Area and Cg For Circular segment

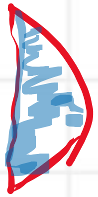
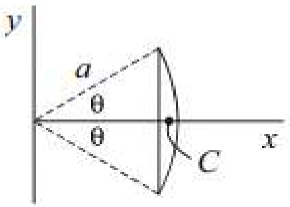
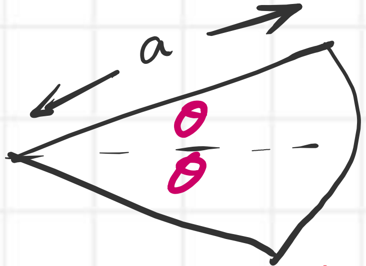


Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
 CIRCULAR SEGMENT	$A = a^2\left[\theta - \frac{\sin 2\theta}{2}\right]$ $x_c = \frac{2a}{3}\frac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$ $y_c = 0$	$I_x = \frac{Aa^2}{4}\left[1 - \frac{2\sin^3\theta\ \cos\theta}{3\theta - 3\sin\theta\ \cos\theta}\right]$ $I_y = \frac{Aa^2}{4}\left[1 + \frac{2\sin^3\theta\ \cos\theta}{\theta - \sin\theta\ \cos\theta}\right]$	$r_x^2 = \frac{a^2}{4}\left[1 - \frac{2\sin^3\theta\ \cos\theta}{3\theta - 3\sin\theta\ \cos\theta}\right]$ $r_y^2 = \frac{a^2}{4}\left[1 + \frac{2\sin^3\theta\ \cos\theta}{\theta - \sin\theta\ \cos\theta}\right]$	$I_{x_c y_c} = 0$ $I_{xy} = 0$

Circular segment =



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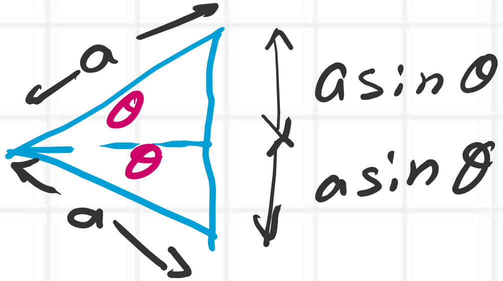
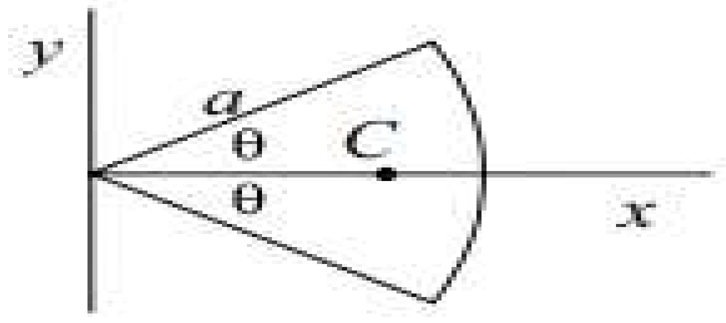


Can be treated as a Circular sector from which two triangles are deducted

Prepared by Eng.Maged Kamel.

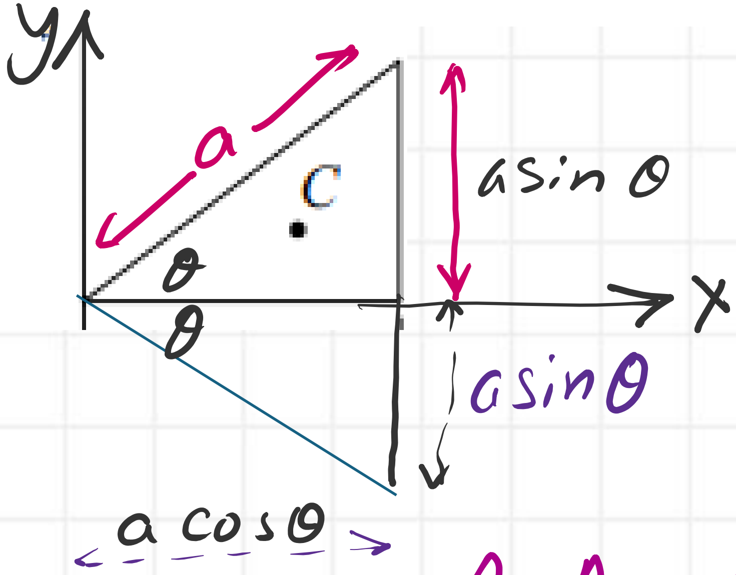
Area of the circular sector.



CIRCULAR SECTOR

$$A = a^2\theta$$
$$x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$$
$$y_c = 0$$

$$A_1 = a^2\theta$$



$$A_2 = 2\left(\frac{1}{2}\right)(a\cos\theta)(a\sin\theta)$$

$$A_2 = a^2 \sin\theta\cos\theta$$

Since $\sin(2\theta) = 2\sin\theta \cdot \cos\theta$

$$A_2 = \frac{a^2}{2}(\sin(2\theta))$$

$$A_1 - A_2 = a^2\left[\theta - \frac{1}{2}\sin(2\theta)\right]$$

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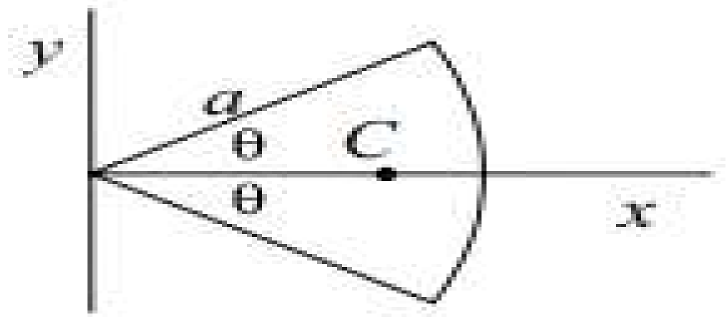
X distance from Cg to y- axis (1/3)



CIRCULAR SECTOR

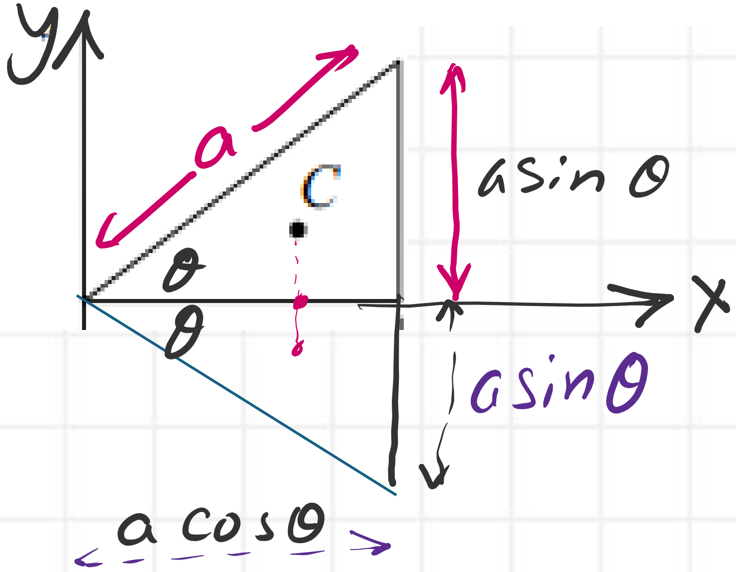
$$A = a^2\theta$$

$$x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$$

$$y_c = 0$$

$$A_1 = a^2\theta$$

$$A_1(x_{c_1}) = \frac{2}{3}a^3\sin\theta$$



$$A_2 = \frac{a^2}{2}(\sin(2\theta))$$

$$X_{2c} = \frac{2}{3}a\cos\theta$$

$$A_2x_2 = \frac{1}{3}a^3(\cos\theta)(\sin 2\theta)$$

$$A_2x_2 = \frac{1}{3}a^3(2)(\sin\theta)(\cos^2\theta)$$

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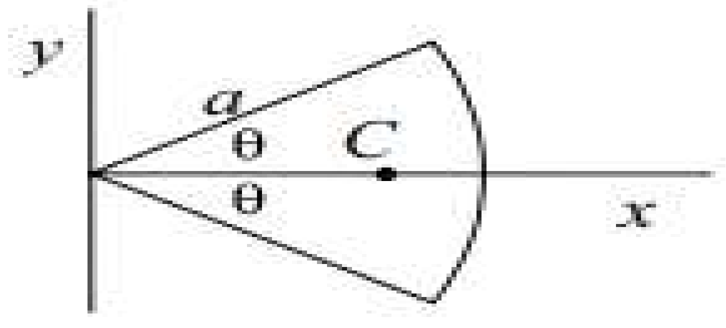
X distance from Cg to y-axis (2/3)



CIRCULAR SECTOR

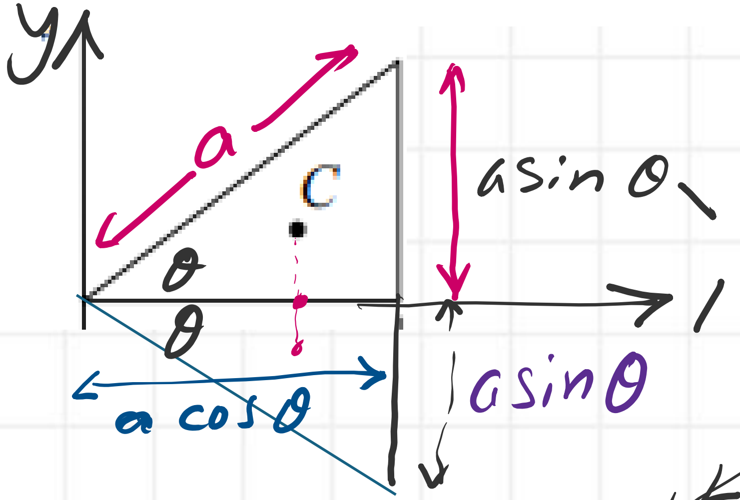
$$A = a^2\theta$$

$$x_c = \frac{2}{3}\frac{a \sin\theta}{\theta}$$

$$y_c = 0$$

$$A_1 = a^2\theta$$

$$A_1(x_{c1}) = \frac{2}{3}a^3 \sin\theta$$



$$A_2x_2 = \frac{1}{3}a^3(2)(\sin\theta)(\cos^2\theta)$$

$$\text{sum }(A\cdot x) = \frac{2}{3}a^3\sin\theta - \frac{2}{3}a^3\sin\theta\cos^2\theta$$

$$= \frac{2}{3}a^3\left[\sin\theta - \cos^2\theta\cdot\sin\theta\right]$$

$$\frac{2}{3}a^3\sin\theta\left[1-\cos^2\theta\right]$$

$$= \frac{2}{3}a^3\sin^3\theta$$

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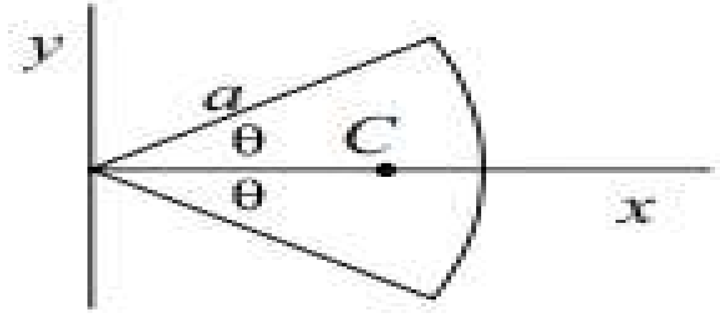
X distance from Cg to y-axis (3/3)



CIRCULAR SECTOR

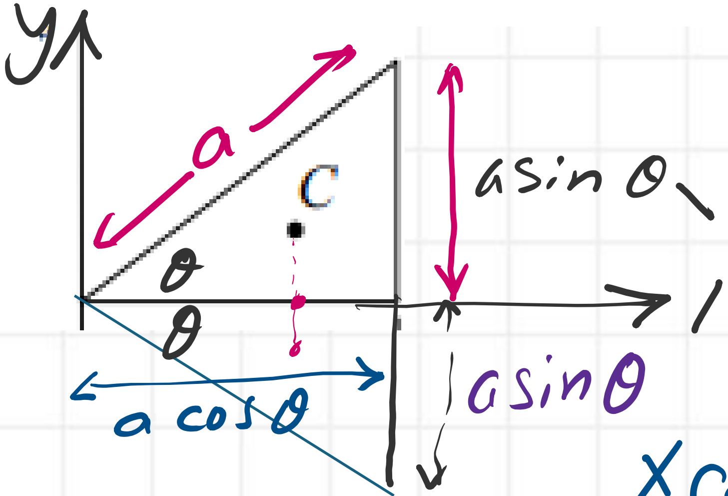
$$A = a^2\theta$$

$$x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$$

$$y_c = 0$$

$$A_1 = a^2\theta$$

$$A_1(x_c) = \frac{2}{3}a^3\sin\theta$$



$$A.x = \frac{2}{3}a^3\left[\sin^3\theta\right]$$

$$A_1 - A_2 = a^2\left[\theta - \frac{1}{2}\sin(2\theta)\right]$$

Divide to get XC

$$X_{cg} = \frac{2}{3}a\,\frac{\sin^3\theta}{(\theta - \frac{1}{2}\sin\theta\cos\theta))}$$

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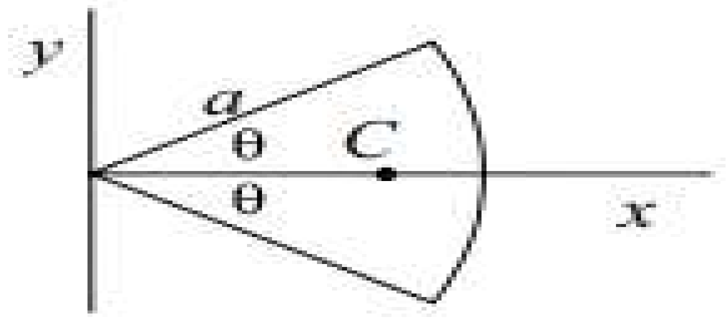
Vertical Cg distance to x-axis.



CIRCULAR SECTOR

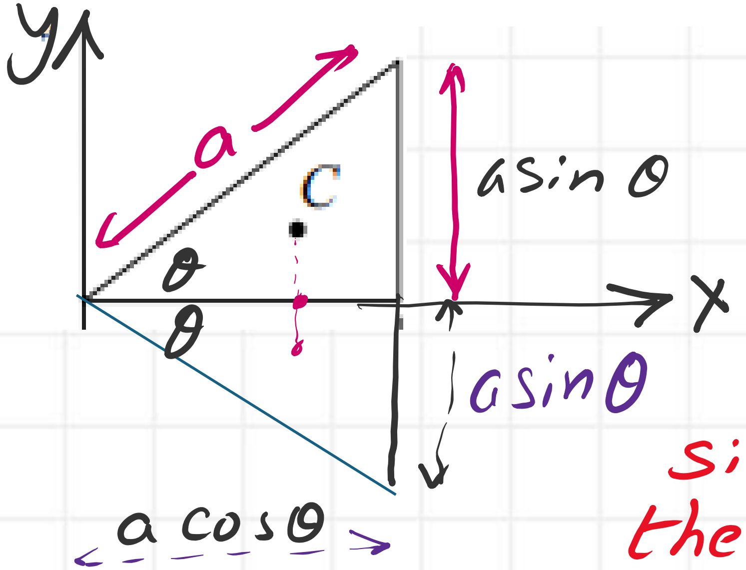
$$A = a^2\theta$$

$$x_c = \frac{2a}{3}\,\frac{\sin\theta}{\theta}$$

$$y_c = 0$$

$A_1 = a^2\theta$

$A_1 y_{1c} = 0\,(a^2\theta) = 0$



$$\left.\begin{array}{l} A_2 = \dfrac{a^2}{2}\left(\sin(2\theta)\right) \\ y_{2c} = 0 \end{array}\right\}\; A_2 \cdot y_{2c} = 0$$

$$A y_c = A_1 y_{c_1} - A_2 y_{2c} = 0$$

since $A \neq 0$

then $y_c = 0$

Prepared by Eng.Maged Kamel.