1. The difference between E & E_T $\rightarrow$ Modulus of Elasticity

$\rightarrow$ Tangent Modulus of Elasticity

2. τ_b : stress reduction Factor.
   What is τ_b ?

Prepared by Eng.Maged Kamel.

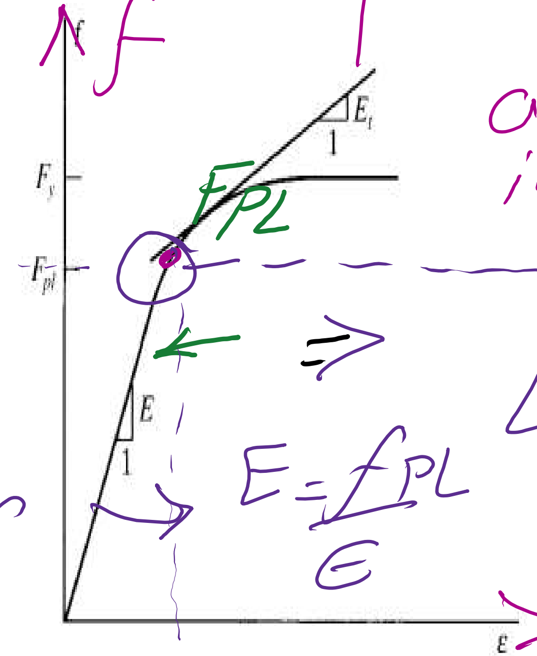
Prof. SEGui P-114

Tangent For Short and intermediate Colums

$$P_{cr} = \frac{\pi^2 E_t I}{L^2}$$

Equation 4.5 is identical to the Euler equation, except that E_t is substituted for *I*

FIGURE 4.5



fPL

Long Column

$E = \frac{f_{PL}}{\varepsilon}$

Long Column

$\rightarrow \varepsilon$

Early researchers soon found that Euler's equation did not give reliable results for stocky, or less slender, compression members. The reason is that the small slenderness ratio for members of this type causes a large buckling stress (from Equation 4.4). If the stress at which buckling occurs is greater than the proportional limit of the material, the relation between stress and strain is not linear, and the modulus of elasticity *E* can no longer be used. (In Example 4.1, the stress at buckling is $P_{cr}/A = 278.9/14.6 = 19.10$ ksi, which is well below the proportional limit for any grade of structural steel.) This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, *E* is not a constant for stresses greater than the proportional limit F_{pl}. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of *f* between F_{pl} and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that

Friedrich Engesser

Relation is Value For E is Value from $0 \rightarrow f_{PL}$ for stress

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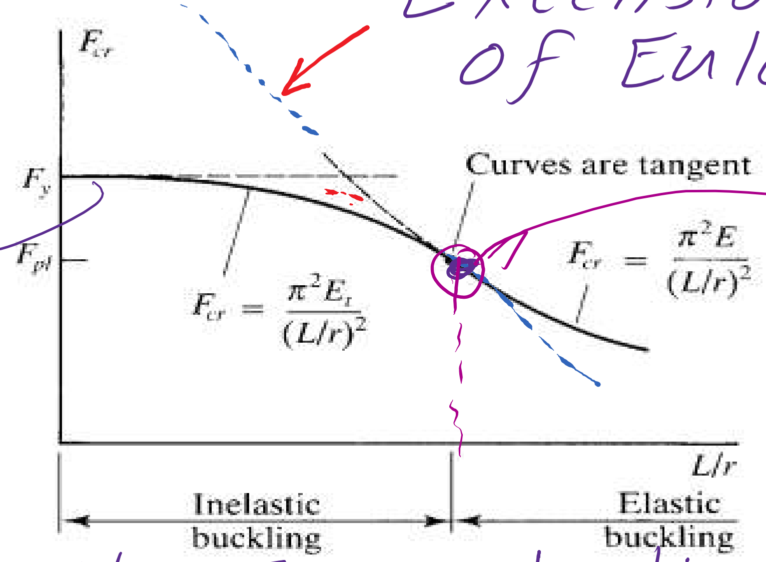
Engesser's tangent modulus theory had its detractors, who pointed out several inconsistencies. Engesser was convinced by their arguments, and in 1895 he refined his theory to incorporate a reduced modulus, which has a value between E and E_t. Test results, however, always agreed more closely with the tangent modulus theory. Shanley (1947) resolved the apparent inconsistencies in the original theory, and today the tangent modulus formula, Equation 4.5, is accepted as the correct one for inelastic buckling. Although the load predicted by this equation is actually a lower bound on the true value of the critical load, the difference is slight (Bleich, 1952).

For any material, the critical buckling stress can be plotted as a function of slenderness, as shown in Figure 4.6. The tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit of the material. The composite curve, called a *column strength curve*, completely describes the strength of any column of a given material. Other than F_y, E, and E_t, which are properties of the material, the strength is a function only of the slenderness ratio.

Extension of Euler graph

$\tau_b = E_T / E < 1$

FIGURE 4.6



$E = 29{,}000$ ksi

Two Curves

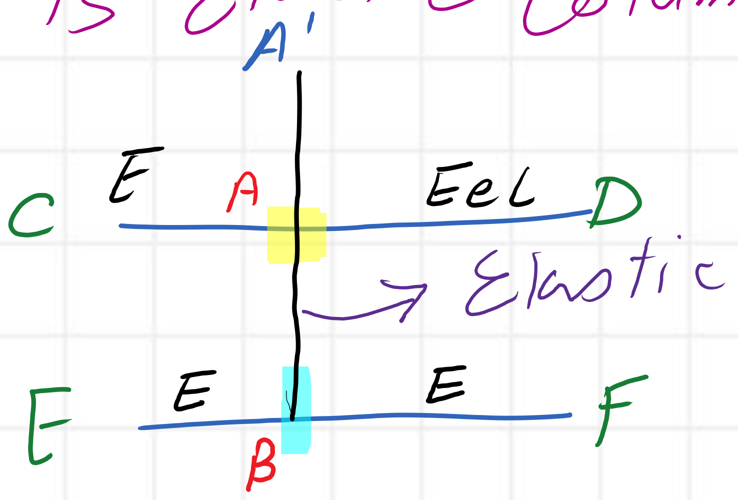
Euler $F_{cr} \rightarrow$

$F_{cr} \rightarrow E_T$

Matches with Test results For inelastic Columns

Prepared by Eng.Maged Kamel.

AB is Elastic Column



$$G_{elastic} = \frac{\Sigma E I_c / L_c}{\Sigma E I_b / L_b}$$

For $G_{elastic}$

Girders (Beams) are Elastic $E = 29000$.

Column is Elastic

$$G_{elastic} = \frac{\Sigma I_c / L_c}{\Sigma I_b / L_b}$$

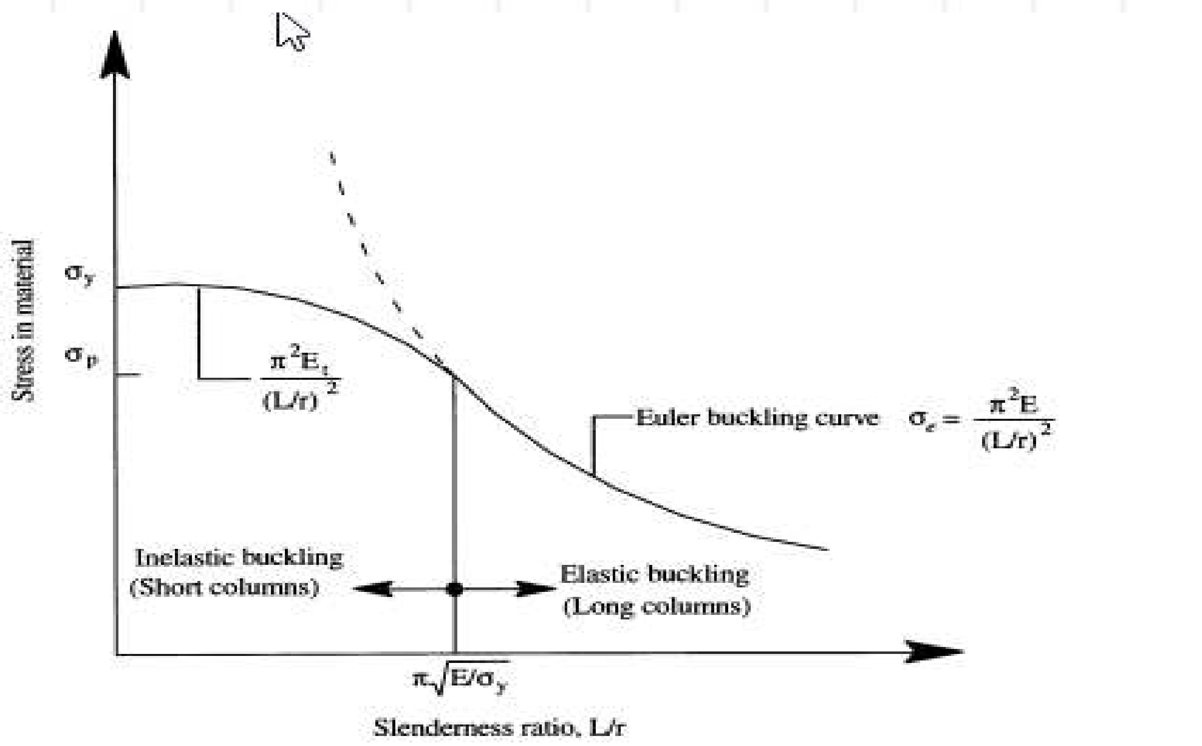


Fig. 10.1. Variation of σ vs. L/r for an ideal axially loaded compression member

$E = E_{Euler}$

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E_T: E at inelastic

A'

C E A Eel D

Einl Elastic

E E E F

Elastic B Elastic

$$G_{A\,inelastic} = \frac{\sum E_T I_c / L_c}{\sum E I_b / L_b} =$$

$$G_{A\,inelastic} = \frac{E_T}{E}\left(\frac{I_c}{I_b}\right)\left(\frac{L_b}{L_c}\right)$$

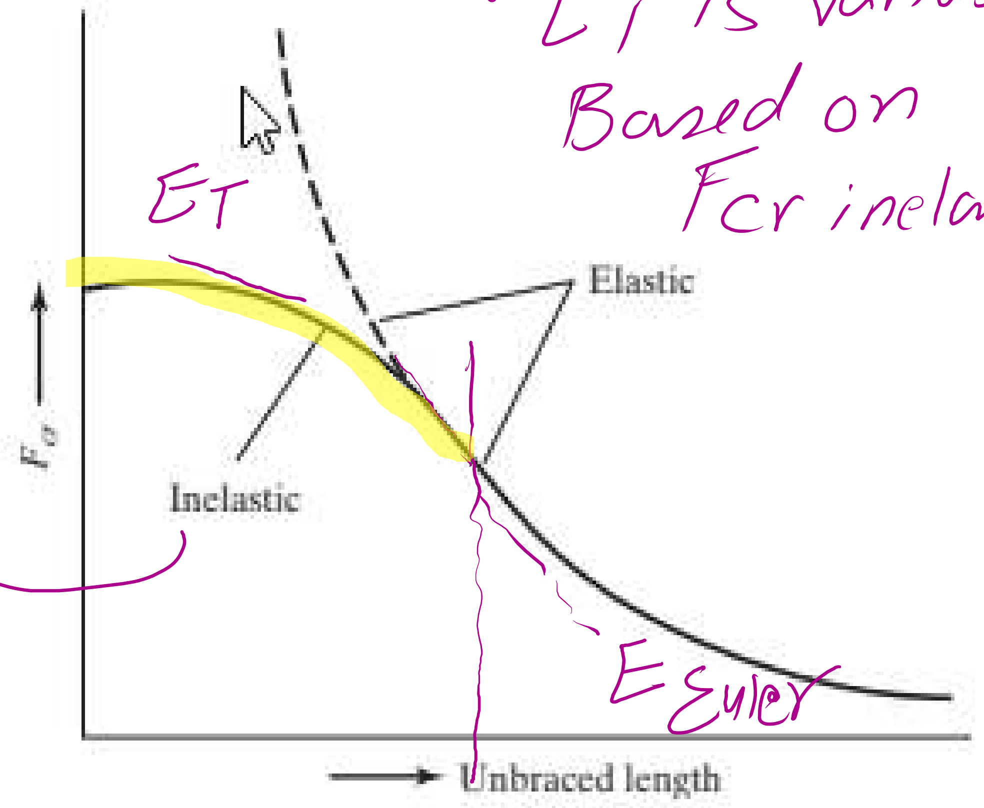
$$G_{A\,inel} = \tau_b \left(\frac{I_c}{I_b}\right)\left(\frac{L_b}{L_c}\right)$$

L_c Column Length

Column is in Inelastic region

E_T is variable Based on Fcr inelastic

$E = E_T$



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The column buckling problem can also be formulated in terms of a fourth-order differential equation instead of Equation 4.2. This proves to be convenient when dealing with boundary conditions other than pinned ends.

For convenience, the equations for critical buckling load will be written as

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2} \qquad (4.6a/4.6b)$$

column given by Equation 4.6b. Dividing it by the cross-sectional area gives the buckling stress:

$$F_{cr} = \frac{\pi^2 E_t}{(KL/r)^2}$$

Chapter 4 – STeel Design
William T. SEGUi

The rotational stiffness of a column in this state would be proportional to $E_t I_c / L_c$, and the appropriate value of G for use in the alignment chart is

$$G_{inelastic} = \frac{\sum E_t I_c / L_c}{\sum E I_g / L_g} = \frac{E_t}{E} G_{elastic}$$

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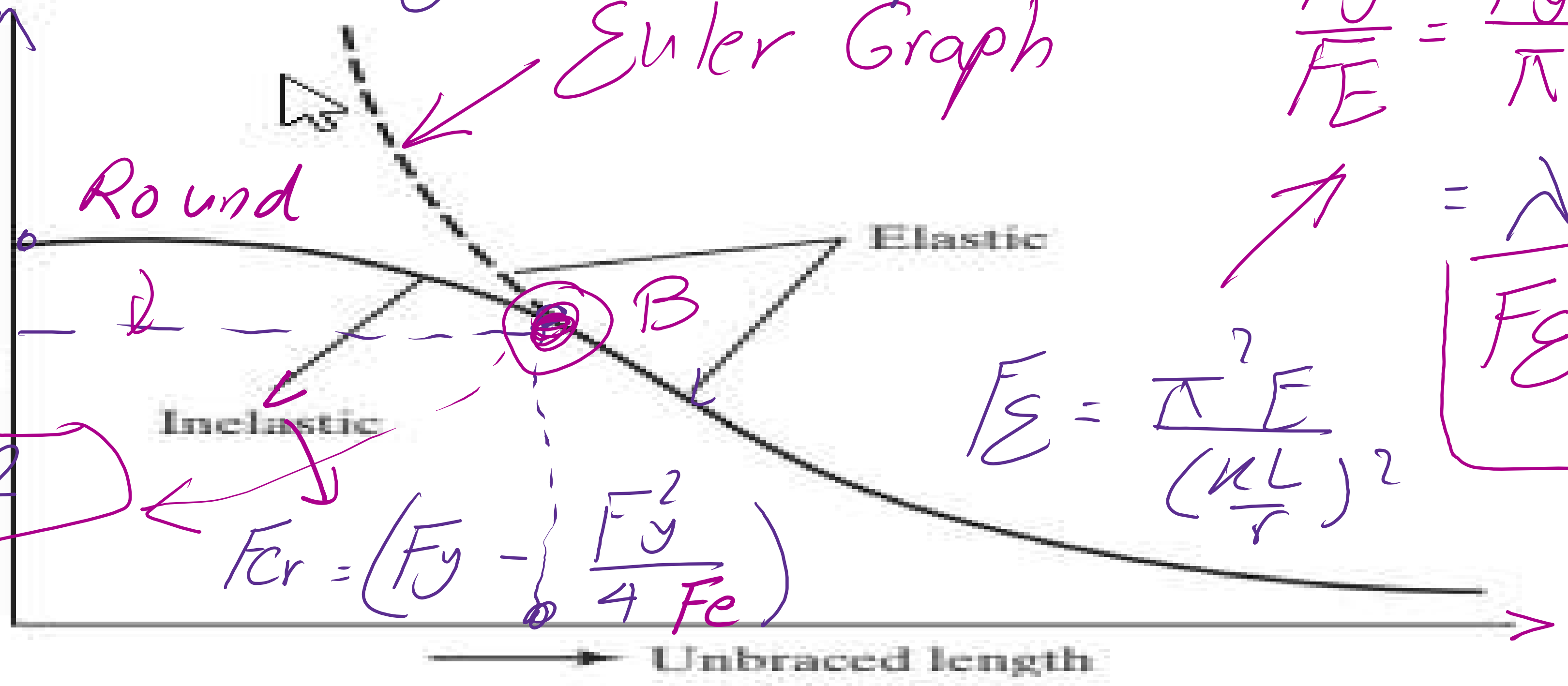
Prepared by Eng.Maged Kamel.

LRFD

Tangent at $F_y/2 = F_{Pl}$



F_{cr}

F_y

$\frac{F_y}{2}$

$\lambda^2 = 2$

$F_{cr} = \left(F_y - \frac{F_y^2}{4 F_e}\right)$

ℓ_c

$$\frac{F_y}{F_E} = \frac{F_y}{\pi^2 E}\left(\frac{KL}{r}\right)^2 = \lambda^2$$

$$\boxed{F_E = \frac{F_y}{\lambda^2}}$$

$$F_E = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

JOHNSON

For inelastic columns

$$F_{cr} = \left[F_y - \left(\frac{F_y^2}{4\pi^2 E}\right)\cdot\left(\frac{KL}{r}\right)^2\right]$$

introduce $\lambda^2 = F_y/F_E$

$$F_{cr} = \left[F_y - F_y\left(\frac{\lambda^2}{4}\right)\right]$$

$$\tau_b = \frac{F_E}{F_{cr}}$$

Point B

$$\frac{F_{cr}}{F_{E1}} = 1$$

Stability Design of Steel Frame

SEGui

in hot-rolled, wide flange shapes which is about $0.3F_y$. To obtain a smooth transition from the parabola to the Euler curve, the constant B was chosen to be $F_y^2/4\pi^2E$. The slenderness ratio that corresponds to $F_{cr} = 0.5F_y$ is designated as C_c in which

$$C_c = \sqrt{\left(\frac{2\pi^2 E}{F_y}\right)} \tag{2.9.3}$$

CRC Column curves

Thus, for columns with slenderness ratios less than or equal to C_c, the CRC curve assumes the shape of a parabola, and for slenderness ratios exceeding C_c, the CRC curve takes the shape of a hyperbola, i.e.

$$F_{cr} = \begin{cases} F_y\left[1 - \dfrac{(KL/r)^2}{2C_c^2}\right], & \text{for } \dfrac{KL}{r} \le C_c \\ \dfrac{\pi^2 E}{(KL/r)^2}, & \text{for } \dfrac{KL}{r} > C_c \end{cases} \tag{2.9.4}$$

Equation (2.9.4) can be rewritten in terms of P_n as follows

$$\frac{P_n}{P_y} = \begin{cases} 1 - 0.25\lambda_c^2, & \lambda_c \le \sqrt{2} \\ \lambda_c^{-2} & \lambda_c > \sqrt{2} \end{cases} \tag{2.9.5}$$

where

$$\lambda_c = \sqrt{(F_y/F_E)} = (KL/r)\sqrt{(F_y/\pi^2 E)} \tag{2.9.6}$$

W. F. Chen

Because E_t is less than E, $G_{\text{inelastic}}$ is less than G_{elastic}, and the effective length factor K will be reduced, resulting in a more economical design. To evaluate E_t/E, called the *stiffness reduction factor* (denoted by τ_b), consider the following relationship.

$$\frac{F_{cr\,(\text{inelastic})}}{F_{cr\,(\text{elastic})}} = \frac{\pi^2 E_t/(KL/r)^2}{\pi^2 E/(KL/r)^2} = \frac{E_t}{E} = \tau_b$$

$$\tau_b = \frac{F_{cr\ \text{inelas}}}{F_{cr\ \text{Elastic}}}$$

From Galambos (1998), $F_{cr\,(\text{inelastic})}$ and $F_{cr\,(\text{elastic})}$ can be expressed as

$$F_{cr\,(\text{inelastic})} = \left(1 - \frac{\lambda_c^2}{4}\right)F_y \tag{4.13}$$

$$F_{cr_{in}} = \left(1 - \frac{\lambda_c^2}{4}\right)(F_y)$$

$$F_{cr\,(\text{elastic})} = \frac{F_y}{\lambda_c^2}$$

$$F_{cr_{El}} = \frac{F_y}{\lambda_c^2}$$

where

$$\lambda_c = \frac{KL}{r}\frac{1}{\pi}\sqrt{\frac{F_y}{E}}$$

$$\lambda_c^2 = \frac{F_y}{F_E} = \frac{F_y}{\pi^2 E}\left(\frac{KL}{r}\right)^2$$

$$\lambda_c = \left(\frac{KL}{r}\right)\left(\frac{1}{\pi}\right)\sqrt{\frac{F_y}{E}}$$

Since

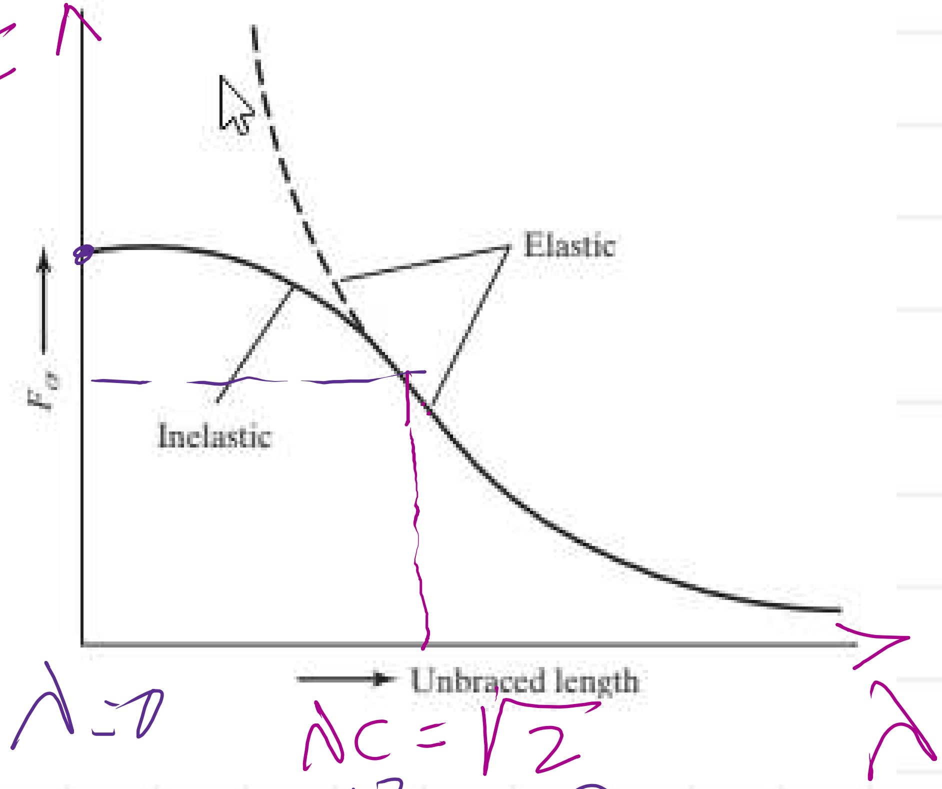
$$\tau_b = \frac{F_{cr\,(\text{inelastic})}}{F_{cr\,(\text{elastic})}}$$

$\lambda_c \le \sqrt{2}$ inelastic

Mccormac

Table 2.6 Column design equations

Column curves	Column equations	
CRC curve	$\dfrac{P_n}{P_y} = 1 - \dfrac{\lambda_c^2}{4}$	$\lambda_c \le \sqrt{2}$
	$\dfrac{P_n}{P_y} = \dfrac{1}{\lambda_c^2}$	$\lambda_c > \sqrt{2}$
AISC-ASD curve	$\dfrac{P_n}{P_y} = \dfrac{1 - \lambda_c^2/4}{\frac{5}{3} + \frac{3}{8}(\lambda_c/\sqrt{2}) - \frac{1}{8}(\lambda_c/\sqrt{2})^3}$	$\lambda_c \le \sqrt{2}$
	$\dfrac{P_n}{P_y} = \dfrac{12}{23}\dfrac{1}{\lambda_c^2}$	$\lambda_c > \sqrt{2}$
AISC-PD curve	$\dfrac{P_n}{P_y} = \dfrac{1.7(1 - \lambda_c^2/4)}{\frac{5}{3} + \frac{3}{8}(\lambda_c/\sqrt{2}) - \frac{1}{8}(\lambda_c/\sqrt{2})^3} \le 1.0$	$\lambda_c \le \sqrt{2}$
AISC-LRFD curve	$\dfrac{P_n}{P_y} = 0.658^{\lambda_c^2}$	$\lambda_c \le 1.5$
	$\dfrac{P_n}{P_y} = \dfrac{0.877}{\lambda_c^2}$	$\lambda_c > 1.5$



$\lambda_c = \sqrt{2}$

$\lambda_c^2 = 2$

For Design $\lambda_c \le 1.5$

> 1.5

$\lambda_c^2 = 2.25$

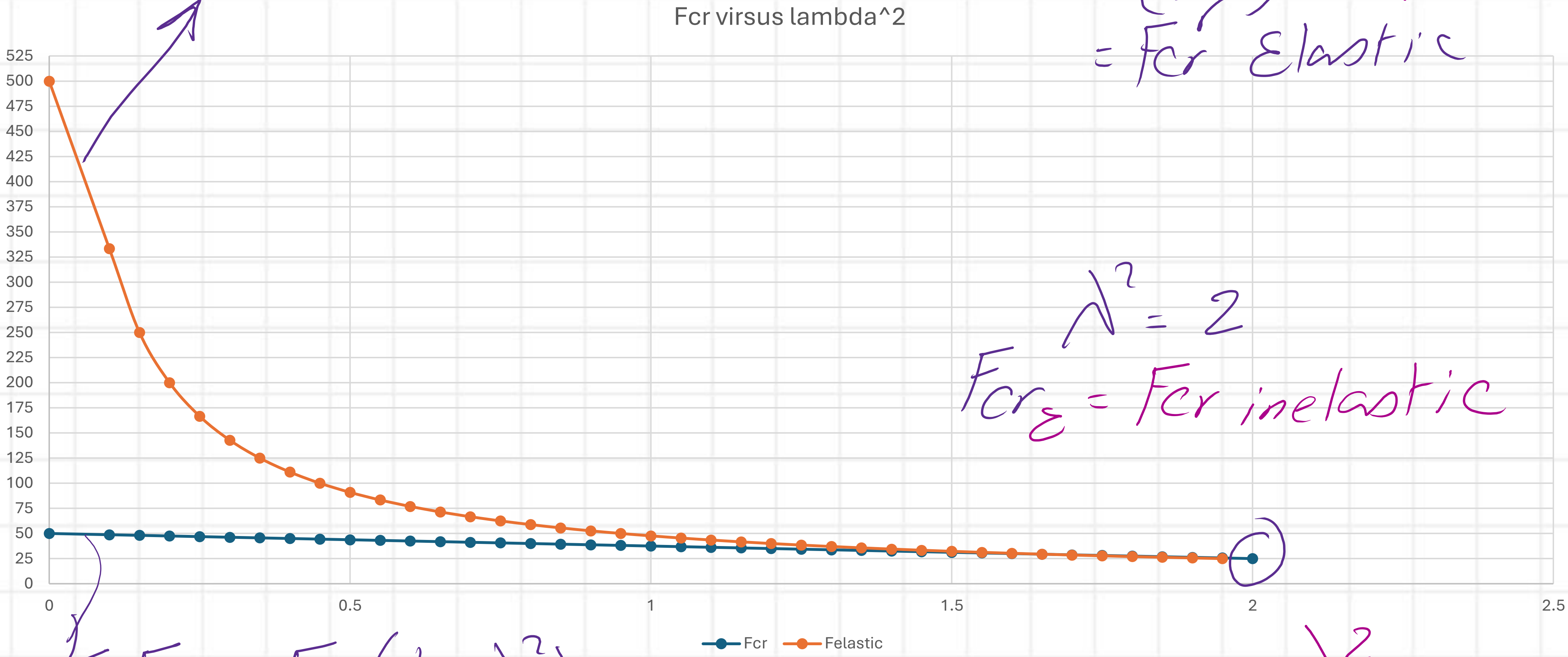
$\dfrac{F_y}{F_e} = \lambda_c^2$

$F_y = 50$ ksi

Euler Graph

$$F_E = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{F_y}{\lambda^2}$$

$= F_{cr}$ Elastic



$\lambda^2 = 2$

$F_{cr_E} = F_{cr}$ inelastic

F_y

$$F_{cr} = F_y\left(1 - \frac{\lambda^2}{4}\right)$$

λ^2 Graph

F_{cr} Elastic $= F_y / \lambda_c^2$

F_{cr} inelastic $= \tau_b\, F_{cr}$ Elastic

Find F_{cr} inelastic

then

$$F_{cr\,(\text{inelastic})} = \tau_b F_{cr\,(\text{elastic})} = \tau_b\left(\frac{F_y}{\lambda_c^2}\right)$$

$$\lambda_c^2 = \frac{\tau_b F_y}{F_{cr\,(\text{inelastic})}}$$

From Equation 4.13,

$$F_{cr\,(\text{inelastic})} = \left(1 - \frac{\lambda_c^2}{4}\right)F_y = \left(1 - \frac{\tau_b F_y}{4F_{cr\,(\text{inelastic})}}\right)F_y$$

F_{cr} inelastic value in Terms of τ_b, F_y

$$F_{cr\,in} = \left(\frac{4F_{cr} - \tau_b F_y}{4F_{cr}}\right)F_y$$

Galambos

F_{cr} inelastic

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$\tau_b F_y^2 = 4 F_y F_{cr} - 4 F_{cr}^2 \Rightarrow \tau_b = 4 \frac{F_{cr}}{F_y} - 4 \frac{F_{cr}^2}{F_y^2}$

Using the notation $F_{cr} = F_{cr\,(\text{inelastic})}$ and solving for τ_b, we obtain

$$\tau_b = 4\left(\frac{F_{cr}}{F_y}\right)\left(1 - \frac{F_{cr}}{F_y}\right)$$

$* \left(\frac{A}{A}\right)$

Find τ_b value in terms of P_n, P_y

This can be written in terms of forces as

$$\tau_b = 4\left(\frac{P_n}{P_y}\right)\left(1 - \frac{P_n}{P_y}\right)$$

where

P_n = nominal compressive strength = $F_{cr} A_g$

P_y = compressive yield strength = $F_y A_g$

$P_n = A F_{cr}$

$P_y = A F_y$

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Substituting the required strength, αP_r, for the available strength, P_n, we have

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left(1 - \frac{\alpha P_r}{P_y}\right) \qquad \text{(AISC Equation C2-2b)}$$

To get LRFD / ASD values use α

where $\alpha = 1.0$ for LRFD and 1.6 for ASD. The required strength is computed at the factored load level, and the 1.6 factor is used to adjust the ASD service load level to a factored load level. The stiffness reduction factor, τ_b, is also used to adjust member stiffnesses for frame analysis. This is discussed in Chapter 6, "Beam–Columns."

Using $\alpha \rightarrow$ $\alpha = 1$ LRFD

$\alpha = 1.6$ ASD

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(b) An additional factor, τ_b, shall be applied to the flexural stiffnesses of all members whose flexural stiffnesses are considered to contribute to the stability of the structure. For noncomposite members, τ_b shall be defined as follows (see Section I1.5 for the definition of τ_b for composite members).

$P_r : P_T = P_D + P_L$ ASD

$P_r = P_{ult} = 1.2D + 1.6L$

(1) When $\alpha P_r / P_{ns} \leq 0.5$

$$\tau_b = 1.0 \tag{C2-2a}$$

(2) When $\alpha P_r / P_{ns} > 0.5$

$P_{ns} = P_y$

$$\tau_b = 4(\alpha P_r / P_{ns})[1 - (\alpha P_r / P_{ns})] \tag{C2-2b}$$

where

α = 1.0 (LRFD); α = 1.6 (ASD)

P_r = required axial compressive strength using LRFD or ASD load combinations, kips (N)

P_{ns} = cross-section compressive strength; for nonslender-element sections, $P_{ns} = F_y A_g$, and for slender-element sections, $P_{ns} = F_y A_e$, where A_e is as defined in Section E7, kips (N)

User Note: Taken together, Sections (a) and (b) require the use of $0.8\tau_b$ times the nominal elastic flexural stiffness and 0.8 times other nominal elastic stiffnesses for structural steel members in the analysis.

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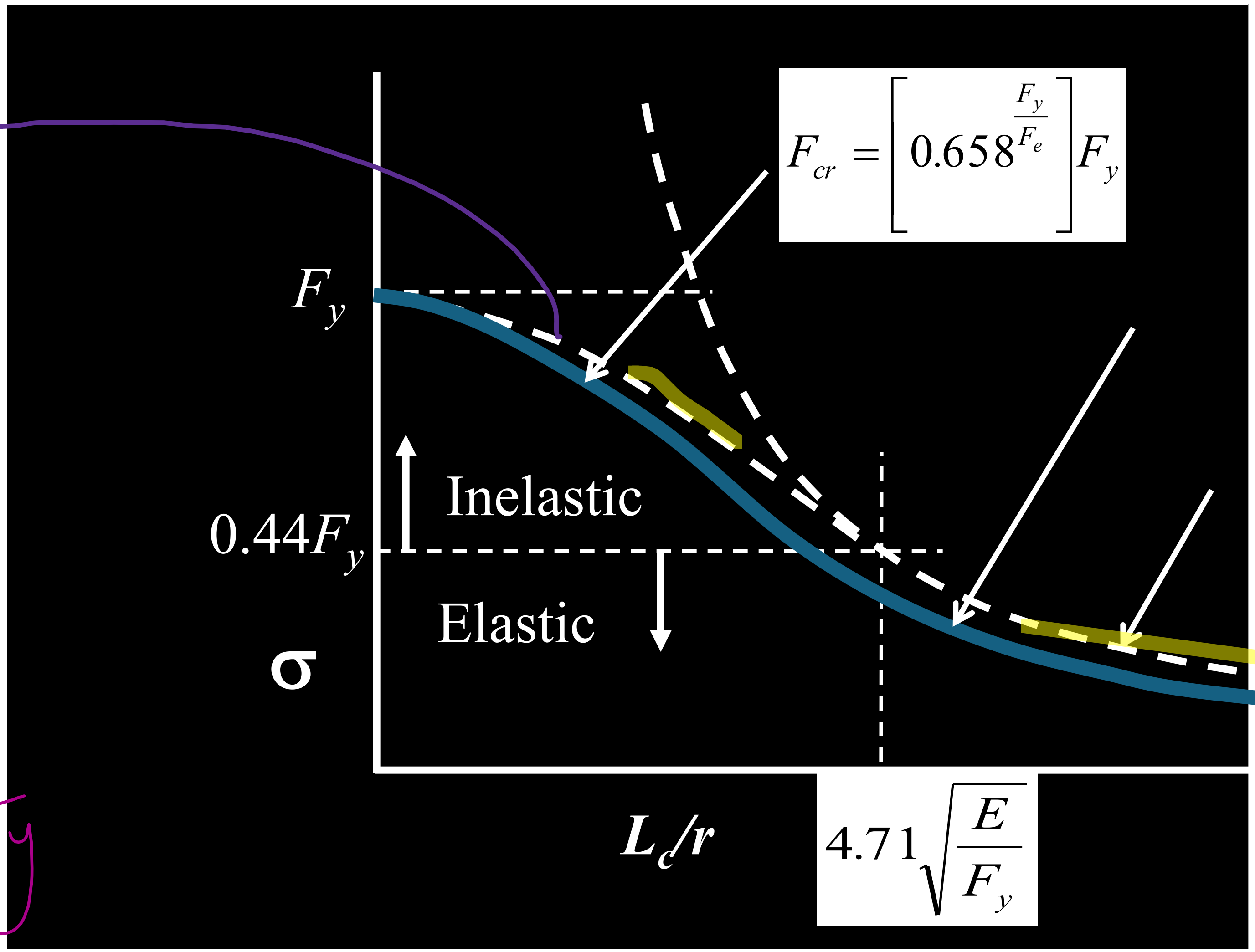
Dotted Graph

- - - Euler

http://www.ecs.umass.edu/cee434/notes/

Inelastic Material Effects

AISC Plot



$$F_{cr} = 0.877F_e$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2}$$

$$F_E = \frac{\pi^2 E}{\left(\frac{Le}{r}\right)^2}$$

$$\lambda^2 = \frac{F_y}{F_E}$$

$F_{cr} = 0.658^{\lambda^2} F_y$

$F_{cr} = 0.877 F_e$

Notes

LRFD Design

Check Table 4-13 values

Table 4-13 $F_y = 50$ ksi; check $\frac{P_u}{A_g} = 45$ ksi $= f_{cr\ inelastic}$

Use $\frac{F_{cr\,in}}{F_y} = \left(1 - \frac{\lambda_c^2}{4}\right) \Rightarrow$ get $\lambda_c^2 \Rightarrow \frac{45}{50} = \left(1 - \frac{\lambda_c^2}{4}\right)$

$\lambda_c^2 \Rightarrow (1 - 0.90)\,4 = 0.40$

Find $F_{cr\,in} = \frac{F_y}{\lambda^2}$ $\Rightarrow F_{cr\,Elas} = 50/0.40 = 125$ ksi

$\tau_b = \frac{F_{cr\,in}}{F_{cr\,E}} = \frac{45}{125} = 0.36 \Rightarrow$ Same as in Table

---

Check $F_{cr} = \frac{P_u}{A}$ ksi $\Rightarrow \lambda_c^2 \Rightarrow \frac{30}{50} = \left(1 - \frac{\lambda_c^2}{4}\right)$

$\lambda_c^2 = 1.60 \Rightarrow F_{crE} = \frac{F_y}{\lambda_c^2} = \frac{50}{1.6} = 31.25$ ksi

$\frac{F_{cr\,in}}{F_{cr\,E}} = \tau_b = \frac{30}{31.25} = 0.960 \Rightarrow$ Same as in Table

Notes LRFD Design

Table 4-13 $F_y = 36$ ksi, check $\frac{P_u}{A_g} = 35$ ksi $= f_{cr}$ inelastic $\Rightarrow F_y = 36$ ksi

Use $\frac{F_{cr\,in}}{F_y} = \left(1 - \frac{\lambda_c^2}{4}\right) \Rightarrow$ get $\lambda_c^2 \Rightarrow \frac{35}{36} = \left(1 - \frac{\lambda_c^2}{4}\right)$

$\lambda_c^2 \Rightarrow (0.0277)\,4 = 0.111$

Find $F_{cr\,in} = \frac{F_y}{\lambda^2} \Rightarrow F_{cr\,Elas} = 36/0.111 = 324.32$ ksi

$\tau_b = \frac{F_{cr\,in}}{F_{cr\,E}} = \frac{35}{324.32} = 0.1079 \Rightarrow$ Same as in Table

Check $F_{cr} = \frac{P_u}{A} = 20$ ksi $\Rightarrow \lambda_c^2 \Rightarrow \frac{20}{36} = \left(1 - \frac{\lambda_c^2}{4}\right)$

$\lambda_c^2 = 1.777 \Rightarrow F_{cr\,E} = \frac{F_y}{\lambda_c^2} = \frac{36}{1.777} = 20.26$ ksi

$\frac{F_{cr\,in}}{F_{cr\,E}} = \tau_b = \frac{20}{20.26} = 0.988 \Rightarrow$ Same as in Table

$\tau_b = 0.988$ $F_y = 36$ ksi

$F_{cr} = 20$ ksi

21	0.154	0.960	0.249	0.972	0.788
20	0.313	0.980	0.395	0.988	0.847

Table 4-13
Stiffness Reduction Factor τ_b

$F_y = 36$ ksi

50 ksi

ASD P_a/A	LRFD P_u/A	F_y, ksi 35 ASD	35 LRFD	36 ASD	36 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD	65 ASD	65 LRFD	70 ASD	70 LRFD
50		–	–	–	–	–	–	–	–				
49		–	–	–	–	–	–	–	0.0784				
48		–	–	–	–	–	–	–	0.154				
47		–	–	–	–	–	–	–	0.226				
46		–	–	–	–	–	–	–	0.294				
45		–	–	–	–	–	0.0851	–	0.360				
44		–	–	–	–	–	0.166	–	0.422				
43		–	–	–	–	–	0.244	–	0.482				
42		–	–	–	–	–	0.318	–	0.538				
41		–	–	–	–	–	0.388	–	0.590				
40		–	–	–	–	–	0.454	–	0.640				
39		–	–	–	–	–	0.516	–	0.686				
38		–	–	–	–	–	0.575	–	0.730				
37		–	–	–	–	–	0.629	–	0.770				
36		–	–	–	–	–	0.681	–	0.806				
35		–	–	–	0.108	–	0.728	–	0.840				
34		–	0.111	–	0.210	–	0.771	–	0.870				
33		–	0.216	–	0.306	–	0.811	–	0.898				
32		–	0.313	–	0.395	–	0.847	–	0.922				
31		–	0.405	–	0.478	–	0.879	0.0317	0.942				
30		–	0.490	–	0.556	–	0.907	0.154	0.960				
29		–	0.568	–	0.627	–	0.932	0.267	0.974				
28		–	0.640	–	0.691	0.102	0.953	0.373	0.986				
27		–	0.705	–	0.750	0.229	0.970	0.470	0.994				
26		–	0.764	–	0.802	0.346	0.983	0.559	0.998				
25		–	0.816	–	0.849	0.454	0.992	0.640	1.00				

$F_{cr} = 35$ ksi

$\tau = 0.108$

$\tau_b = 0.0784$

$F_{cr} = 49$ ksi

$\tau_b = 0.360$

$F_{cr} = 45$ ksi

$\tau_b = 0.96$

$F_{cr} = 30$ ksi

$\lambda^2 = 2$

$\tau_b = 1$ $F_{cr} = F_y/\lambda^2$

Notes ASD Design $\boxed{\alpha = 1.6}$ Check Table 4-13 values

Table 4-13 $F_y = 50$ ksi, check

(1) $\lambda^2 = 4\left(1 - \frac{F_{cr\,in}}{F_y}\right) \rightarrow 4\left(1 - \frac{\alpha}{\alpha}\frac{F_{cr\,in}}{F_y}\right)$

Take $\frac{F_{cr}}{\alpha} = f'_{cr} \rightarrow \lambda^2 = 4\left(1 - \frac{F'_{cr}}{F_y}(\alpha)\right)$

Consider $\frac{P}{A} = 48$ ksi $\rightarrow$ Divide by $\alpha \rightarrow$

Consider $F'_{cr} = \frac{48}{1.6} = 30$ ksi $\rightarrow$ value in Table

$\lambda^2 = 4\left(1 - \frac{(1.6)(30)}{50}\right) = 0.16$

$F_{cr\varepsilon} = F_y / \alpha\lambda^2 = 50 / (1.6)(0.16) = 195.312$ ksi

$\tau = \frac{F'_{cr}}{F_{cr\varepsilon}} = \frac{30}{195.312} = 0.1536 \rightarrow 0.154$ in Table

ASD $\alpha = 1.6$

Table 4-13
Stiffness Reduction Factor τ_b

$F_y = 36\,ksi$ 50 ksi

ASD P_a/A / LRFD P_u/A	F_y, ksi: 35 ASD	35 LRFD	36 ASD	36 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD	65 ASD	65 LRFD	70 ASD	70 LRFD
31	–	0.405	–	0.478	–	0.879	0.0317	0.942				
30	–	0.490	–	0.556	–	0.907	0.154	0.960				
29	–	0.568	–	0.627	–	0.932	0.267	0.974				
28	–	0.640	–	0.691	0.102	0.953	0.373	0.986				
27	–	0.705	–	0.750	0.229	0.970	0.470	0.994				
26	–	0.764	–	0.802	0.346	0.983	0.559	0.998				
25	–	0.816	–	0.849	0.454	0.992	0.640	1.00				
24	–	0.862	–	0.889	0.552	0.998	0.713					
23	–	0.901	–	0.923	0.640	1.00	0.777					
22	–	0.934	0.087	0.951	0.719		0.834					
21	0.154	0.960	0.249	0.972	0.788		0.882					
20	0.313	0.980	0.395	0.988	0.847		0.922					
19	0.457	0.993	0.525	0.997	0.896		0.953					
18	0.583	0.999	0.640	1.00	0.936		0.977					
17	0.693	1.00	0.739		0.967		0.992					
16	0.786	1.00	0.822		0.987		0.999					
15	0.862	1.00	0.889		0.998		1.00					
14	0.922	1.00	0.940		0.999							

$\frac{F_{cr}}{\alpha} = \frac{48}{1.6} = 30\,ksi$

$F_E = 195.31\,ksi$

$\tau = 0.154$

$\lambda^2 = 4\left(1 - \frac{30}{50}(1.6)\right) = 0.16$

$\frac{F_{cr}}{\alpha} = \frac{24}{1.6} = 15\,ksi$

$\lambda^2 = 4\left(1 - \frac{15}{50}(1.6)\right) = 2.08$

$F_E = \frac{50}{1.6(2.08)} = 15\,ksi$

$\tau = 1$