Summary

(A) Review of the plate theory

get Exprenion of Fcr For Plates under Lateral Comprenion

(B) What are λ_r values for stiffened and Unstiffened Parts?

Plate theory review

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Column is Composed of plates

When Column is Slender

⇓

Plates will have Local buckling

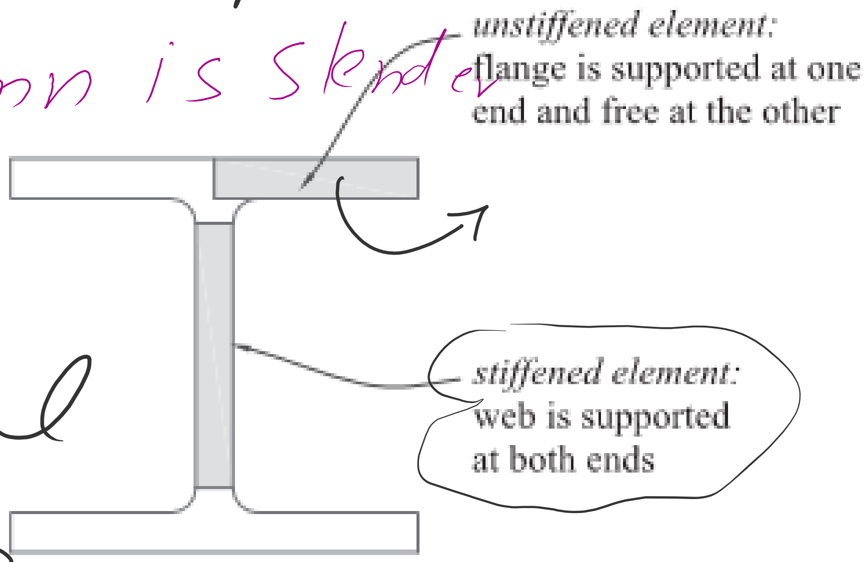


Figure 5-7 Stiffened and unstiffened elements.

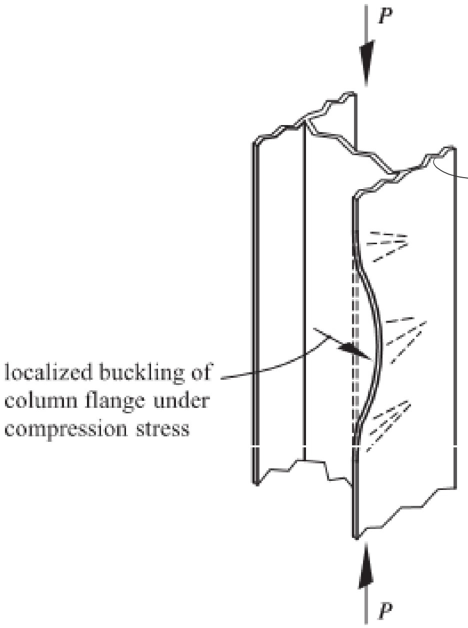


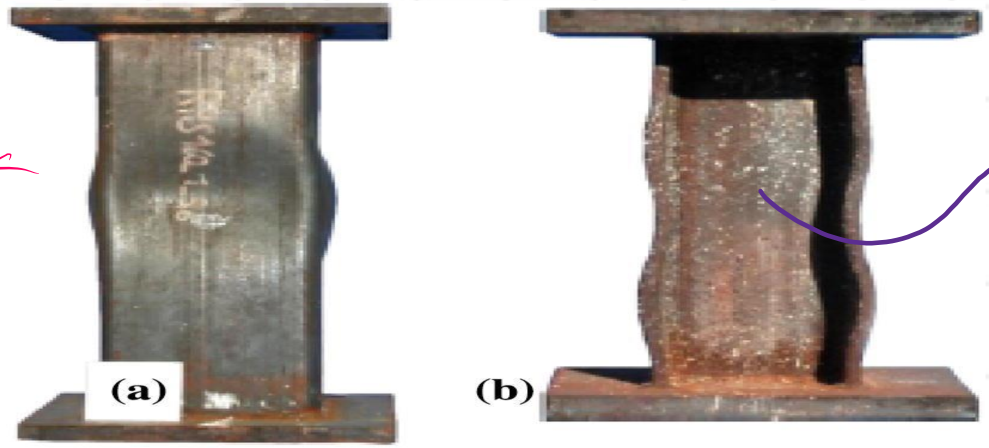
Figure 5-6 Local buckling of column under axial compression load.

Flange Local buckling

Local buckling For Column

Local buckling

Unstiffend Part



Local buckling for Web

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Cases for local buckling

Figure 6.1.3
Theoretical Maximum Compressive Stress
Click on image for larger view

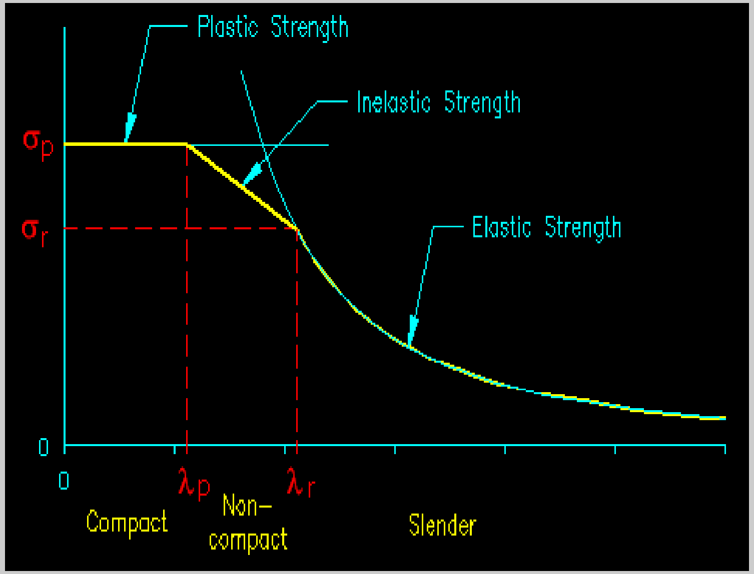
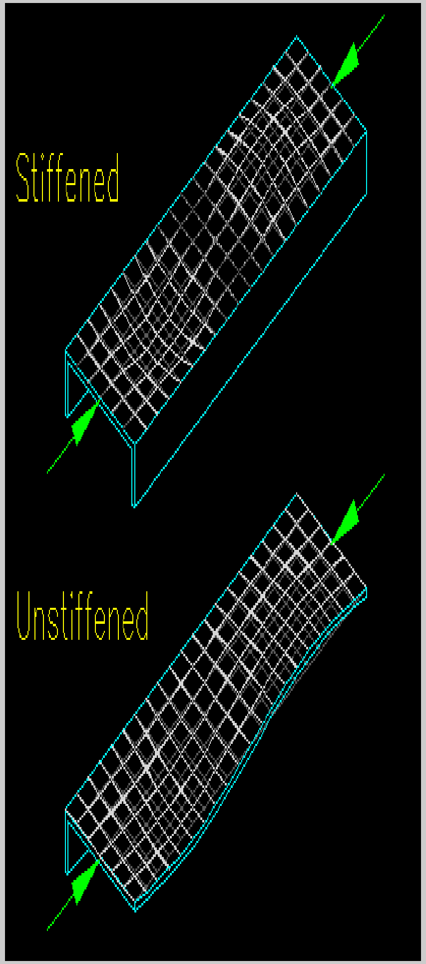
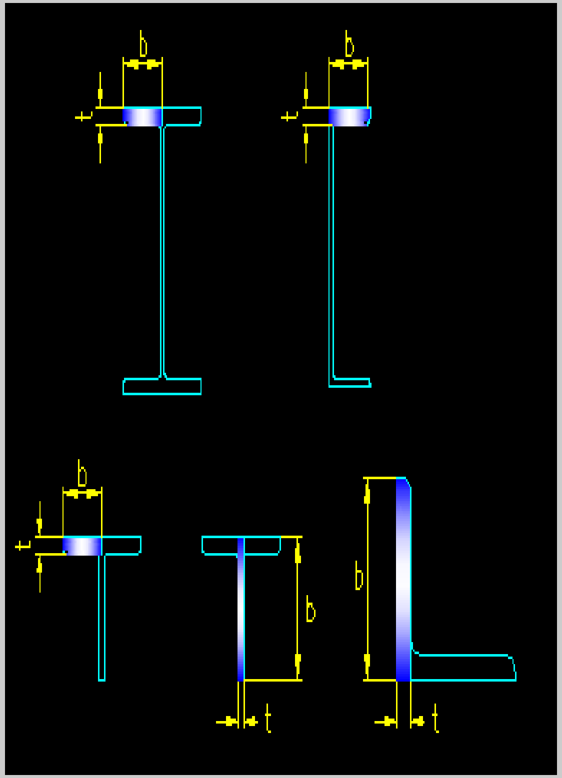


Figure 6.3.3
Plate Buckling Modes
Click on image for larger view



Unstiffend elements

Figure 6.3.5
Unstiffened Elements
Click on image for larger view



$\left(\frac{KL}{r}\right)$ # 2010

http://www.bgstructuralengineering.com/BGSCM14/BGSCM006/BGSCM00603.htm

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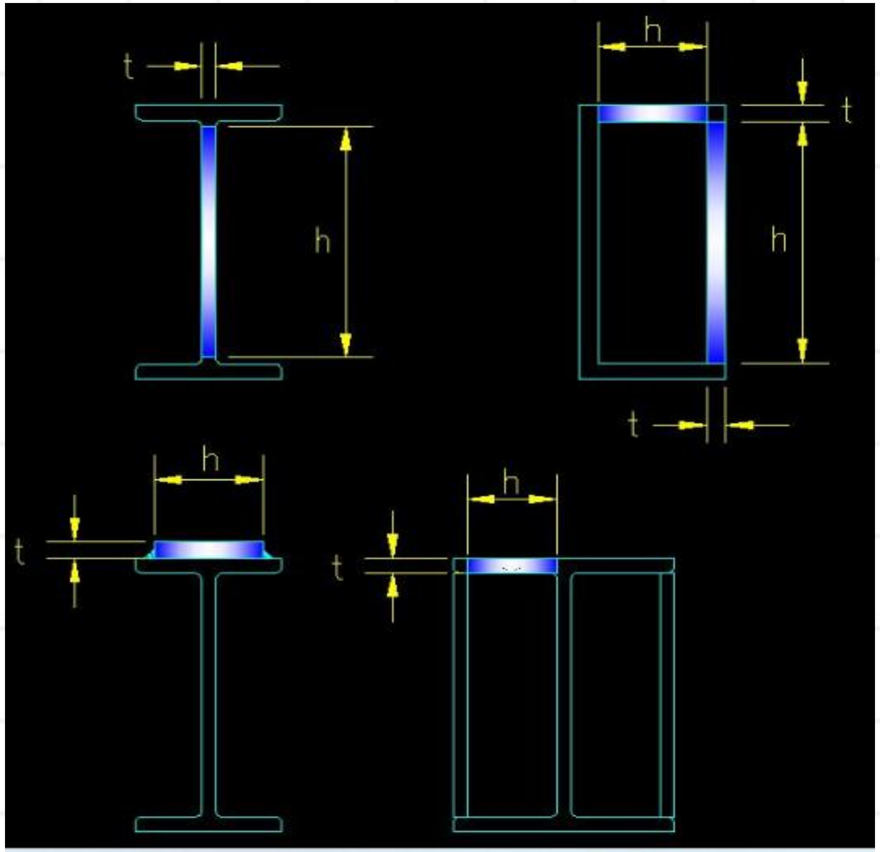
half sin waves

Samples

Cases for local buckling for stiffened and unstiffened

Figure 6.3.6 shows the *stiffened elements* on some typical steel sections and the measurement of the element width, h, and thickness, t. Note that "W" shapes and channels each have one stiffened plate element in their cross-section. A square or rectangular HSS has four stiffened elements in its cross-section. Normally, unstiffened plate elements can be stiffened with the addition of plate elements as seen in the figure.



https://www.bgstructuralengineering.com/BGSCM15/BGSCM006/BGSCM00603.htm

→ Sample of stiffened elements

https://www.bgstructuralengineering.com/BGSCM15/BGSCM006/Stiff.jpg

Links to graphs

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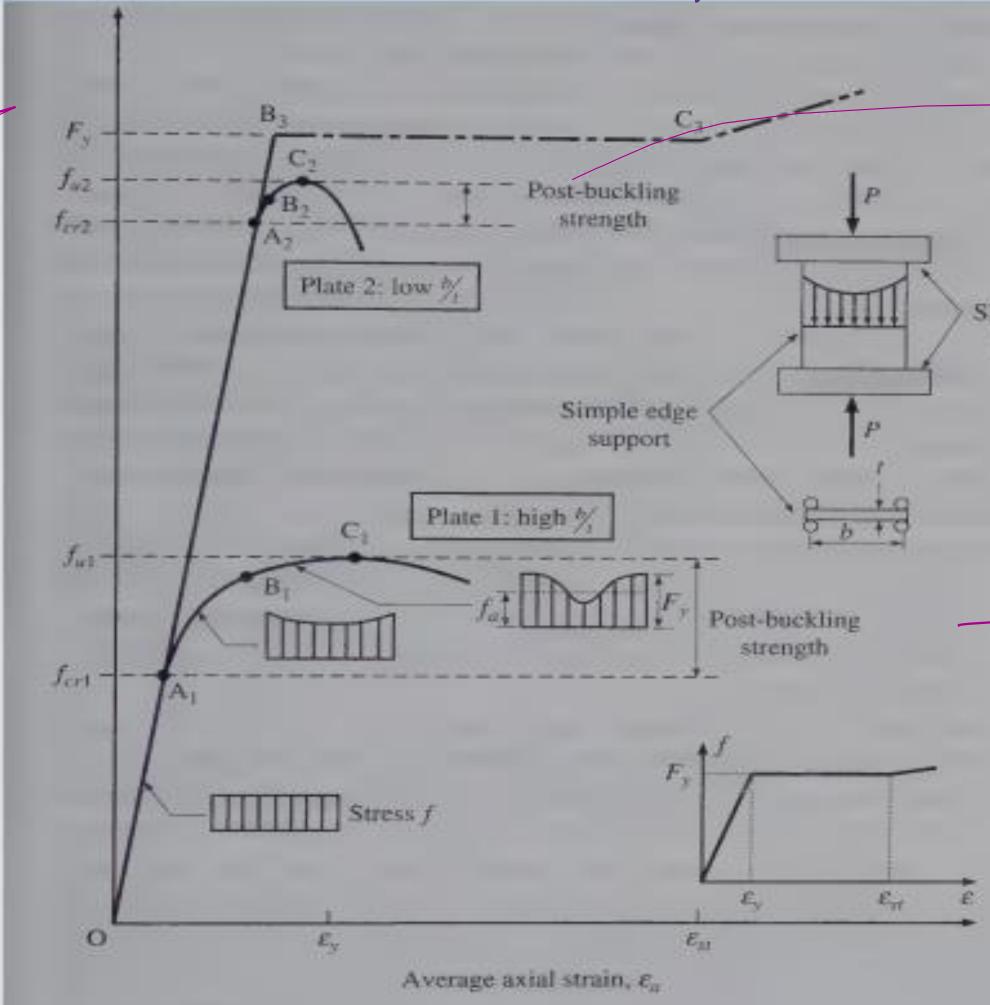
Behavior of plates under Loads

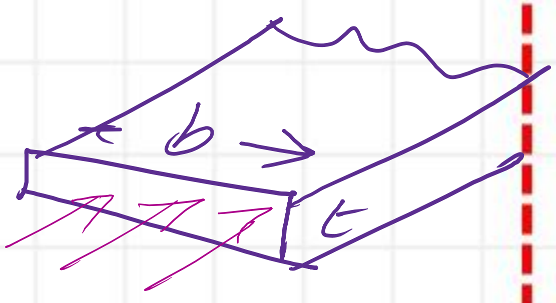
F_c



Stress shape

higher $\frac{b}{t}$

Smaller $\frac{b}{t}$ higher f_{cr}



Slender

Lower F_{cr}

Post buckling strength

Strain against stress for plate

Average strain

Figure 8.9.2: Behavior of rectangular plates under edge compression.

the edges to remain straight in the process of loading. The plate is made of linearly elastic, plastic, strain-hardening material containing no residual stresses. A diagram of plate behavior is obtained by plotting the average compressive stress f_a versus the average strain ε_a, where

Forces acting on a plate and their notations

Forces and stress notations

In-plane normal forces and bending moments, Fig. 6.1.7:

$$N_x = \int_{-h/2}^{+h/2} \sigma_{xx} dz, \qquad N_y = \int_{-h/2}^{+h/2} \sigma_{yy} dz$$
$$M_x = -\int_{-h/2}^{+h/2} z\sigma_{xx} dz, \quad M_y = -\int_{-h/2}^{+h/2} z\sigma_{yy} dz \tag{6.1.1}$$

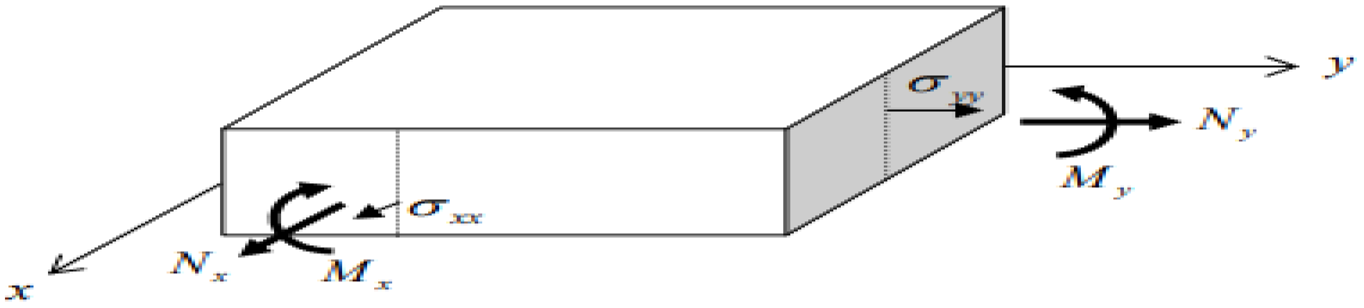


Fig. 6.1.7: in-plane normal forces and bending moments

We are dealing with Nx or Nxx

Solid Mechanics – II

Piaras Kelly

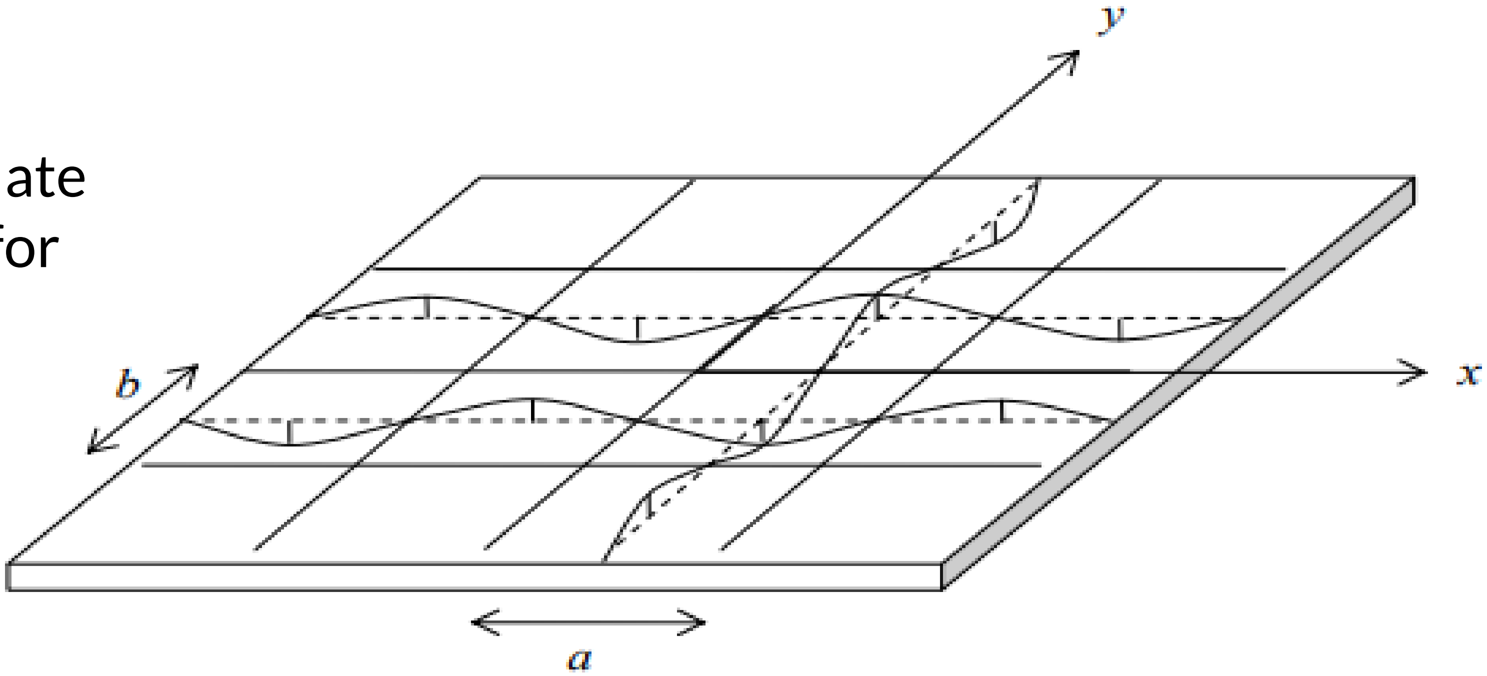
6.5.3 An Infinite Plate with Sinusoidal Deflection

Consider next the classic plate problem addressed by Navier in 1820. It consists of an infinite plate with an undulating "up/down" sinusoidal deflection, Fig. 6.5.5,

Deflection

$$w(x,y) = w_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{6.5.24}$$

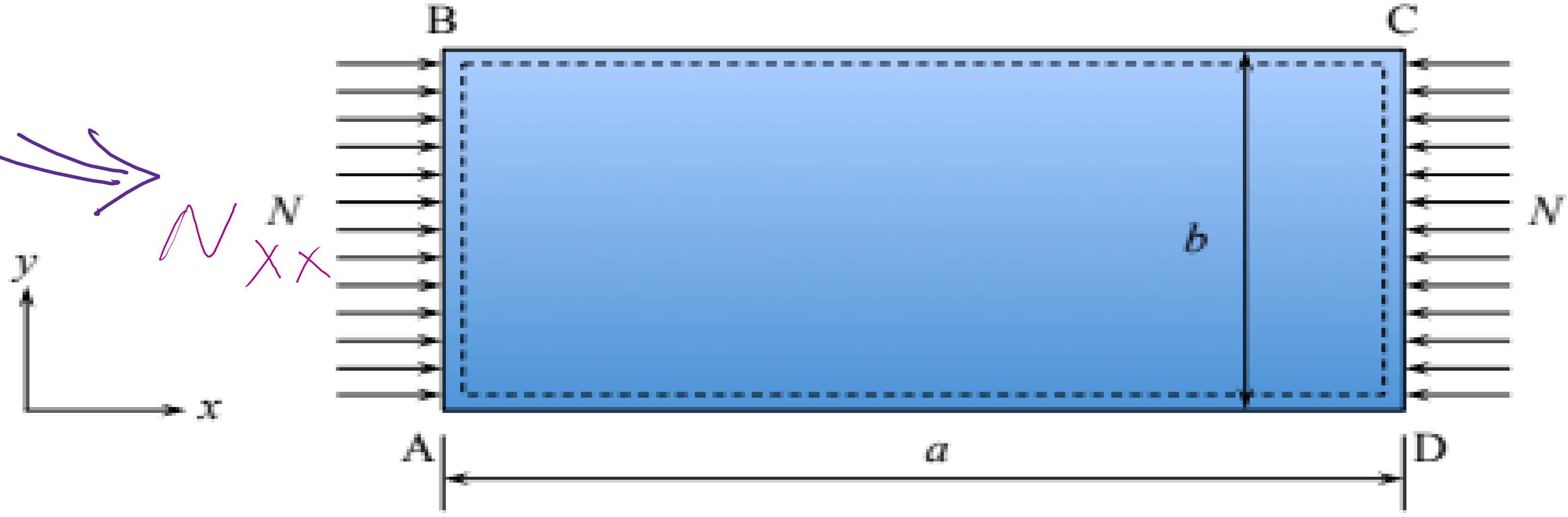
Deflection for plate and expression for w



Column is Composed of Plates → Stiffened / → Unstiffened

Governing equations

Load acts as shown → N_{xx}



N_{xx} Load/m or load/ft

Figure 11.1: Geometry and loading of the classical plate buckling problem.

$EI\,w^{IV} + N w^{II} = 0$

11.1 Governing Equations and Boundary Conditions

In the present notes the column buckling was extensively studied in Lecture 9. The governing equation for a geometrically perfect column is

w : Lateral deflection

$$EIw^{\mathrm{IV}} + Nw'' = 0 \qquad (11.1)$$

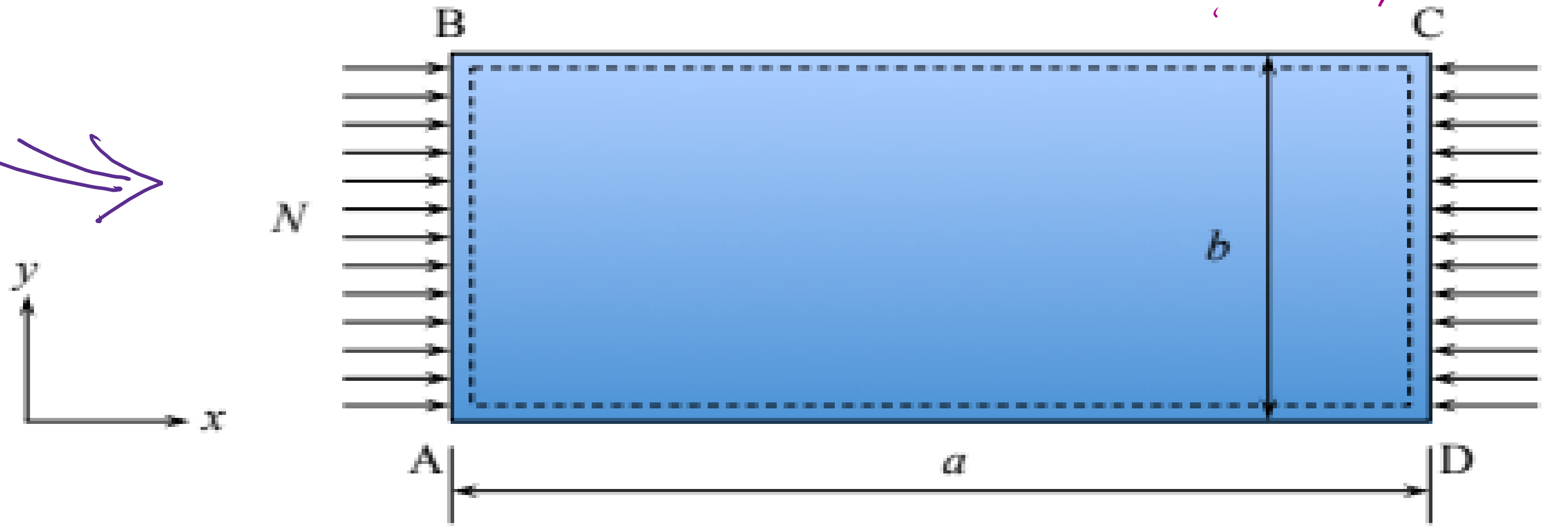
$M = -Nw$

$EIw'' = -Nw$

Column is Composed of Plates → Stiffened / Unstiffened

Critical stresses for plate

Load acts as shown



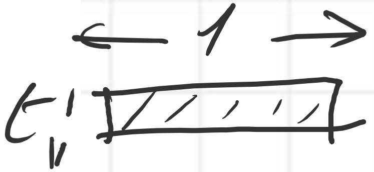


Figure 11.1: Geometry and loading of the classical plate buckling problem.

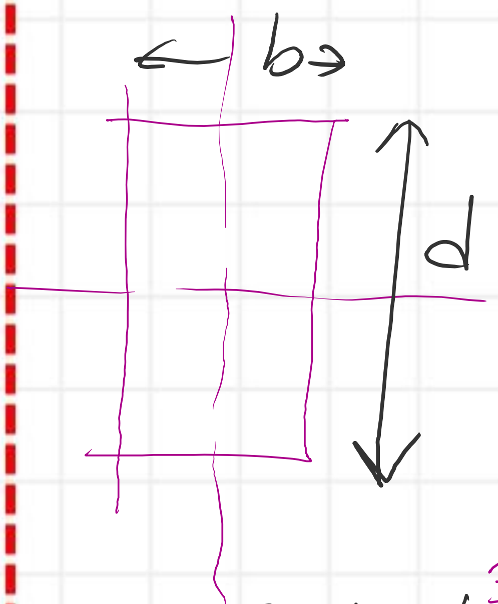
flexural strength

$$N_{xx} = \frac{K\pi^2 D}{b^2} \quad D \rightarrow EI \qquad I = \frac{1}{12}(t)^3$$

$$F_{cr} = \frac{N_{xx}}{(1\,t)} = \frac{K\pi^2}{t\,b^2}\left[\frac{t^3 E}{12(1-\nu^2)}\right] = \frac{K\pi^2}{12}\left(\frac{t}{b}\right)^2\left(\frac{1}{1-\nu^2}\right)$$

Flexural rigidity beam

EI



$b \neq 1$

No ν

Possions

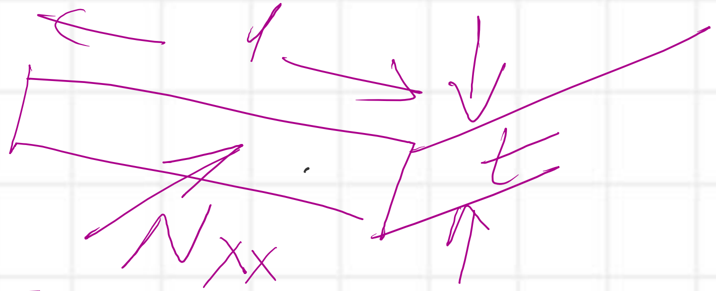
Flexural rigidity for beam and plates

$$E\left(\frac{bd^3}{12}\right)$$

Flexural rigidity Plate

D

$b = 1$



Flexural rigidity for plate

$$EI = \frac{E \cdot t^3 (1)}{12(1-\nu^2)}$$

Comparison between EI For a beam / plate

K_C Equation which is based on $\frac{b}{a}$ & m

$$\sigma = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2\left[\left(\frac{mb}{a}\right)^2 + 2n^2 + n^4\left(\frac{a}{mb}\right)^2\right] \qquad (2.40)$$

We now desire the values of the coefficients m and n that result in the lowest characteristic value of stress and at which the buckled form first becomes stable. It is evident that the bracketed term in Eq. (2.30) increases as n increases. Consequently, the lowest integer value of $n = 1$ will result in the critical compressive stress

Kc and m relation

$$\sigma_{cr} = \frac{\pi^2 k_c E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \qquad (2.41)$$

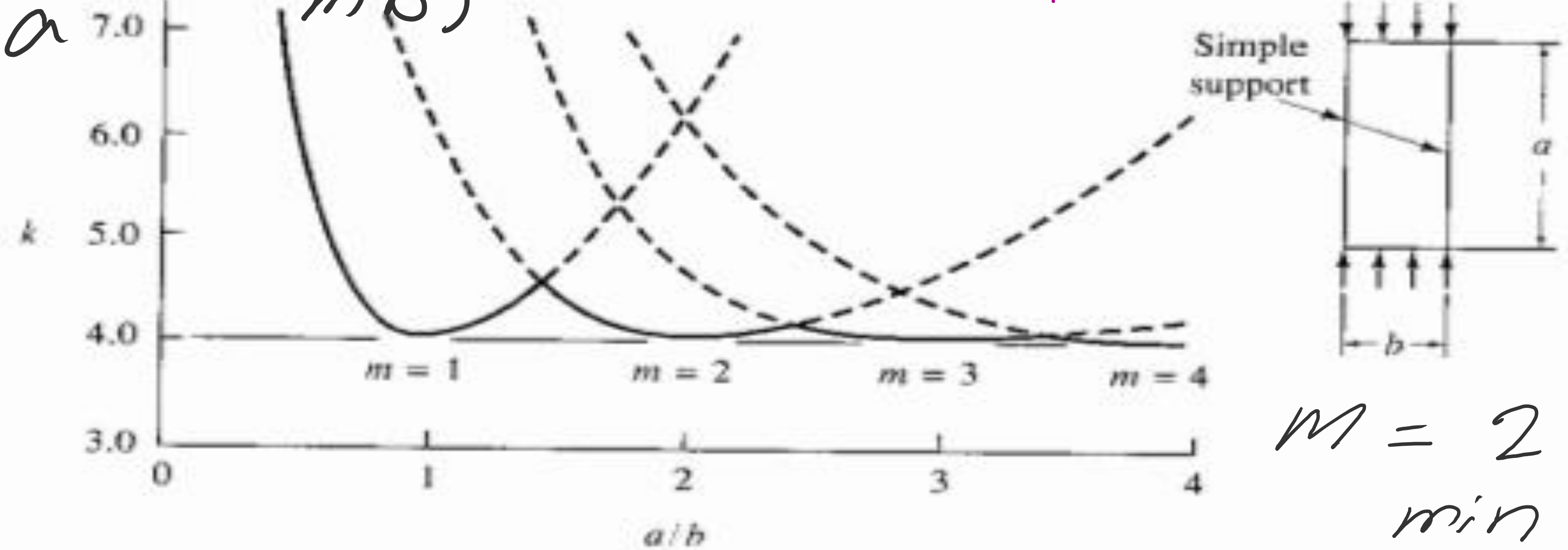
where $$k_c = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \qquad (2.42)$$

The term k_c is commonly referred to as the buckling coefficient. By inserting $n = 1$ into Eq. (2.37), we find that the buckle pattern of a simply supported plate is such that one half-wave forms across the width b of the plate.

K_c Value for $m = 1$ For $\frac{a}{b} = 1$ $m=1$

$$K_c = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 = (1+1)^2 = 4 \quad K_c = 4$$

square plate



$m = 2 \Rightarrow k = 4$ min. value

Figure 6.14.8 Buckling coefficient for uniformly compressed plate—simple support longitudinal edges (Eq. 6.14.29).

Pinned in all directions

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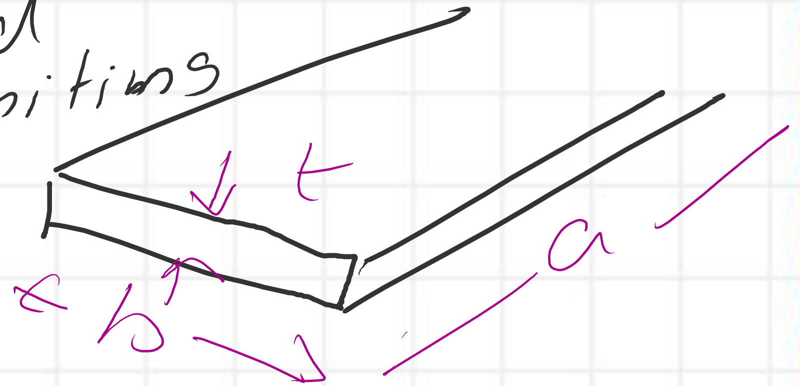
a/b versus K

$$\frac{F_{cr}}{F_y} = \frac{1}{\lambda^2}$$

$$\lambda_c^2 < 1$$

Find Limitings $\frac{b}{t}$



$$F_{cr} = \frac{F_y}{\lambda^2} \geq \left(\frac{\pi K_c E}{12(1-\nu^2)}\right)\left(\frac{t}{b}\right)^2$$

$$\left(\frac{b}{t}\right)^2 \leq \frac{\lambda_c^2 \pi K_c E}{12(1-\nu^2)\lambda_c^2 F_y}$$

$$F_{cr} \geq F_y$$

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}}$$

b/t value

$$\frac{b}{t} \leq \lambda_c \frac{(\pi)\sqrt{K_c}\sqrt{E}}{\sqrt{12(0.91)}\sqrt{F_y}}$$

$\nu = 0.30$ For steel

$\Rightarrow \lambda_c = 1$

$$\frac{b}{t} \leq 0.9506\,\lambda_c \sqrt{\frac{K_c}{F_y}}\sqrt{E}$$

reduce λ_c to increase F_{cr}

But for $\lambda_c = 0.70$ multiply R.H.S

For design F_{cr} plate $\geq F_{cr}$ Column

$F_{cr} \geq F_y$ $\qquad \lambda_c^2 = F_y / F_{cr}$

$F_{cr} = F_y$

$\lambda_c^2 = 1$

actual behavior of plate

$\frac{F_{cr}}{F_y}$

$\frac{F_{cr}}{F_y} = \frac{1}{\lambda_c^2}$

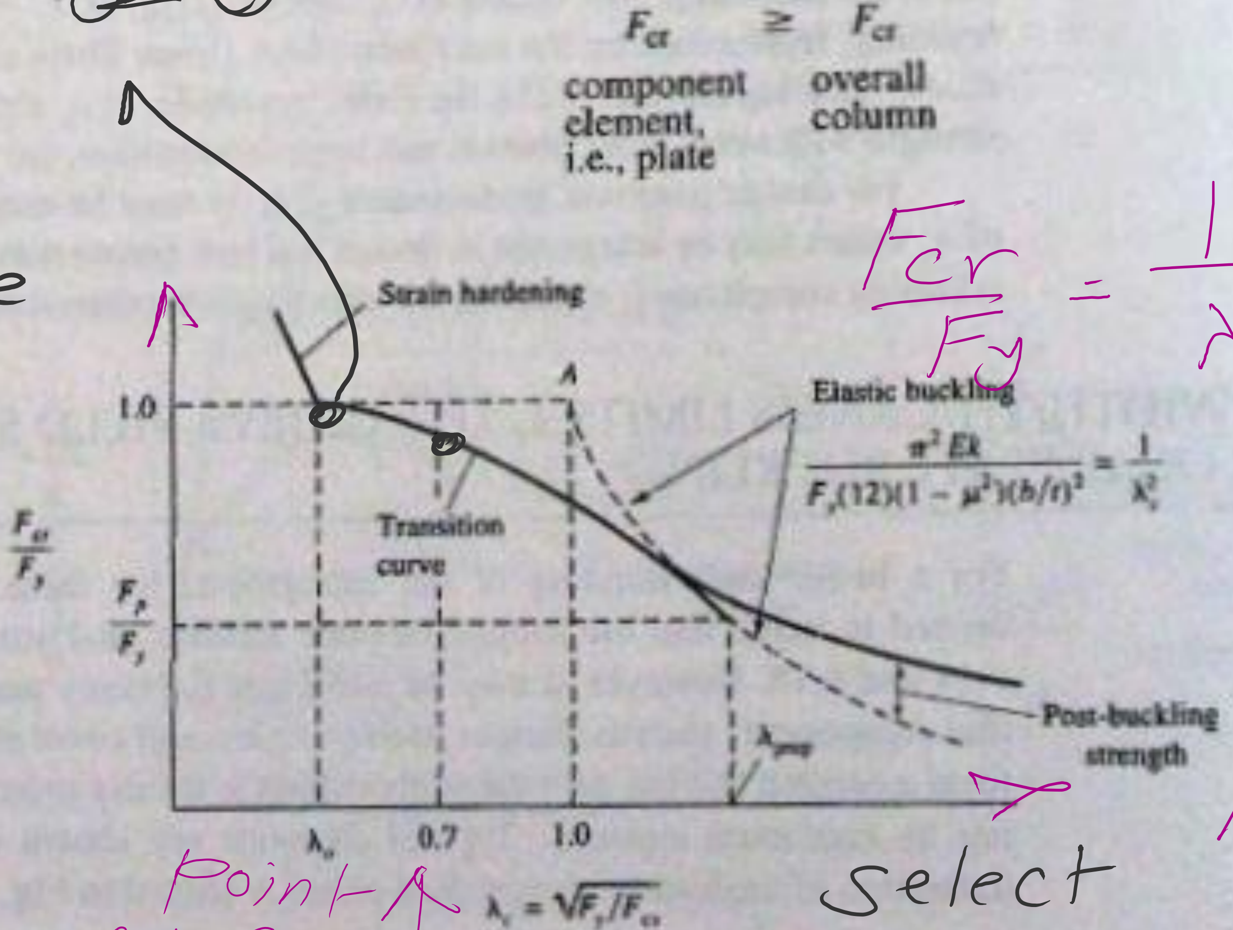


Figure 6.16.2
Dimensionless representation of plate strength in edge compression.

What is lambda C?

$\lambda_c = \sqrt{\frac{F_y}{F_c}}$

Point our

select 0.70

Choose $\lambda_c = 0.70$ due to actual behavior

$\not< b/t \Rightarrow$ increme

Use Transition Curve

Back

$$\frac{b}{t} \leq 0.666 \sqrt{k_c} \left(\sqrt{\frac{E}{F_y}} \right)$$

dependend on stiffened or unstiffened

$\rightarrow$ k_c values

$$\lambda_r \leq 0.666 \sqrt{k_c} \sqrt{\frac{E}{F_y}}$$

What is lambda r?

Five Types of End Conditions

Different End Conditions

A
B
C
D
E

ALL Loading Edges are pinned

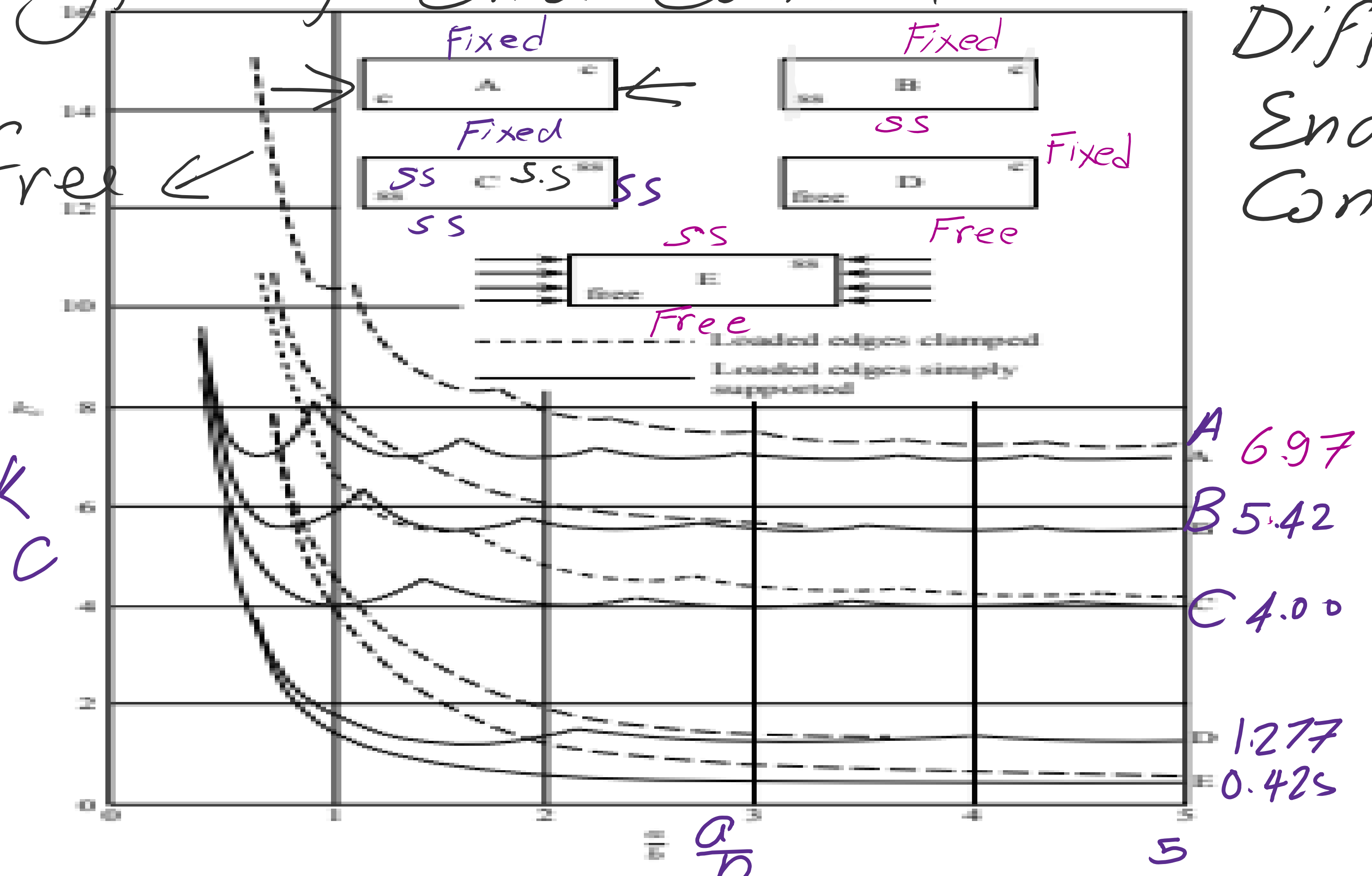


Figure 11.4: Effect of boundary conditions on the buckling coefficient of rectangular plates subjected to in-plane boundary conditions.

Unsported edge
Fixed - Fixed
Fixed - pinned
Pinned - pinned
→ Fixed - Free
Pinned - Free

$\frac{a}{b} > 5$

Kc and a/b for different end conditions

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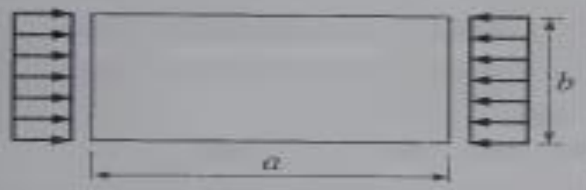
K or Kc Values

Axial Compression	Conditions at Nonloaded Edges	k_c
All loaded edges simply supported  $a/b > 4.0$	Case 1: Both fixed	6.97
	Case 2: One fixed, one simply supported	5.42
	Case 3: Both simply supported	4.00
	Case 4: One fixed, one free	1.277
	Case 5: One simply supported, one free	0.425

Stiffened

Kc and a/b for stiffened and unstiffened

unstiffened

From the previous Graph

Local buckling-limits-report _aisc_adhoctg

Table 9 Assumptions underlying AISC 360-16 *w/t* limits - λ_r Compression Only

		k	$\lambda^* = \sqrt{\frac{F_y}{F_{cr}}}$	Eq. (10)	AISC
	Unstiffened			λ_r	λ_r
1	Rolled Flange	0.70[b]	0.70[a]	$0.56\sqrt{\frac{E}{F_y}}$	$0.56\sqrt{\frac{E}{F_y}}$
2	Built-up Flange	0.35~0.76	0.70[a]	$0.39{\sim}0.58\sqrt{\frac{E}{F_y}}$	$0.38{\sim}0.56\sqrt{\frac{E}{F_y}}$
3	Angle leg, other	0.425[e]	0.70[a]	$0.43\sqrt{\frac{E}{F_y}}$	$0.45\sqrt{\frac{E}{F_y}}$
4	Stem of tee	1.277[f]	0.70[a]	$0.75\sqrt{\frac{E}{F_y}}$	$0.75\sqrt{\frac{E}{F_y}}$

a. non-dimensional slenderness to achieve a plate strength approaching F_y
b. ~1/2 way between pinned and fixed k values
c. ~1/3 of the way between pinned and fixed k values
d. this k factor back-calculated from λ^* and *w/t* limit
e. ideal case for simple-free longitudinal edge conditions
f. ideal case for fixed-free longitudinal edge condition

AISC adhoc

Unstiffened → (a) Rolled Flange

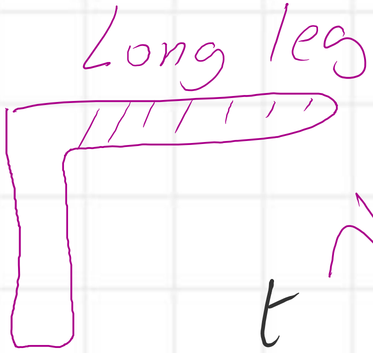
Lambda r for Unstiffened

→ 1/3 distance $= 0.425 + \frac{1}{3}(1.277 - 0.425)$

Pinned - Fixed $= 0.70$ ⇒ $\lambda_r = 0.666\sqrt{0.70}\sqrt{\frac{E}{F_y}}$

$$\lambda_r = 0.557\sqrt{\frac{E}{F_y}} \simeq 0.56\sqrt{\frac{E}{F_y}}$$

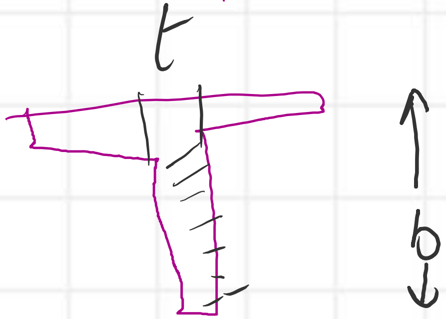
(b)



$K_c = 0.425$ → hinged - Free

$$\lambda_r = 0.666\sqrt{0.425}\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{E}{F_y}}$$

(c)



$K_c = 1.277$ Fixed - Free

$$\lambda_r = 0.75\sqrt{E/F_y}$$

Local buckling-limits-report _aisc_adhoctg

	Stiffened	K	λ		AISC
5	Rolled Web	5.0[c]	0.70[a]	$1.49\sqrt{\frac{E}{F_y}}$	$1.49\sqrt{\frac{E}{F_y}}$
6	HSS Wall	4.4[d]	0.70[a]	$1.40\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$
7	Cover plate	4.4[d]	0.70[a]	$1.40\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$
8	Other	5.0[c]	0.70[a]	$1.49\sqrt{\frac{E}{F_y}}$	$1.49\sqrt{\frac{E}{F_y}}$

a. non-dimensional slenderness to achieve a plate strength approaching F_y
b. ~1/2 way between pinned and fixed k values
c. ~1/3 of the way between pinned and fixed k values
d. this k factor back-calculated from λ^* and w/t limit
e. ideal case for simple-free longitudinal edge conditions
f. ideal case for fixed-free longitudinal edge condition

Lambda for Stiffened elements

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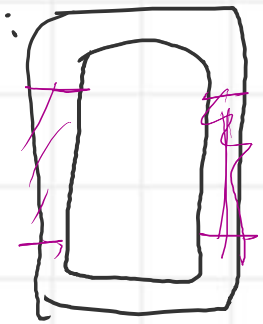
What are the values b/t selected ?

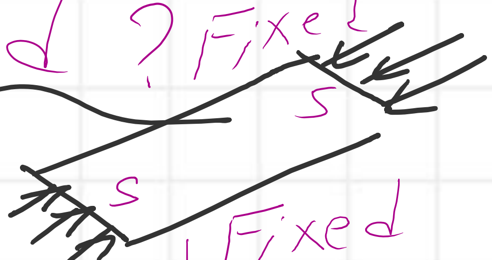
Stiffened

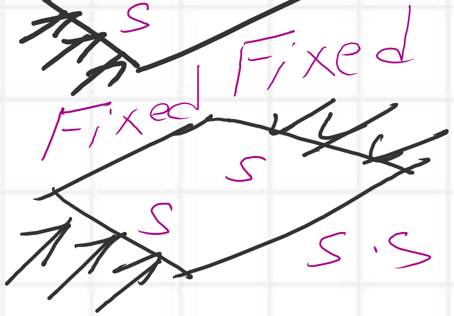
HSS



$K_c = 6.97$

$= 5.42$

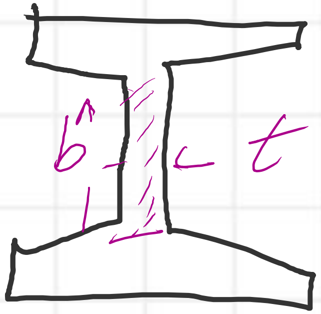




$K_c = 4.00$

$K_c = 4.40 \Rightarrow \lambda_r = 0.666\sqrt{4.40}\sqrt{E/F_y} \cong 1.40\sqrt{\frac{E}{F_y}}$

(b)



Rolled web

$K_c = \frac{1}{3}$ Pinned $\rightarrow$ Fixed

$$4 + \frac{1}{3}(6.97 - 4.00) \cong 5.00$$

$$\lambda_r = 0.666\sqrt{5}\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{E}{F_y}}$$

Kc and lambda r for stiffened elements