stability theory, or alternatively the equivalent elastic column buckling stress,  $F_c = \pi^2 E / (L_c / r)^2$ , is used to calculate an axial compressive strength,  $P_c$ , through an empirical column curve that accounts for geometric imperfections and distributed yielding (including the effects of residual stresses). This column strength is then combined with the available flexural strength,  $M_c$ , and second-order member forces,  $P_r$  and  $M_r$ , in the beam-column interaction equations.

$L_c$	Effective length of member, in. (mm) . . . . .	E2
$L_{cx}$	Effective length of member for buckling about $x$ -axis, in. (mm) . . . . .	E4
$L_{cy}$	Effective length of member for buckling about $y$ -axis, in. (mm) . . . . .	E4

Table CA-7.1

<b>TABLE C-A-7.1</b> <b>Approximate Values of Effective Length Factor, <math>K</math></b>						
Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical $K$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	 <ul style="list-style-type: none"> <li> Rotation fixed and translation fixed</li> <li> Rotation free and translation fixed</li> <li> Rotation fixed and translation free</li> <li> Rotation free and translation free</li> </ul>					

$L_c$	Effective length of member, in. (mm) . . . . .	E2
$L_{cx}$	Effective length of member for buckling about $x$ -axis, in. (mm) . . . . .	E4
$L_{cy}$	Effective length of member for buckling about $y$ -axis, in. (mm) . . . . .	E4

$F_e$  = elastic buckling stress determined according to Equation E3-4, as specified in Appendix 7, Section 7.2.3(b), or through an elastic buckling analysis, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \quad (E3-4)$$

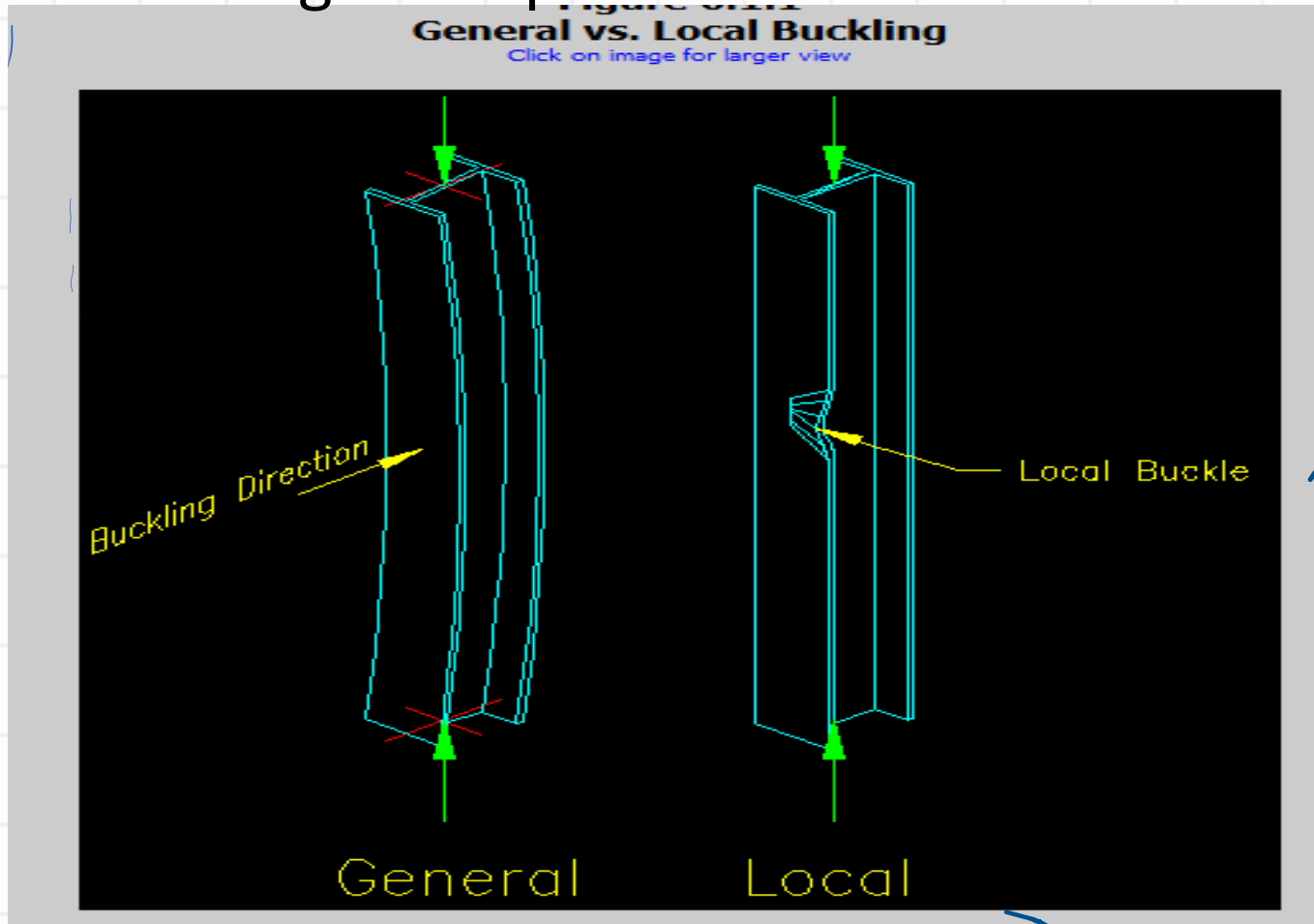
$F_y$  = specified minimum yield stress of the type of steel being used, ksi (MPa)

$r$  = radius of gyration, in. (mm)

Fe -Euler stress equation

<http://www.bgstructuralengineering.com/BGSCM15/BGSCM006/index.htm>  
Buckling concepts

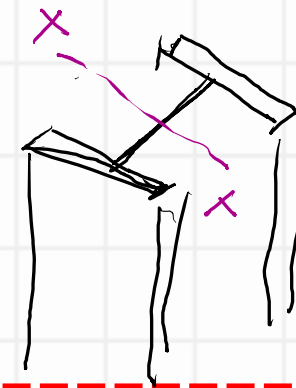
CM # 15



A Beginner's Guide to the Steel Construction Manual, 15<sup>th</sup> ed.  
**Chapter 6 - Buckling Concepts**  
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→ This is a Local Buckling

This is a buckling about x-x axis



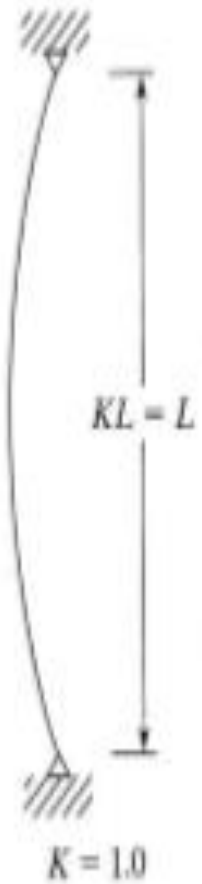
General

Difference between

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Different K values

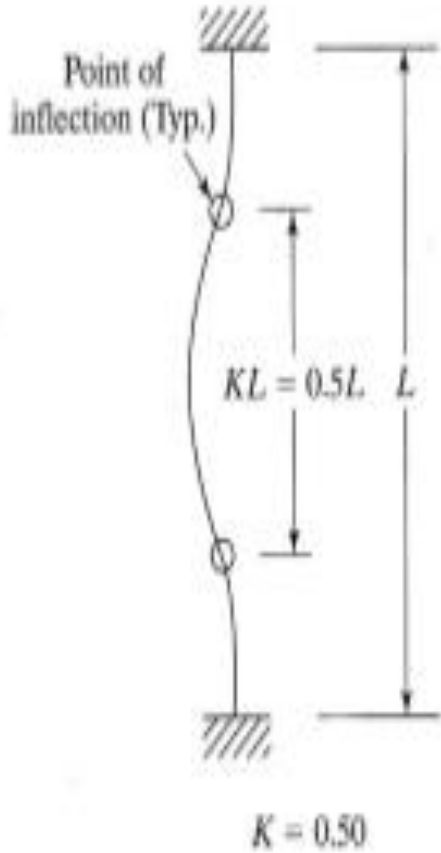
Pinned



pinned

$K = 1.0$

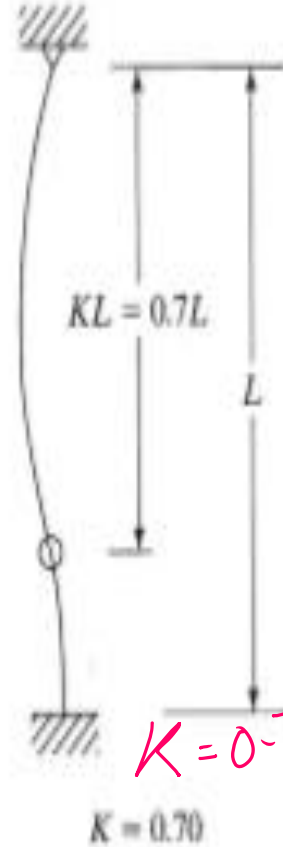
Fixed



Fixed

$k = 0.5$

hinged



Fixed

$K = 0.70$

Difference between x-x

& y-y

axes

Figure 6.2.1  
Compression Member Principle Axis

Click on image for larger view

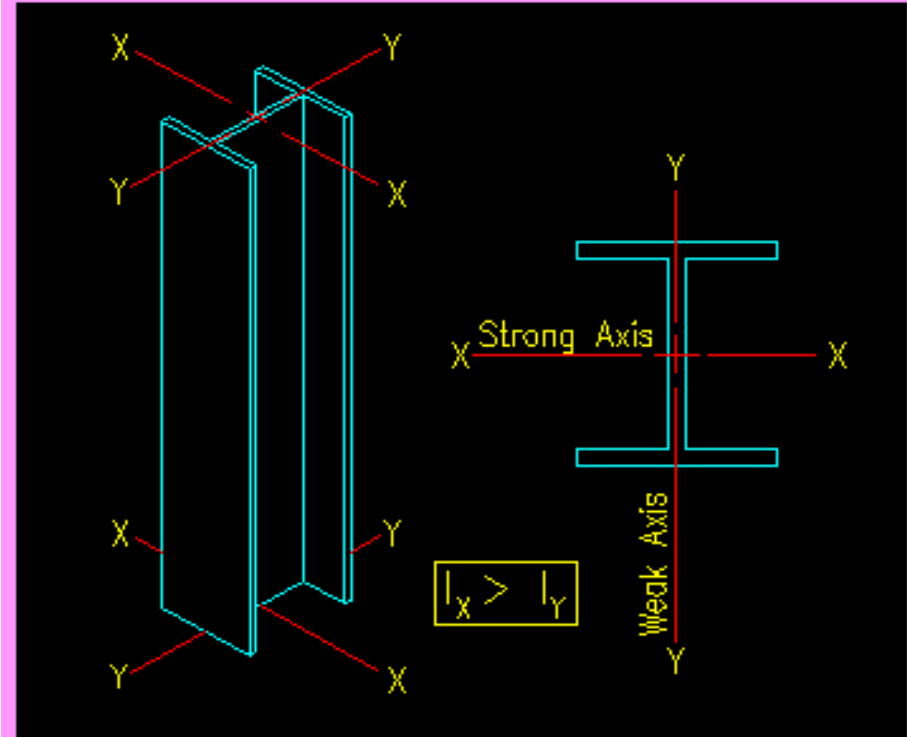


Figure 6.2.2  
Compression Member Principle Planes

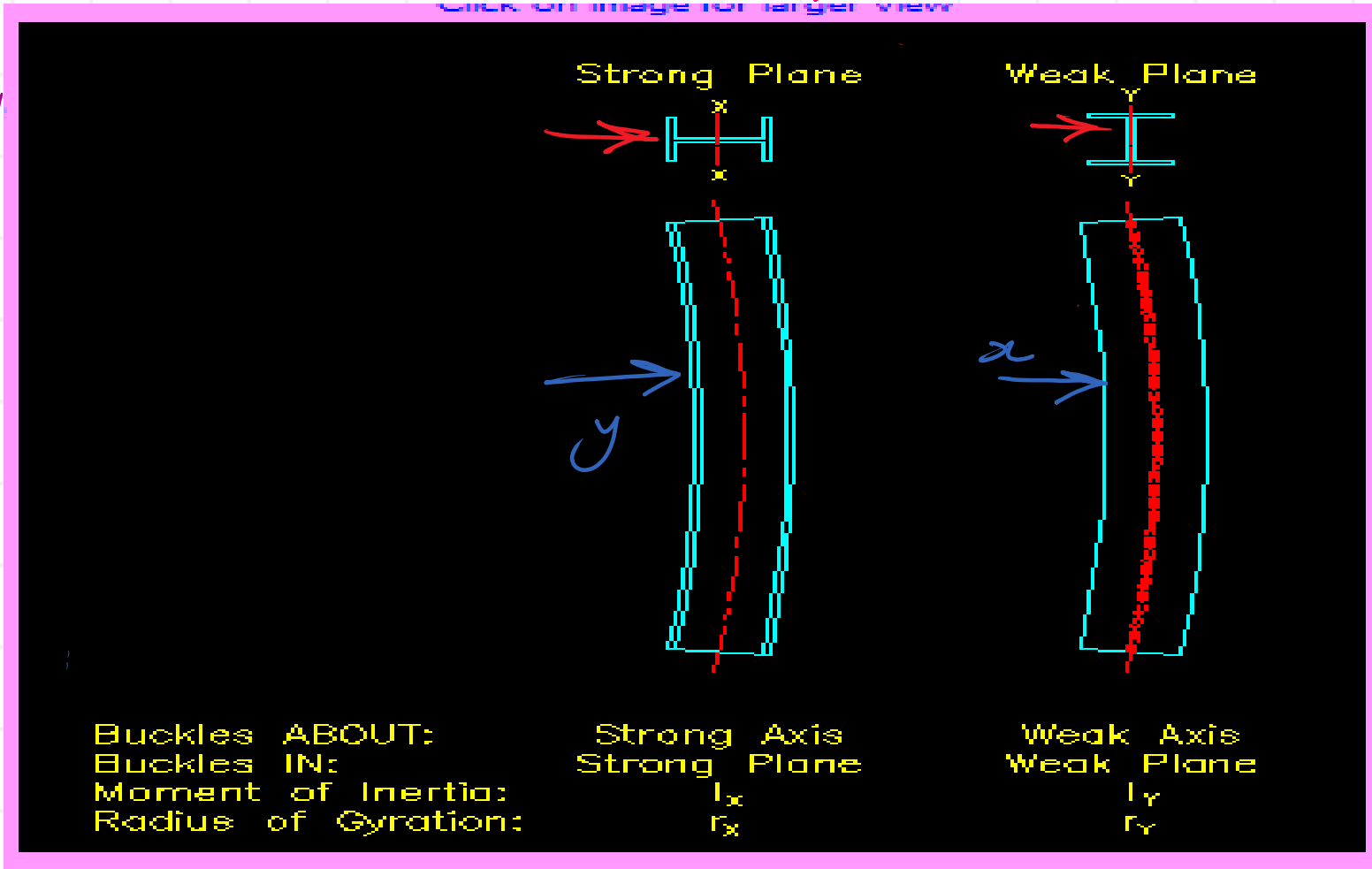
Click on image for larger view

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x-x major  
y-y minor

Strong axis Movement in y

Buckling



Weak axis buckling  
Accompanied by movement in x-direction

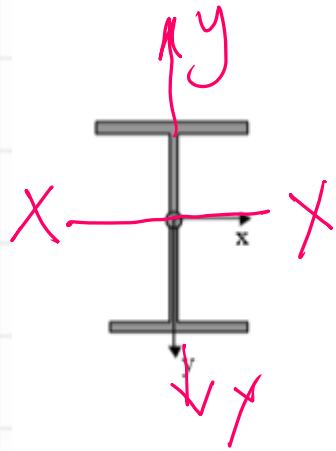
General buckling

Differentiate between strong/weak axis buckling

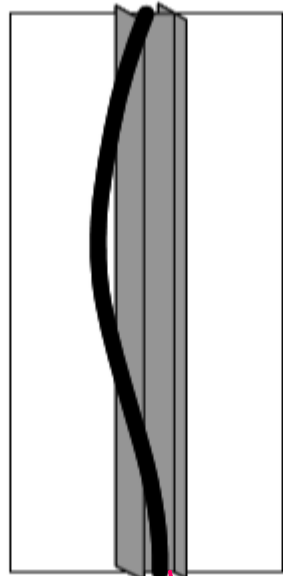
Prepared by Eng. Maged Kamel.

Determine the buckling strength of a W12 x 50 column. Its length is 20 Ft. For minor axis buckling, it is pinned at both ends. For major buckling, is it pinned at one end and fixed at the other end.

Solution



Pinned

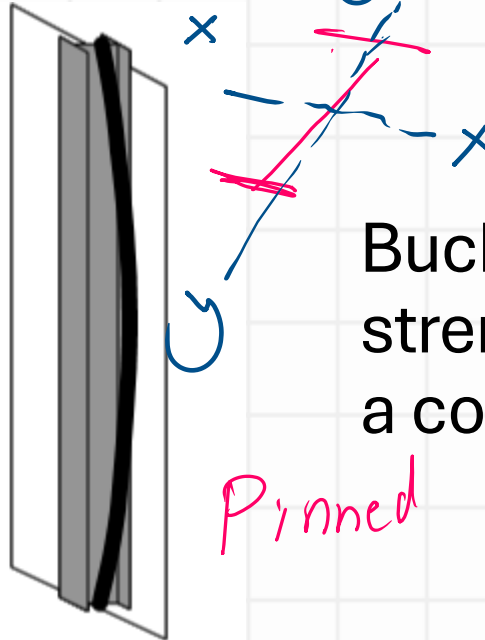


Fixed

$$K = 0.70$$

$$\Rightarrow K = 0.80$$

Pinned



Pinned

$$K = 1$$

W12 x 50

$$h = 20'$$

movement in x-direction

Buckling strength for a column

Bending about y-y

minor buckling

$$(KL)_x = 0.8(20) = 16'$$

$$(KL)_y = 1(20) = 20'$$

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W12 x 50  
Table 1-1 (continued)  
W-Shapes  
Properties



Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y				$r_{ts}$ in.	$h_o$ in.	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$I$ in. <sup>4</sup>	$S$ in. <sup>3</sup>	$r$ in.	$Z$ in. <sup>3</sup>	$I$ in. <sup>4</sup>	$S$ in. <sup>3</sup>	$r$ in.	$Z$ in. <sup>3</sup>			$J$ in. <sup>4</sup>	$C_w$ in. <sup>6</sup>
	58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51			32.5	2.81
53	8.69	28.1	425	70.6	5.23	77.9	95.8	19.2	2.48	29.1	2.79	11.5	1.58	3160
50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1880

From Table 1-1

$$I_x = 391 \text{ inch}^4$$

$$I_y = 56.3 \text{ inch}^4$$

$$A = 14.60 \text{ inch}^2$$

$$Ar_x^2 = I_x$$

$L = 20'$   
E value = 29000 ksi:

$$P_{cr_x} = \frac{\pi^2 E I_x}{(KL)_x^2}$$

Buckling about x-x major

$$k = 0.80 \quad (KL)_x = (0.8)(20)(12) = 192''$$

$$P_{cr_x} = (3.14159)^2 (29)(10^3) (391) / (192)^2 = 3035.90 \text{ kips}$$

Critical load-x direction

Prepared by Eng. Maged Kamel.

Critical load y direction

Table 1-1 (continued)

W-Shapes

W12 x 50

Properties



Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y				$r_{ts}$ in.	$h_o$ in.	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$I$	$S$	$r$	$Z$	$I$	$S$	$r$	$Z$			$J$	$C_w$
			in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>	in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>			in. <sup>4</sup>	in. <sup>6</sup>
58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.81	11.6	2.10	3570
53	8.69	28.1	425	70.6	5.23	77.9	95.8	19.2	2.48	29.1	2.79	11.5	1.58	3160
50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1880

From Table 1-1

$I_x = 391 \text{ inch}^4$

$I_y = 56.3 \text{ inch}^4$

$A = 14.60 \text{ inch}^2$

$Ar_x^2 = I_x$

$E \text{ value} = 29000 \text{ Ks}$   
 $L = 20'$

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

Buckling about y-y minor

$K = 1$   
 $(KL) = 1 (20)(12) = 240''$

$P_{cry} = (3.14159)^2 (29)(10^3) (56.3) / (240)^2 = 279.76 \text{ kips}$

$P_{CF} = \min(3035.90, 279.76) = 279.75 \text{ kips}$

Buckling y governs

## Case of yielding prior to buckling

W 12 x 50

In the previous Example  $P_{cr} = 279.76 \text{ kips}$   
Area = 14.60 inch<sup>2</sup>

$$(F_E)_y = 279.76 / 14.60 = 19.60 \text{ ksi} < F_y$$

where  $F_y = 50 \text{ ksi}$ : Buckling occurs prior to Buckling

Case of yielding prior to buckling

But if Column is reduced to 10' For same problem

$$F_E = \frac{\pi^2 E I_y}{(KL)^2 A} = \frac{\pi^2 (29000) (56.30)}{(120)^2 (14.6)} = 76.65 \text{ ksi} > 50 \text{ ksi}$$

## Case of yielding prior to buckling

- **Notes:**

- Minor axis buckling usually governs for all doubly symmetric cross-sections. However, for some cases, major (x) axis buckling can govern.
- Note that the steel yield stress was irrelevant for calculating this buckling strength.

### 3.3 INELASTIC COLUMN BUCKLING

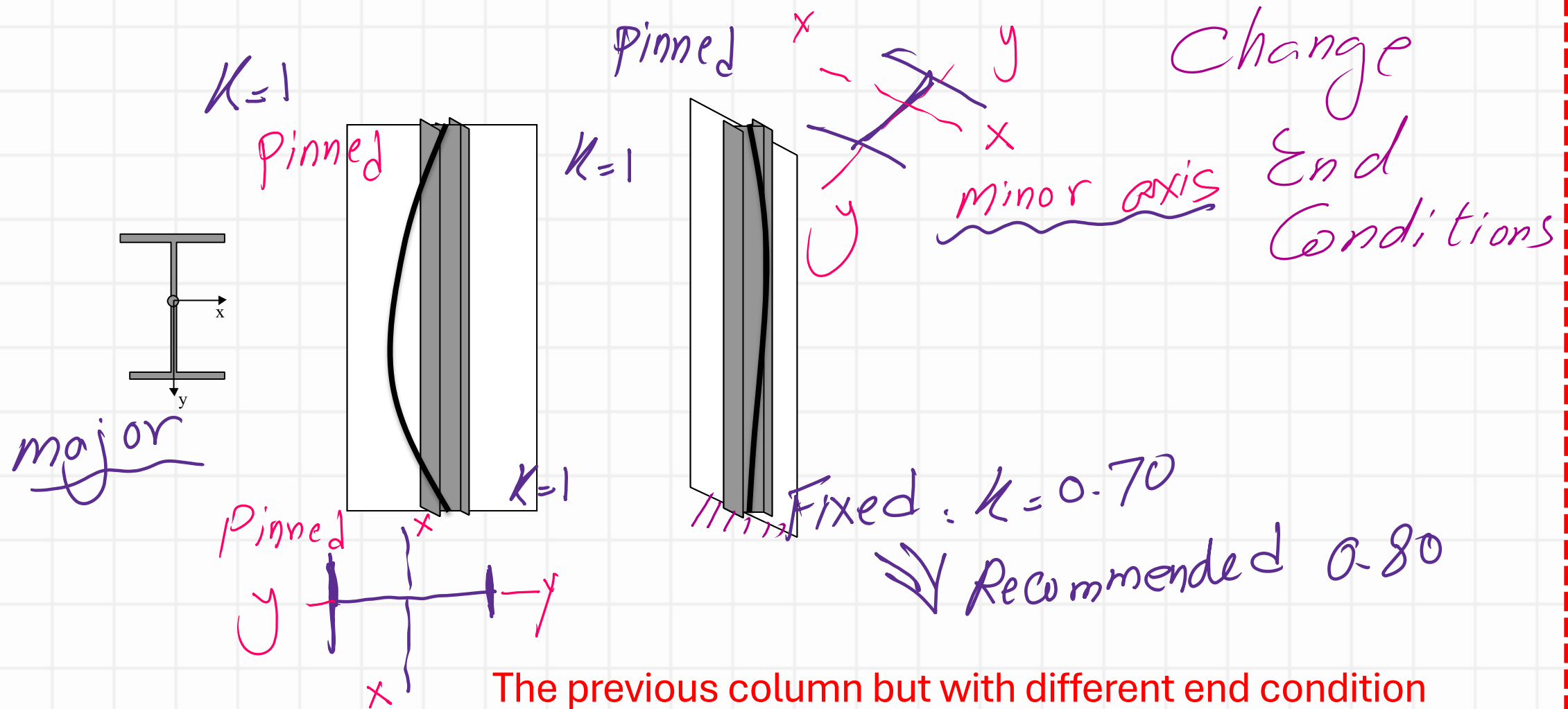
Prof. Varma notes

- Let us consider the previous example. According to our calculations  $P_{cr} = 279.8$  kips. This will cause a uniform stress  $f = P_{cr}/A$  in the cross-section
- For W12 x 50,  $A = 14.6 \text{ in}^2$ . Therefore, for  $P_{cr} = 279.8$  kips;  $f = 19.16$  ksi  
The calculated value of  $f$  is within the elastic range for a 50 ksi yield stress material.
- However, if the unsupported length was only 10 ft.,  $P_{cr} = \frac{\pi^2 E I_y}{(K_y L_y)^2}$  would be calculated as 1119 kips, and  $f = 76.6$  kips.

Check whether yielding occur prior to buckling

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Determine the buckling strength of a W12 x 50 column. Its length is 20 ft. For major axis buckling, it is pinned at both ends. For minor buckling, it is pinned at one end and fixed at the other end.



W12 x 50  
Table 1-1 (continued)  
W-Shapes  
Properties



Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y				$r_{ts}$ in.	$h_o$ in.	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$I$	$S$	$r$	$Z$	$I$	$S$	$r$	$Z$			$J$	$C_w$
			in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>	in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>			in. <sup>4</sup>	in. <sup>6</sup>
58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.81	11.6	2.10	3570
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$$I_x = 391 \text{ inch}^4$$

$$I_y = 56.3 \text{ inch}^4$$

$$A = 14.60 \text{ inch}^2$$

$$Ar_x^2 = I_x$$

$$E \text{ value} = 29000 \text{ Ks}; \quad L = 20'$$

$$P_{cr_x} = \frac{\pi^2 E I_x}{(KL)_x^2}$$

Buckling about x-x major

$$K = 1 \quad (KL)_x = (1)(20)(12) = 240'$$

$$P_{cr_x} = (3.14159)^2 (29)(10^3) (391) / (240)^2 = 1942.90 \text{ Kips}$$

Critical load-x direction

Prepared by Eng. Maged Kamel.

# Final load

Final load  
W12 x 50

Table 1-1 (continued)  
W-Shapes  
Properties



Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y				$r_{ts}$ in.	$h_o$ in.	Torsional Properties	
	$b_f/2t_f$	$h/t_w$	$I$ in. <sup>4</sup>	$S$ in. <sup>3</sup>	$r$ in.	$Z$ in. <sup>3</sup>	$I$ in. <sup>4</sup>	$S$ in. <sup>3</sup>	$r$ in.	$Z$ in. <sup>3</sup>			$J$ in. <sup>4</sup>	$C_w$ in. <sup>6</sup>
	58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51			32.5	2.81
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From Table 1-1

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$$A = 14.60 \text{ inch}^2$$

$$Ar_x^2 = I_x$$

$L = 20'$   
E value = 29000 Ks:

$$P_{cr} = \frac{\pi^2 E I_x}{(KL)^2}$$

Buckling about y-y minor

$$K = 0.80 \quad (KL) = 0.8 (20)(12) = 192''$$

$$P_{cry} = (3.14159)^2 (29)(10^3) (56.3) / (192)^2 = 437.12 \text{ K}$$

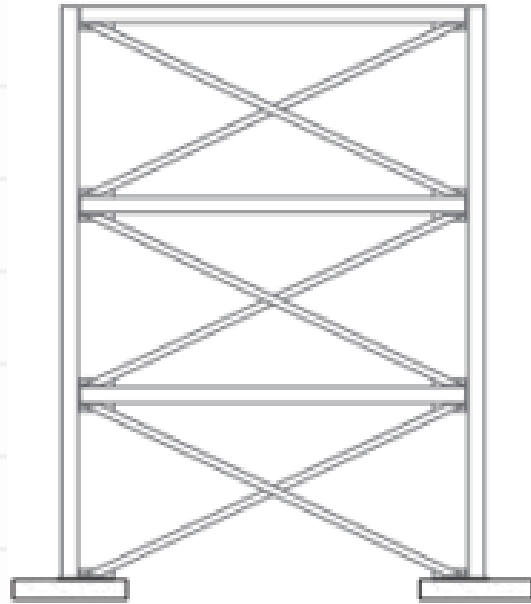
$$P_{CF} = \min(1942.9, 437.12) = 437.12 \text{ K}$$

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Buckling y governs

# Types of bracings

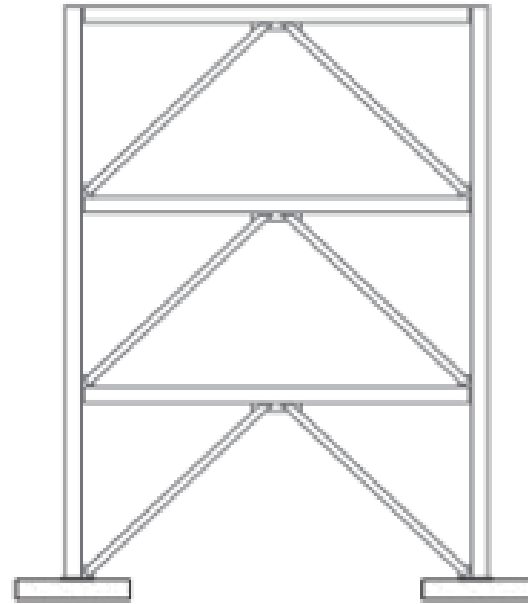
X-brace



X-brace

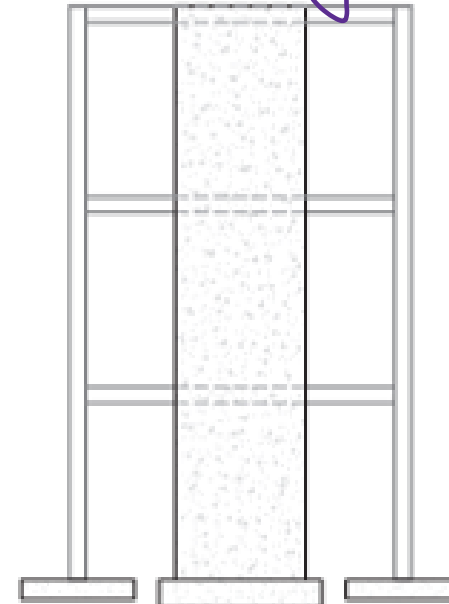
Types of bracing

K-brace



chevron or K-brace

Bracing using shear wall

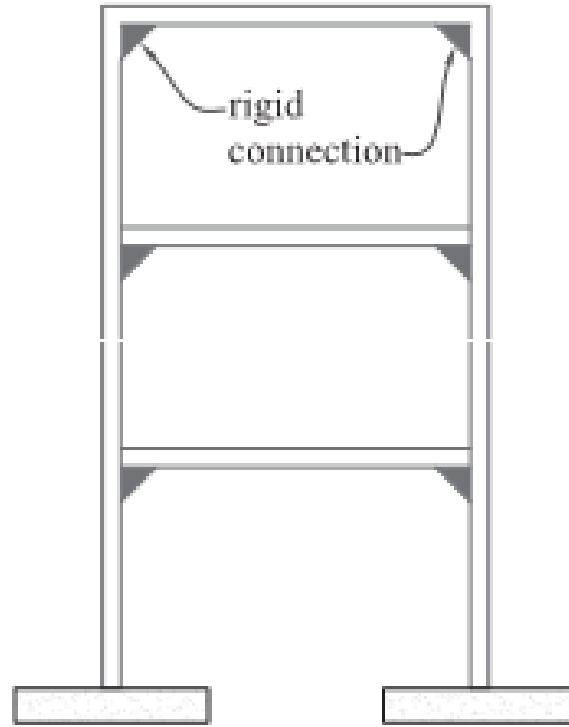


shear wall

a. braced frames

Prepared by Eng. Maged Kamel.

# Braced and unbraced frames

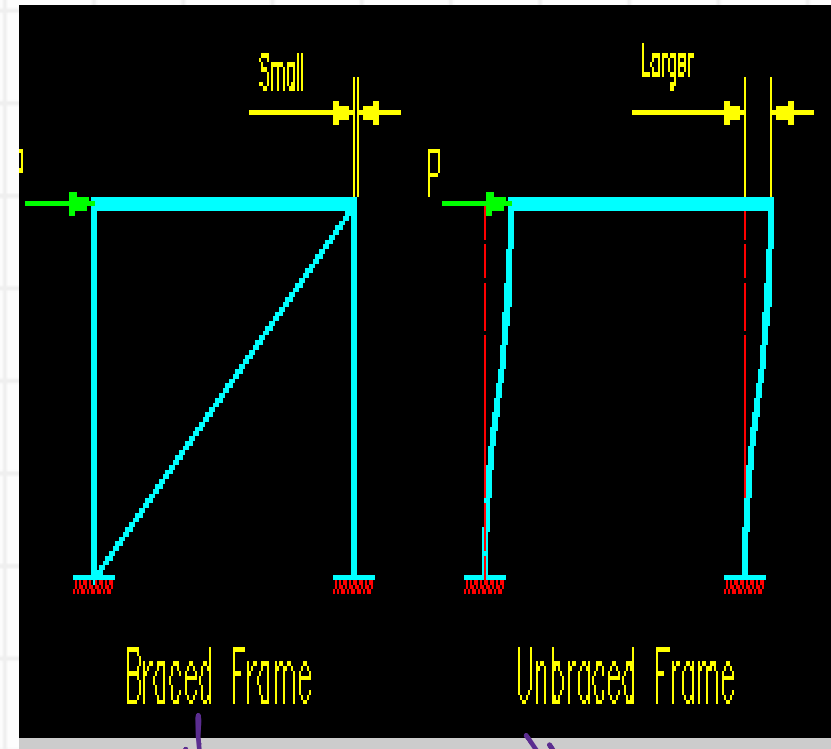


*b. unbraced frames*

**Figure 5-4** Braced and unbraced frames.

*Small deformation*

*Large deformation*



*Un braced  
Frame*