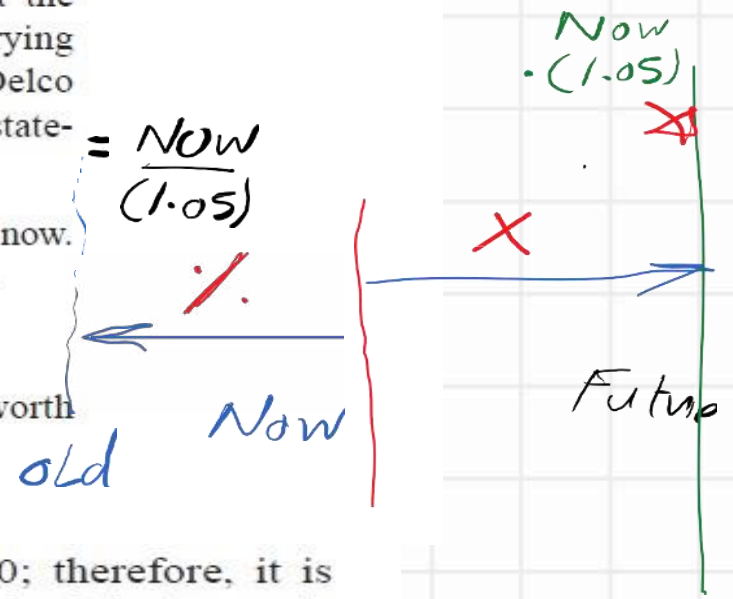
EXAMPLE 1.3

AC-Delco makes auto batteries available to General Motors dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Delco facility. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

a. The amount of $98 now is equivalent to a cost of $105.60 one year from now.
b. A truck battery cost of $200 one year ago is equivalent to $205 now.
c. A $38 cost now is equivalent to $39.90 one year from now.
d. A $3000 cost now is equivalent to $2887.14 one year ago.
e. The carrying charge accumulated in 1 year on an investment of $2000 worth of batteries is $100.



Solution

a. Total amount accrued = 98(1.05) = $102.90 ≠ $105.60; therefore, it is false. Another way to solve this is as follows: Required original cost is 105.60/1.05 = $100.57 ≠ $98.
b. Required old cost is 205.00/1.05 = $195.24 ≠ $200; therefore, it is false.
c. The cost 1 year from now is $38(1.05) = $39.90; true.
d. Cost now is 2887.14(1.05) = $3031.50 ≠ $3000; false.
e. The charge is 5% per year interest, or $2000(0.05) = $100; true.

Example of an Interest Transaction

As an example of how the elements we have just defined are used in a particular situation, let us suppose that an electronics manufacturing company buys a machine for $25,000 and borrows $20,000 from a bank at a 9% annual interest rate. In addition, the company pays a $200 loan origination fee when the loan commences. The bank offers two repayment plans as follows:

- **Plan 1.** The principal amount P is $20,000, and the interest rate i is 9%. The interest period is one year, and the duration of the transaction is five years, which means there are five interest periods ($N = 5$). As mentioned earlier, whereas one year is a common interest period, interest is frequently calculated at other intervals: monthly, quarterly, or semiannually, for instance. For this reason, we used the term **period** rather than **year** when we defined the preceding list of variables. The receipts and disbursements planned over the duration of this transaction yield a cash flow pattern of five equal payments A of $5,141.85 each, paid at year's end during years 1 through 5. (You will have to accept these amounts on faith for now—the next section presents the formula

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used to arrive at the amount of these equal payments, given the other elements of the problem.)

- **Plan 2.** This plan has most of the elements of Plan 1, except that the partial payment of principal is not allowed. Instead, only interest is paid each year and the principal is paid in a lump sum when the loan matures.

These two payment plans are summarized in Table 3.1.

TABLE 3.1 **Repayment Plans for Example Given in Text (for $N = 5$ years and $i = 9\%$)**

End of Year	Receipts	Payment Plan	
		Plan 1	Plan 2
Year 0	$20,000.00	$ 200.00	$ 200.00
Year 1		5,141.85	1,800.00
Year 2		5,141.85	1,800.00
Year 3		5,141.85	1,800.00
Year 4		5,141.85	1,800.00
Year 5		5,141.85	21,800.00

Note: An origination fee is an up-front fee charged by a lender for processing a new loan. You actually borrow $19,800 with the origination fee of $200, but you pay back on the basis of $20,000.

EXAMPLE 3.6 Equivalence Calculations with Multiple Payments

Recall the repayment plans in Table 3.1. Determine whether these repayment plans are equivalent to the bank with a lending rate of 9%.

STRATEGY: Notice that under Plan 1, we will pay a total of $25,909.25, whereas under Plan 2, we will pay a total of $29,200. Before concluding that we prefer Plan 1, remember that a comparison of the two cash flows is based on a *combination of payment amounts and the timing of those payments.* To make our comparison, we must

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compare the equivalent value of each option at a single point in time. It is simplest to convert the cash flow pattern of both plans to a single value at $n = 5$. To do this, we must convert the six disbursements of each plan to their respective equivalent values at $n = 5$. At that point, since they share a time in common, we can simply sum them in order to compare them (Figure 3.8).

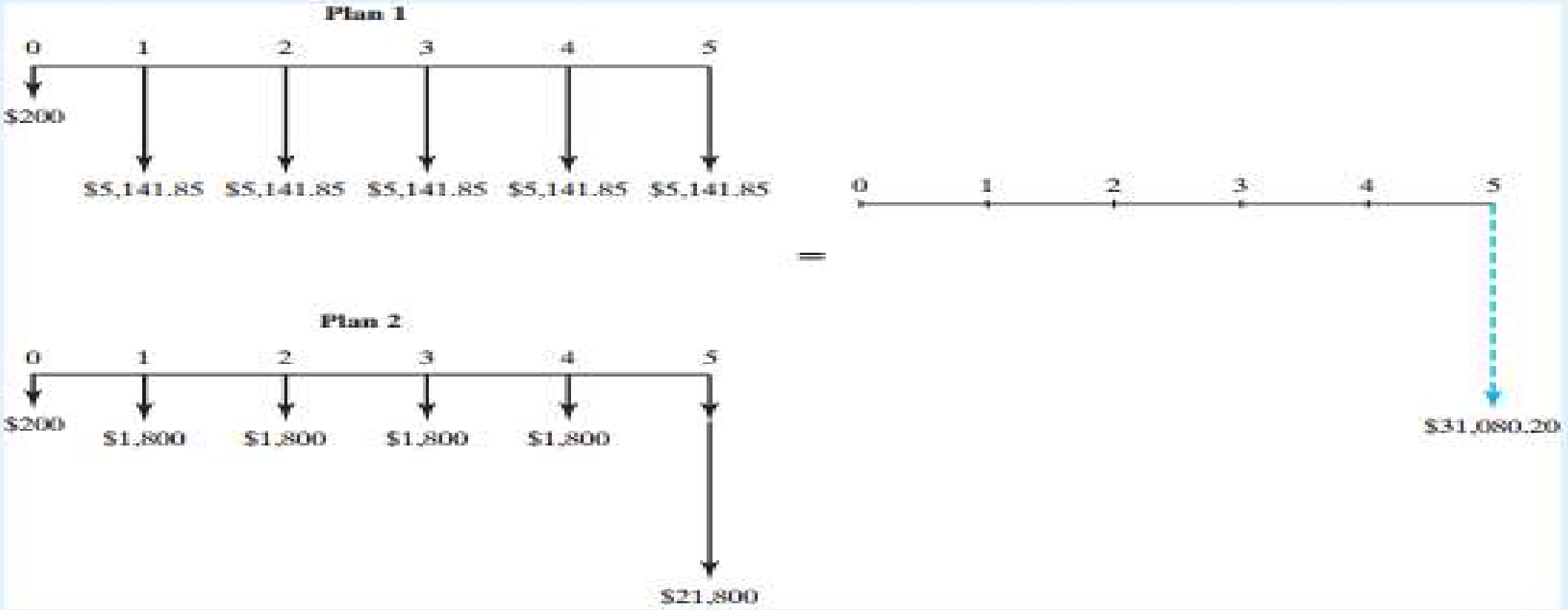

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Figure 3.8 Loan repayment schedules for two options (Example 3.6).

SOLUTION

Given: Loan repayment plans; $i = 9\%$ per year and $N = 5$

Find: A single future value F of the cash flows at the end of year 5.

Equation: $F = P(1 + i)^N$, applied to each disbursement in the cash flow diagram, where N is the number of periods during which interest is in effect, and n is the period number (i.e., for year 1, $n = 1$). We determine the value of F by finding the number of interest periods applicable for each payment. By converting the cash flow in Plan 2 to a single future payment at year 5, we can compare Plans 1 and 2.

- Plan 1

$$
\begin{aligned}
F_5 \text{ for \$200 paid at } n = 0: &\quad \$200(1 + 0.09)^{5-0} = \$ \quad 307.72 \\
F_5 \text{ for \$5,141.85 paid at } n = 1: &\quad \$5{,}141.85(1 + 0.09)^{5-1} = \$ \quad 7{,}258.14 \\
F_5 \text{ for \$5,141.85 paid at } n = 2: &\quad \$5{,}141.85(1 + 0.09)^{5-2} = \$ \quad 6{,}658.84 \\
F_5 \text{ for \$5,141.85 paid at } n = 3: &\quad \$5{,}141.85(1 + 0.09)^{5-3} = \$ \quad 6{,}109.03 \\
F_5 \text{ for \$5,141.85 paid at } n = 4: &\quad \$5{,}141.85(1 + 0.09)^{5-4} = \$ \quad 5{,}604.62 \\
F_5 \text{ for \$5,141.85 paid at } n = 5: &\quad \$5{,}141.85(1 + 0.09)^{5-5} = \$ \quad 5{,}141.85 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \$31{,}080.20
\end{aligned}
$$

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- Plan 2

$$
\begin{aligned}
F_5 \text{ for \$200 paid at } n = 0: &\quad \$200(1 + 0.09)^{5-0} = \$ \quad 307.72 \\
F_5 \text{ for \$1,800 paid at } n = 1: &\quad \$1{,}800(1 + 0.09)^{5-1} = \$ \ 2{,}540.85 \\
F_5 \text{ for \$1,800 paid at } n = 2: &\quad \$1{,}800(1 + 0.09)^{5-2} = \$ \ 2{,}331.05 \\
F_5 \text{ for \$1,800 paid at } n = 3: &\quad \$1{,}800(1 + 0.09)^{5-3} = \$ \ 2{,}138.58 \\
F_5 \text{ for \$1,800 paid at } n = 4: &\quad \$1{,}800(1 + 0.09)^{5-4} = \$ \ 1{,}962.00 \\
F_5 \text{ for \$1,800 paid at } n = 5: &\quad \$21{,}800(1 + 0.09)^{5-5} = \$21{,}800.00 \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \$31{,}080.20
\end{aligned}
$$

We see that the two repayment schedules are equivalent. Thus, the bank would be economically indifferent to a borrower's choice between the two plans. Note that the final payment in each plan does not accrue any compound interest.

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