Solved Example 4-13 For inelastic Column

Find stiffness reduction Factor τ_b Prof. SEGui

For a given W10x54 Considered as inelastic Column

Service DL = 100 kips

Service LL = 200 kips

A992 steel $F_y = 50$ ksi

No given Height for Column

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(b) An additional factor, τ_b, shall be applied to the flexural stiffnesses of all members whose flexural stiffnesses are considered to contribute to the stability of the structure. For noncomposite members, τ_b shall be defined as follows (see Section I1.5 for the definition of τ_b for composite members).

check $\alpha P_r / P_{ns} < 0.5$

(1) When $\alpha P_r/P_{ns} \leq 0.5$

$$\tau_b = 1.0 \tag{C2-2a}$$

$\rightarrow \tau_b = 1.0$

(2) When $\alpha P_r/P_{ns} > 0.5$

$$\tau_b = 4(\alpha P_r/P_{ns})[1-(\alpha P_r/P_{ns})] \tag{C2-2b}$$

where

α = 1.0 (LRFD); α = 1.6 (ASD)

P_r = required axial compressive strength using LRFD or ASD load combinations, kips (N)

P_{ns} = cross-section compressive strength; for nonslender-element sections, $P_{ns} = F_y A_g$, and for slender-element sections, $P_{ns} = F_y A_e$, where A_e is as defined in Section E7, kips (N)

User Note: Taken together, Sections (a) and (b) require the use of $0.8\tau_b$ times the nominal elastic flexural stiffness and 0.8 times other nominal elastic stiffnesses for structural steel members in the analysis.

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Substituting the required strength, αP_r, for the available strength, P_n, we have

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left(1-\frac{\alpha P_r}{P_y}\right) \qquad \text{(AISC Equation C2-2b)}$$

where $\alpha = 1.0$ for LRFD and 1.6 for ASD. The required strength is computed at the factored load level, and the 1.6 factor is used to adjust the ASD service load level to a factored load level. The stiffness reduction factor, τ_b, is also used to adjust member stiffnesses for frame analysis. This is discussed in Chapter 6, "Beam–Columns."

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EXAMPLE 4.13

A W10 × 54 of A992 steel is used as a column. It is subjected to a service dead load of 100 kips and a service live load of 200 kips. If the slenderness ratio makes this member an inelastic column, what is the stiffness reduction factor, τ_b?

Solution

A992 steel $\Rightarrow F_y = 50$ ksi

$D = 100$ kips & $L = 200$ kips

LRFD

$1.2D = 120$

$1.6L = 320$

440 kips

$1.4D = 1.4(100) = 140$ kips

max of

$W10 \times 54 = A_g = 15.80 \text{ inch}^2$

inelastic Column

τ_b? $P_{ns} = P_y$

$\frac{\alpha P_r}{P_{ns}} \leq \frac{1}{2}$ $\tau = 1.00$

$\frac{\alpha P_r}{P_{ns}} > \frac{1}{2}$ $\tau < 1$

P_r: required Load

$= \max$ of $(440, 140) = 440$ kips

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$\alpha = 1$ $\quad P_r = 440$ kips $\quad$ LRFD

$P_{ns} = A_g F_y \rightarrow$ For non slender

(1) When $\alpha P_r / P_{ns} \le 0.5$

$$\tau_b = 1.0$$

(2) When $\alpha P_r / P_{ns} > 0.5$

$$\tau_b = 4(\alpha P_r / P_{ns})[1 - (\alpha P_r / P_{ns})]$$

$\rightarrow A_g = 15.80 \text{ inch}^2$

$F_y = 50$ ksi

$P_{ns} = 15.8(50) = 790$ kips

$$\frac{\alpha P_r}{P_{ns}} = \frac{1(440)}{790} = 0.557 > 0.5$$

$$\tau_b = 4\left(\frac{\alpha P_r}{P_{ns}}\right)\left[1 - \frac{\alpha P_r}{P_{ns}}\right]$$

$$= 4(0.557)(1 - 0.557) = 0.987$$

(2) When $\alpha P_r/P_{ns} > 0.5$

$P_r = P_{ult} = 440$

$P_{ns} = P_y = 50(15.80) = 790$ kips

$$\tau_b = 4(\alpha P_r/P_{ns})[1-(\alpha P_r/P_{ns})] \quad \text{(C2-2b)}$$

our Case

$\alpha = 1$

Use Table 4-13

LRFD

$\frac{P_u}{A} = \frac{440}{15.80} = 27.848$ ksi

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LRFD DeSign

Table 4-13
Stiffness Reduction Factor τ_b

50↓

$\frac{P_u}{A} = 50$ ksi

Pult → F_{strom}

27.85 ⇉

ASD	LRFD	F_y, ksi											
$\frac{P_a}{A}$	$\frac{P_u}{A}$	35		36		46		50		65		70	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
50		—	—	—	—	—	—	—	—	—	0.710	—	0.816
49		—	—	—	—	—	—	—	0.0784	—	0.742	—	0.840
48		—	—	—	—	—	—	—	0.154	—	0.773	—	0.862
47		—	—	—	—	—	—	—	0.226	—	0.801	—	0.882
46		—	—	—	—	—	—	—	0.294	—	0.827	—	0.901
45		—	—	—	—	—	0.0851	—	0.360	—	0.852	—	0.918
44		—	—	—	—	—	0.166	—	0.422	—	0.875	—	0.934
43		—	—	—	—	—	0.244	—	0.482	—	0.896	0.0674	0.948
42		—	—	—	—	—	0.318	—	0.538	—	0.915	0.154	0.960
41		—	—	—	—	—	0.388	—	0.590	—	0.932	0.236	0.971
40		—	—	—	—	—	0.454	—	0.640	0.0606	0.947	0.313	0.980
39		—	—	—	—	—	0.516	—	0.686	0.154	0.960	0.387	0.987
38		—	—	—	—	—	0.575	—	0.730	0.242	0.971	0.457	0.993
37		—	—	—	—	—	0.629	—	0.770	0.325	0.981	0.522	0.997
36		—	—	—	—	—	0.681	—	0.806	0.404	0.988	0.583	0.999
35		—	—	—	0.108	—	0.728	—	0.840	0.477	0.994	0.640	1.00
34		—	0.111	—	0.210	—	0.771	—	0.870	0.546	0.998	0.693	
33		—	0.216	—	0.306	—	0.811	—	0.898	0.610	1.00	0.741	
32		—	0.313	—	0.395	—	0.847	—	0.922	0.669		0.786	
31		—	0.405	—	0.478	—	0.879	0.0317	0.942	0.723		0.826	
30		—	0.490	—	0.556	—	0.907	0.154	0.960	0.773		0.862	
29		—	0.568	—	0.627	—	0.932	0.267	0.974	0.817		0.894	
28		—	0.640	—	0.691	0.102	0.953	0.373	0.986	0.857		0.922	
27		—	0.705	—	0.750	0.229	0.970	0.470	0.994	0.892		0.945	
26		—	0.764	—	0.802	0.346	0.983	0.559	0.998	0.922		0.964	
25		—	0.816	—	0.849	0.454	0.992	0.640	1.00	0.947		0.980	
24		—	0.862	—	0.889	0.552	0.998	0.713		0.967		0.991	
23		—	0.901	—	0.923	0.640	1.00	0.777		0.982		0.997	
22		—	0.934	0.087	0.951	0.719		0.834		0.993		1.00	

27 ksi ⇒ 0.994
28 ksi → 0.986 ⇒

$\tau_b = 0.994 - (0.85)\left(\frac{0.008}{1}\right)$
$= 0.9872$

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A W10 × 54 of A992 steel is used as a column. It is subjected to a service dead load of 100 kips and a service live load of 200 kips. If the slenderness ratio makes this member an inelastic column, what is the stiffness reduction factor, τ_b?

Solution A992 steel $\Rightarrow F_y = 50$ ksi

$D = 100$ kips & $L = 200$ kips

ASD

$D + L = 100 + 200 = 300$ kips

$W10 \times 54 = A_g = 15.80 \text{ inch}^2$

inelastic Column

τ_b? $P_{ns} = P_y$

$\frac{\alpha P_r}{P_{ns}} \leq \frac{1}{2}$ $\tau = 1.00$

$\frac{\alpha P_r}{P_{ns}} > \frac{1}{2}$ $\tau < 1$

P_r: required Load $= 300$ kips

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$\alpha = 1.6 \quad P_r = 300$ kips

ASD

$P_{ns} = A_g F_y \rightarrow$ For non slender

(1) When $\alpha P_r / P_{ns} \leq 0.5$

$$\tau_b = 1.0 \rightarrow$$

(2) When $\alpha P_r / P_{ns} > 0.5$

$$\tau_b = 4(\alpha P_r / P_{ns})[1 - (\alpha P_r / P_{ns})]$$

$A_g = 15.80\ \text{inch}^2$

$F_y = 50$ ksi

$P_{ns} = 15.8(50) = 790$ kips

$$\frac{\alpha P_r}{P_{ns}} = \frac{1.6(300)}{790} = 0.6076 > 0.50$$

$$\tau_b = 4\left(\frac{\alpha P_r}{P_{ns}}\right)\left[1 - \frac{\alpha P_r}{P_{ns}}\right]$$

$$\tau_b = 4(0.6076)(1 - 0.6076) = 0.9537$$

(2) When $\alpha P_r/P_{ns} > 0.5$

$$\tau_b = 4(\alpha P_r/P_{ns})[1-(\alpha P_r/P_{ns})] \quad \text{(C2-2b)}$$

our Case $\alpha = 1.60$

ASD

Use Table 4-13 ASD

$\frac{P_n}{A} = \frac{300}{15.80} = 18.987$ ksi

ASD

$F_y = 50$ ksi

Table 4-13
Stiffness Reduction Factor τ_b

$\frac{P_n}{A}$ ksi 18.987 → Close to 19 ksi

ASD P_a/A / LRFD P_u/A	F_y, ksi 35 ASD	35 LRFD	36 ASD	36 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD	65 ASD	65 LRFD	70 ASD	70 LRFD
20	0.313	0.980	0.395	0.988	0.847	↓	0.922	↓	1.00	↓	↓	↓
19	0.457	0.993	0.525	0.997	0.896		0.953					
18	0.583	0.999	0.640	1.00	0.936		0.977					
17	0.693	1.00	0.739	↓	0.967		0.992					
16	0.786	1.00	0.822		0.987		0.999					
15	0.862	1.00	0.889		0.998		1.00					
14	0.922	1.00	0.940		0.999		↓					
13	0.964	1.00	0.976		1.00							
12	0.991	1.00	0.996		↓							
11	1.00	1.00	1.00									
10	1.00	1.00	1.00									

– Indicates the stiffness reduction parameter is not applicable because the required strength exceeds the available strength for $L_c/r = 0$.
$A = A_g$ for members not controlled by slender element buckling, in.^{2}
$= A_e$ as defined in AISC *Specification* Section E7 for members controlled by slender element buckling, in.^{2}

18.00 ksi → 0.977
19 ksi → 0.953

$\tau_b \simeq 0.953$

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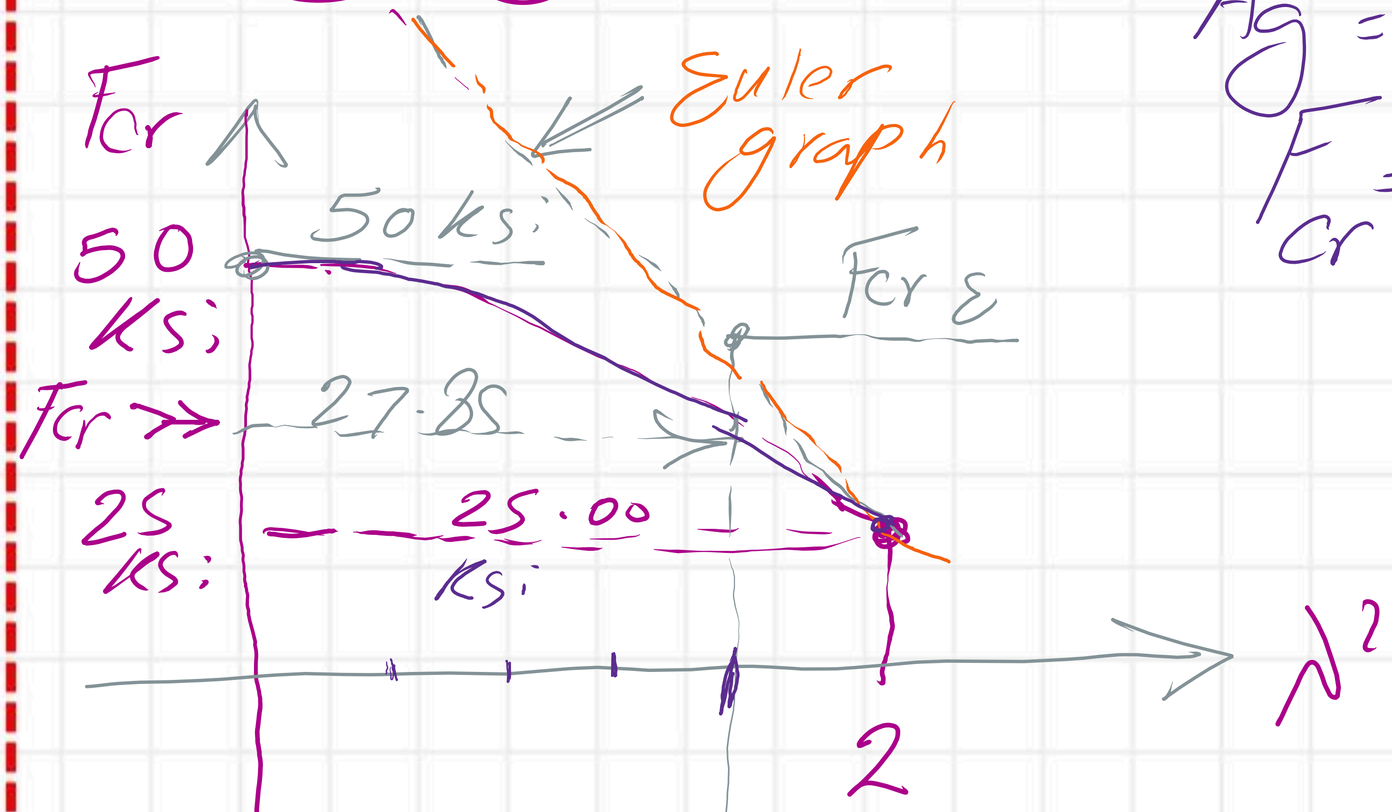
For LRFD



Get λ

√Felastic

λ^2?

$\tau = F_{cr\,in} / F_{cr\,E}$

$P_{ult} = 440$ Kips

$A_g = 15.80$ inch²

$F_{cr} = 27.848$ kips

$F_y = 50$ ksi

$$F_{cr} = F_y\left(1 - \frac{\lambda^2}{4}\right)$$

$$\frac{27.85}{50} = 1\left(1 - \frac{\lambda^2}{4}\right)$$

$$\lambda^2 = 4\left(1 - \frac{27.85}{50}\right)$$

$$\lambda^2 = 1.772$$

$$F_{cr_E} = F_y / \lambda^2$$

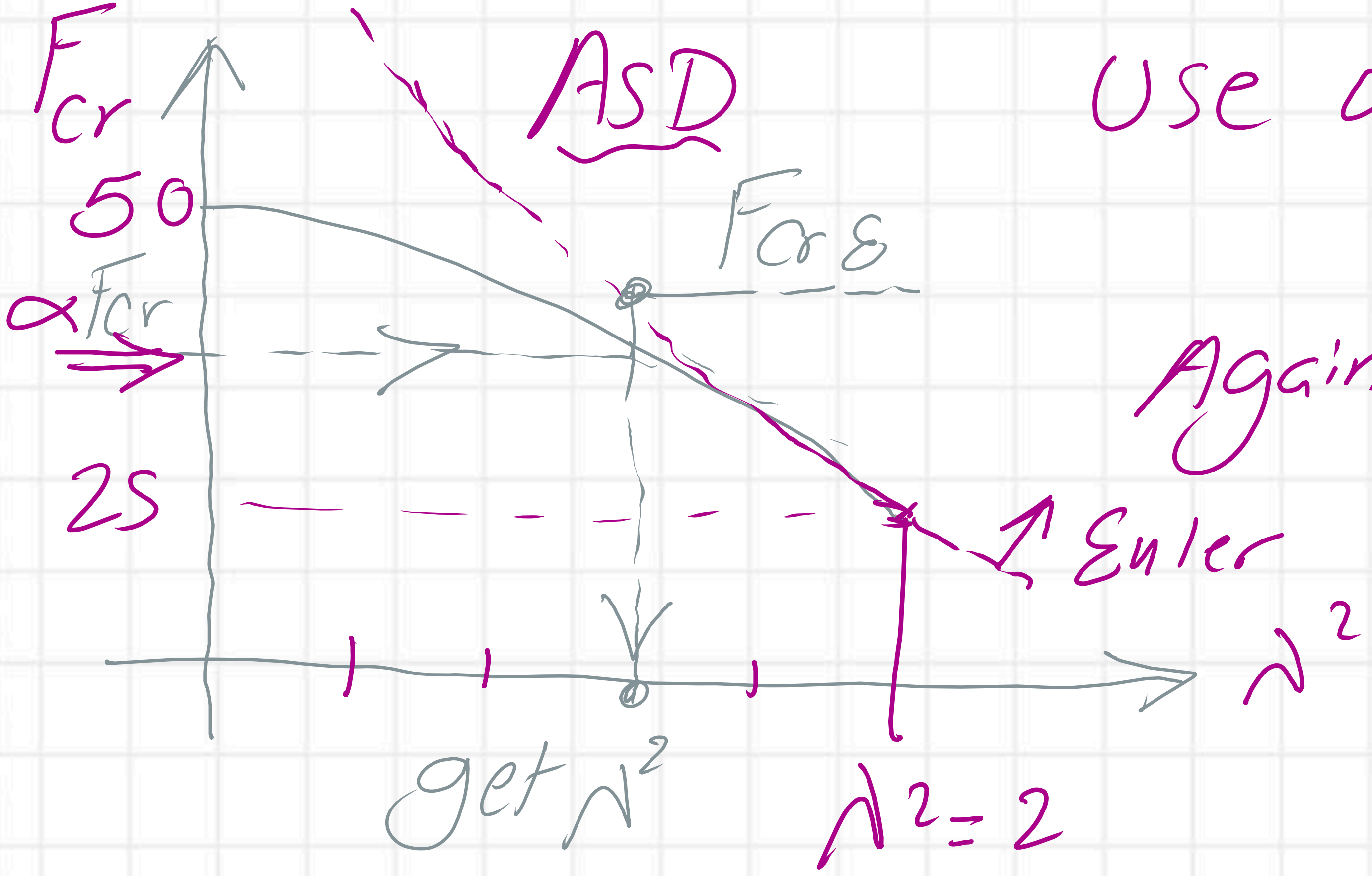
$$= \frac{50}{1.772} = 28.22 \text{ ksi}$$

$$\tau = \frac{F_{cr\,in}}{F_{cr\,E}} = \frac{27.85}{28.22} = 0.987 \Rightarrow \text{matches}$$

While For ASD $P_T = 300$ kips

$A_g = 15.80$ kips

$$F_{cr} = \frac{300}{15.80} = 18.99 \simeq 19 \text{ Ksi}$$



Use $\alpha \frac{P_n}{A} = 1.6(19)$

$= 30.40$ ksi

Again $(\alpha F_{cr}) = F_y \left(1 - \frac{\lambda^2}{4}\right)$

$\frac{30.40}{50} = 1 \left(1 - \frac{\lambda^2}{4}\right)$

$\lambda^2 = 4\left(1 - \frac{30.40}{50}\right)$

$\lambda^2 = 1.568$

$F_{cr\,Elastic} = \frac{F_y}{\lambda^2} = \frac{50}{1.568} = 31.89$ ksi

$\tau = \frac{F_{cr\,in}}{F_{cr\,E}} = \frac{30.40}{31.89} = 0.953 \Rightarrow$ matches

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