Summary of Post 8 - Compression

Solved Problem   From   Iqbal;book

F.E. Exam

@ Problem 13.28   Preparation
Multiple choice selection

$W_{12} \times 50$ Value of available strength in kips

or $\phi_c P_n$ F.E Handbook 10-4

Value   For the given End Conditions

Guided roller
Fixed support

Question 13.28 – A W12×50 is used as a column as shown in Fig. 13.13.

← Guided support (handwritten)

15'

W12x50

Fixed (handwritten)

Case C in Table (handwritten)

Table C.A-7-1 (handwritten)
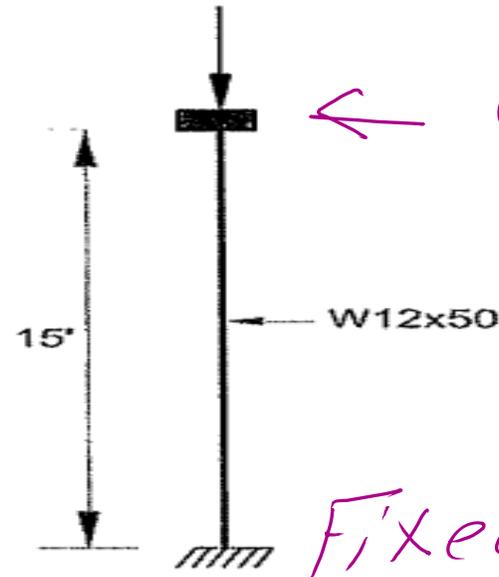
Fig. 13.13

The steel yield stress is 50 ksi. The available strength in axial compression (kips) of the column is most nearly:

A. 90
B. 270
C. 360
D. 660

Solution : The support at the top is a Guided support (handwritten)

- The critical buckling load P_{cr} for columns is theoretically given by Equation (3.1)

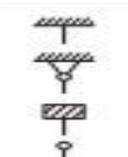$$P_{cr} = \frac{\pi^2 E I}{(K\,L)^2} \qquad (3.1)$$

where, I = moment of inertia about axis of buckling

K = effective length factor based on end boundary conditions

- Effective length factors are given on page 16.1-189 of the AISC manual.

Civil Engineering

TABLE C-A-7.1 AISC APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, *K*						
BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(a)	(b)	(c)	(d)	(e)	(f)
THEORETICAL *K* VALUE	0.5	0.7	1.0	1.0	2.0	2.0
AISC-RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0
END CONDITION CODE	ROTATION FIXED AND TRANSLATION FIXED					
	ROTATION FREE AND TRANSLATION FIXED					
	ROTATION FIXED AND TRANSLATION FREE					
	ROTATION FREE AND TRANSLATION FREE					

Based on Case C

$$K_x = K_y = 1$$

Both X & Y has a Guided Support at Top and a Fixed support at bottom

But it is recommended to have

$$K_x = K_y = 1.2 \quad , \quad h = 15'$$

$$Le_x = Le_y = (KL) = 1.2(15) = 18'$$

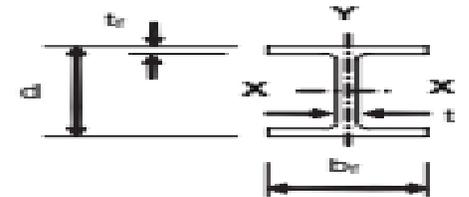Use Table 4-1 For the available strength

Design of Steel Components
(ANSI/AISC 360-16) LRFD, ASD   E = 29,000 ksi

Part - 1

Table 1-1: W Shapes Dimensions and Properties

Shape	Area A In.^{2}	Depth d In.	Web t_w In.	Flange b_f In.	Flange t_f In.	Axis X-X I In.^{4}	Axis X-X S In.^{3}	Axis X-X r In.	Axis X-X Z In.^{3}	Axis Y-Y I In.^{4}	Axis Y-Y r In.
W24X68[c]	20.1	23.7	0.415	8.97	0.585	1830	154	9.55	177	70.4	1.87
W24X62[c]	18.2	23.7	0.430	7.04	0.590	1550	131	9.23	153	34.5	1.38
W24X55[c,v]	16.3	23.6	0.395	7.01	0.505	1350	114	9.11	134	29.1	1.34
W21X73[c]	21.5	21.2	0.455	8.30	0.740	1600	151	8.64	172	70.6	1.81
W21X68[c]	20.0	21.1	0.430	8.27	0.685	1480	140	8.60	160	64.7	1.80
W21X62[c]	18.3	21.0	0.400	8.24	0.615	1330	127	8.54	144	57.5	1.77
W21X55[c]	16.2	20.8	0.375	8.22	0.522	1140	110	8.40	126	48.4	1.73
W21X57[c]	16.7	21.1	0.405	6.56	0.650	1170	111	8.36	129	30.6	1.35
W21X50[c]	14.7	20.8	0.380	6.53	0.535	984	94.5	8.18	110	24.9	1.30
W21X48[c,f]	14.1	20.6	0.350	8.14	0.430	959	93.0	8.24	107	38.7	1.66
W21X44[c]	13.0	20.7	0.350	6.50	0.450	843	81.6	8.06	95.4	20.7	1.26
W18X71	20.8	18.5	0.495	7.64	0.810	1170	127	7.50	146	60.3	1.70
W18X65	19.1	18.4	0.450	7.59	0.750	1070	117	7.49	133	54.8	1.69
W18X60[c]	17.6	18.2	0.415	7.56	0.695	984	108	7.47	123	50.1	1.68
W18X55[c]	16.2	18.1	0.390	7.53	0.630	890	98.3	7.41	112	44.9	1.67
W18X50[c]	14.7	18.0	0.355	7.50	0.570	800	88.9	7.38	101	40.1	1.65
W18X46[c]	13.5	18.1	0.360	6.06	0.605	712	78.8	7.25	90.7	22.5	1.29
W18X40[c]	11.8	17.9	0.315	6.02	0.525	612	68.4	7.21	78.4	19.1	1.27
W16X67[c]	19.7	16.3	0.395	10.2	0.67	954	117	6.96	130	119	2.46
W16X57	16.8	16.4	0.430	7.12	0.715	758	92.2	6.72	105	43.1	1.60
W16X50[c]	14.7	16.3	0.380	7.07	0.630	659	81.0	6.68	92.0	37.2	1.59
W16X45[c]	13.3	16.1	0.345	7.04	0.565	586	72.7	6.65	82.3	32.8	1.57
W16X40[c]	11.8	16.0	0.305	7.00	0.505	518	64.7	6.63	73.0	28.9	1.57
W16X36[c]	10.6	15.9	0.295	6.99	0.430	448	56.5	6.51	64.0	24.5	1.52

Part-2 **W 12 x 50   is not slender Column**

No C Foot note →

Table 1-1: W Shapes Dimensions and Properties

Shape	Area A in.²	Depth d in.	Web t_w in.	Flange b_f in.	Flange t_f in.	Axis X-X I in.⁴	Axis X-X S in.³	Axis X-X r in.	Axis X-X Z in.³	Axis Y-Y I in.⁴	Axis Y-Y r in.
W12X79	23.2	12.4	0.470	12.1	0.735	662	107	5.34	119	216	3.05
W12X72	21.1	12.3	0.430	12.0	0.670	597	97.4	5.31	108	195	3.04
W12X65[f]	19.1	12.1	0.390	12.0	0.605	533	87.9	5.28	96.8	174	3.02
W12X58	17.0	12.2	0.360	10.0	0.640	475	78.0	5.28	86.4	107	2.51
W12X53	15.6	12.1	0.345	9.99	0.575	425	70.6	5.23	77.9	95.8	2.48
W12X50	14.6	12.2	0.370	8.08	0.640	391	64.2	5.18	71.9	56.3	1.96
W12X45	13.1	12.1	0.335	8.05	0.575	348	57.7	5.15	64.2	50.0	1.95
W12X40	11.7	11.9	0.295	8.01	0.515	307	51.5	5.13	57.0	44.1	1.94
W10x60	17.6	10.2	0.420	10.1	0.680	341	66.7	4.39	74.6	116	2.57
W10x54	15.8	10.1	0.370	10.0	0.615	303	60.0	4.37	66.6	103	2.56
W10x49	14.4	10.0	0.340	10.0	0.560	272	54.6	4.35	60.4	93.4	2.54
W10x45	13.3	10.1	0.350	8.02	0.620	248	49.1	4.32	54.9	53.4	2.01
W10x39	11.5	9.92	0.315	7.99	0.530	209	42.1	4.27	46.8	45.0	1.98

Notes:

c Shape is slender for compression with $F_y = 50$ ksi.

r Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

v Shape does not meet the h/t_w limit for shear in AISC Specifications Section G2.1(a) with $F_y = 50$ ksi.

C For Slender Column (Local buckling Parameter)

$r_x = 5.18"$

$r_y = 1.96"$

$A_g = 14.60$ inch^2

To Use Table 4-1 Consider

The tabulated values are given for the effective length with respect to the y-axis, L_{cy}. However, the effective length with respect to the x-axis, L_{cx}, must also be investigated. To determine the available strength in axial compression, the table should be entered at the larger of L_{cy} and $L_{cy\ eq}$, where

$$L_{cy\ eq} = \frac{L_{cx}}{\frac{r_x}{r_y}}$$

$$L_{cy}\ Eq = \left(\frac{L_{cx}}{\frac{r_x}{r_y}} \right)$$

$$\Rightarrow P_{cr_x} = P_{cr_y}$$

$$\frac{\pi^2 E I_x}{(L_{cx})_x^2} = \frac{\pi^2 E I_y}{L_{cy}^2}$$

$$L_{cy} = \sqrt{\frac{I_y}{I_x}}\ L_{cx}$$

$$L_{cy} = L_{cx} \left(\frac{r_y}{r_x} \right)$$

$$L_{cy} = \frac{L_{cx}}{\frac{r_x}{r_y}}$$

$$L_{cx} = (KL)_x = 1.2(15) = 18`$$

$$r_x = 5.18"$$

$$r_y = 1.96" =$$

$$\frac{L_{cx}}{\frac{r_x}{r_y}} = \frac{18}{\left(\frac{5.18}{1.96} \right)} = 6.81`$$

$$L_{Cy} = 1.2(15) = 18'$$

Choose max value of $(6.81', 18') = 18'$

Table 4-1 in the F.E Handbook 10.4

will give ϕP_n LRFD Value

LoG in $L_{Cy} = 18'$, W12×50

Select the available strength value at the intersection.

$\phi c\, P_n = 270\ kips$ W12×50 $F_y = 50\ ksi$

$\phi_c = 0.9$

Selected W14, W12, W10

AISC Table 4–1
Available Strength in Axial Compression, kips—W shapes
LRFD: ϕP_n

$F_y = 50$ ksi
$\phi_c = 0.90$

Shape wt/ft	W14					W12					W10				
	74	68	61	53	48	58	53	50	45	40	60	54	49	45	39
11	797	728	652	497	449	627	569	471	422	375	655	586	533	435	373
12	766	700	626	465	420	603	547	443	396	351	631	565	513	410	351
13	734	670	599	433	391	578	525	413	370	328	606	543	493	384	328
14	701	639	572	401	361	553	501	384	343	304	581	520	471	358	305
15	667	608	543	369	332	527	477	354	317	280	555	496	450	332	282
16	632	576	515	338	304	500	452	326	291	257	528	472	428	306	260
17	598	544	486	308	276	473	427	297	265	234	501	448	405	281	238
18	563	512	457	278	250	446	402	270	241	212	474	423	383	256	216
19	528	480	428	250	224	420	378	244	217	191	447	399	360	233	195
20	494	448	400	226	202	393	353	220	196	172	420	375	338	210	176
22	428	387	345	186	167	342	306	182	162	142	367	327	295	174	146
24	365	329	293	157	140	293	261	153	136	120	317	282	254	146	122
26	311	281	250	133	120	249	222	130	116	102	270	241	216	124	104
28	268	242	215	115	103	215	192	112	99.8	88.0	233	208	186	107	90.0
30	234	211	187	100	89.9	187	167	97.7	87.0	76.6	203	181	162	93.4	78.4

Effective length KL (ft) with respect to least radius of gyration r_y

$L_{cy} \Rightarrow = 18`$

Option (B) is Correct
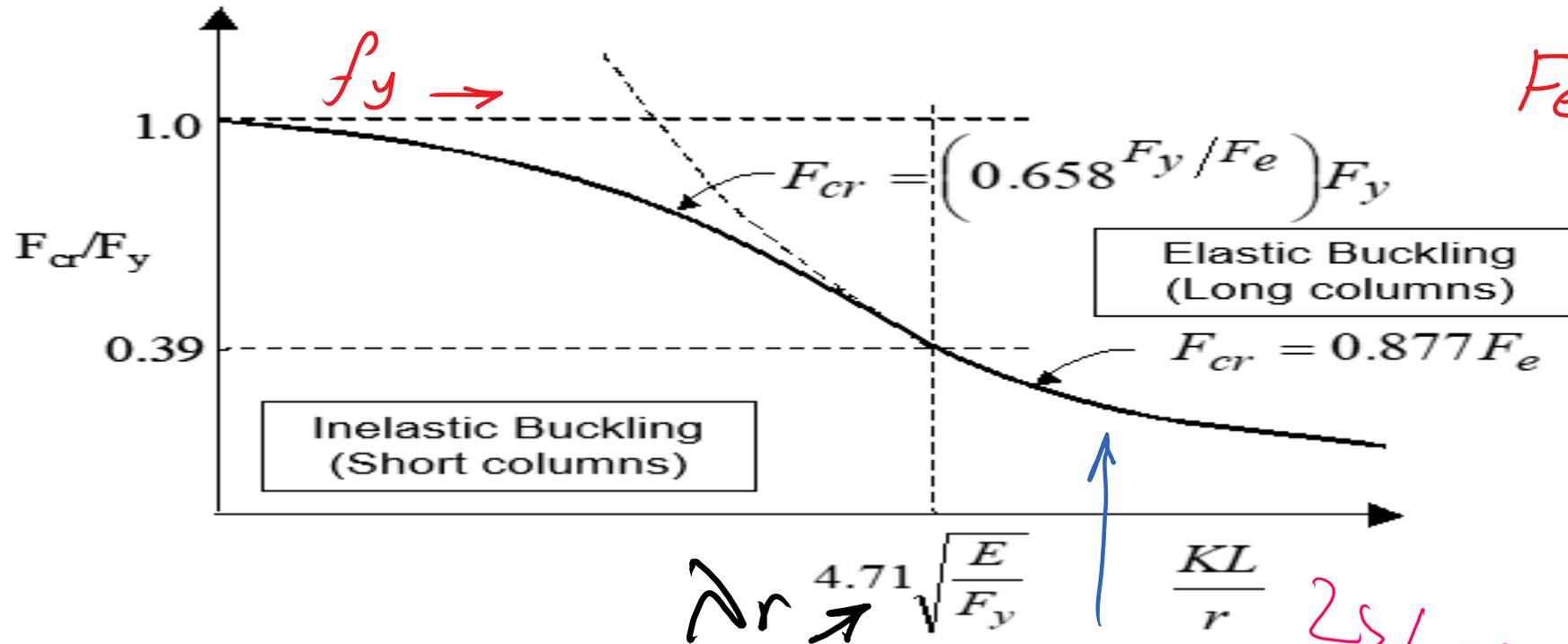
GENERAL PROVISIONS

The *design compressive strength*, $\phi_c P_n$, and the *allowable compressive strength*, $P_n \Omega_c$, are determined as follows.

The *nominal compressive strength*, P_n, shall be the lowest value obtained based on the applicable *limit states* of *flexural buckling*, *torsional buckling*, and *flexural-torsional buckling*.

$$\phi_c = 0.90 \text{ (LRFD)} \qquad \Omega_c = 1.67 \text{ (ASD)}$$

Global buckling

$$F_e = \frac{P_e}{A_g} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad \text{min.}$$

$f_y \rightarrow$

$$1.0$$

$$F_{cr} = \left(0.658^{F_y/F_e}\right)F_y$$

$$F_{cr}/F_y$$

Elastic Buckling (Long columns)

$$0.39$$

$$F_{cr} = 0.877 F_e$$

Inelastic Buckling (Short columns)

$\lambda r \nearrow \quad 4.71\sqrt{\dfrac{E}{F_y}}$

$$\frac{KL}{r}$$

$\left(\dfrac{L_e}{r}\right)$

Prepared by Eng.Maged Kamel.

if you wish to check your answer From

AISC. E-1 equations $h = 15'$, $k = 1.2$

@ Check whether $W_{12 \times 50}$ is Long or short

$$\lambda r = 4.71 \sqrt{\frac{E}{Fy}} = 4.71 \left(\frac{29000}{50}\right)^{1/2} = 113.43$$

$r_x = 5.18''$

$r_y = 1.96$

$$L_{cx} : \left(\frac{KL}{r_x}\right) = \frac{1.2(15)(12)}{(5.18)} = 41.70$$

$$L_{cy} = \left(\frac{KL}{r_y}\right) = \frac{1.2(15)(12)}{1.96} = 110.204 \Rightarrow y \text{ direction governs}$$

$110.204 > 41.7$

(Hint) L_{cy} Final which We Use For Table

4-1 Equals
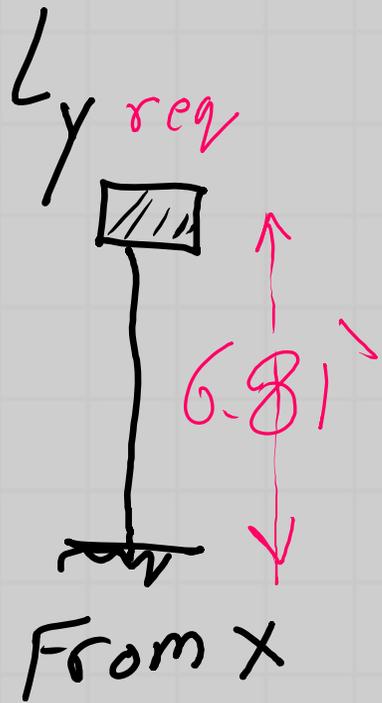
max of $\left[\begin{array}{c}\left(\dfrac{KL}{r_{DL}}\right)r_y \\[2em] \left(\dfrac{KL}{r_y}\right)(r_y)\end{array}\right.$

r_y

$= 41.7(1.96) =$

$= 81.732" \searrow /12 = 6.81'$

$\searrow = 110.264(1.96) = 216$

$= 216 \searrow /12 = 18'$

$L_{y\,req}$

6.81'

From X

18'

original $\Rightarrow$ Bigger

y-direction Height

Column is short $\left(\dfrac{L_c}{r_y}\right)$

$Ag = 14.6 \text{ inch}^2$

$Fy = 50 \text{ ksi}$

$110.204 < 113.43$

$Fcr = 0.658^{\lambda_c^2} \, Fy$ where $\lambda_c^2 = \dfrac{Fy}{Fe}$

$Fe = \dfrac{\pi^2 E}{\left(\dfrac{Le}{r}\right)^2_y} = \dfrac{(3.14159)^2 (29000)}{(110.204)^2} = 23.567 \text{ ksi}$

$Fcr = 0.658^{\left(\frac{50}{23.567}\right)} (50) = 20.574 \text{ ksi}$

$\phi_c Fcr Ag = 0.90 (20.574)(14.6) = 270.34$

$\simeq 270 \text{ kips}$
option B