EXAMPLE 5.10 ***Strength of Column with Slender Elements***	**Goal:**	Determine the available strength of a compression member with a slender web.
	Given:	Use a W16×26 as a column with L_{cy} = 5.0 ft. 6.0 FT

Solution W16 X 26 is marked C Table 1-1

Section is slender, $A_g = 7.68$ inch²

$\frac{b_f}{2t_f} = 7.97$, $\frac{h}{t_w} = 56.8$

r_x : 6.26"

I_x : 301 inch⁴

r_y : 1.12"

I_y : 9.59 inch⁴

$L_{cy} = 6.0\ FT = (KL_y)$

$\left(\frac{L_{cy}}{r_y}\right) = \frac{6.0(12)}{1.12} = 64.29$

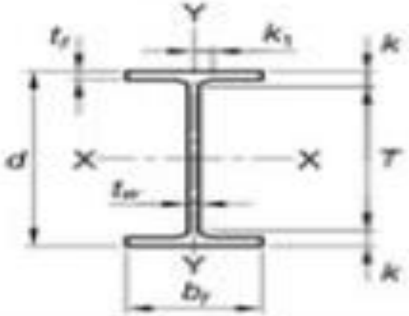


Table 1-1 (continued)
W-Shapes
Dimensions

Part-1

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W16×31[c]	9.13	15.9	15 7/8	0.275	1/4	1/8	5.53	5 1/2	0.440	7/16	0.842	1 1/8	3/4	13 5/8	3 1/2
×26[c,v]	7.68	15.7	15 3/4	0.250	1/4	1/8	5.50	5 1/2	0.345	3/8	0.747	1 1/16	3/4	13 5/8	3 1/2

[c] Shape is slender for compression with F_y = 50 ksi. C

[f] The actual size, combination and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 50 ksi.

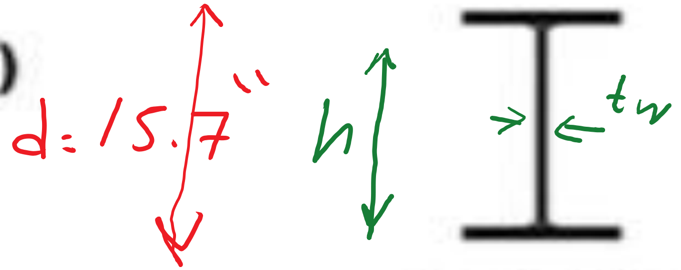
V

W16x26 with Footnote C Shape is slender For Compression

Prepared by Eng.Maged Kamel.

Find h from $\frac{h}{t_w}$ (t_w)

Table 1-1 (continued)
W-Shapes
Properties



W18–W16

$r_y = 1.12$ inch

Nom-inal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
31	6.28	51.6	375	47.2	6.41	54.0	12.4	4.49	1.17	7.03	1.42	15.5	0.461	739
26	7.97	56.8	301	38.4	6.26	44.2	9.59	3.49	1.12	5.48	1.38	15.4	0.262	565

W16 x 26

$\frac{b_f}{2t_f} = 7.97$

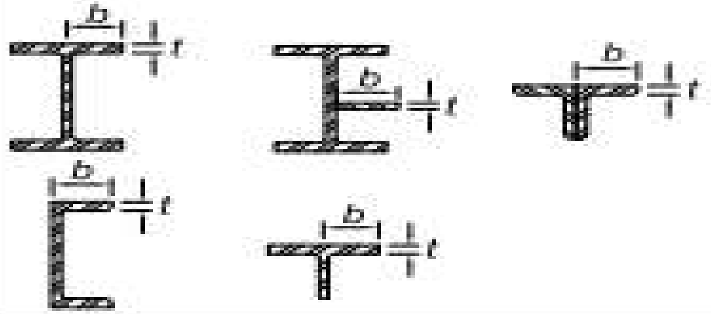
$h/t_w = 56.80$

$h = t_w(56.8)$
$= 0.25(56.8) = 14.20$

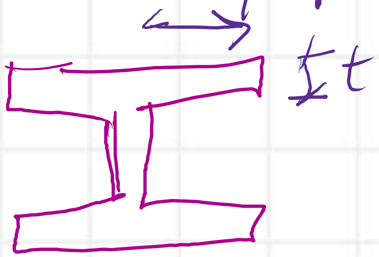
Prepared by Eng.Maged Kamel.

For Flange → Case-1

TABLE B4.1a
Width-to-Thickness Ratios: Compression Elements Members Subject to Axial Compression

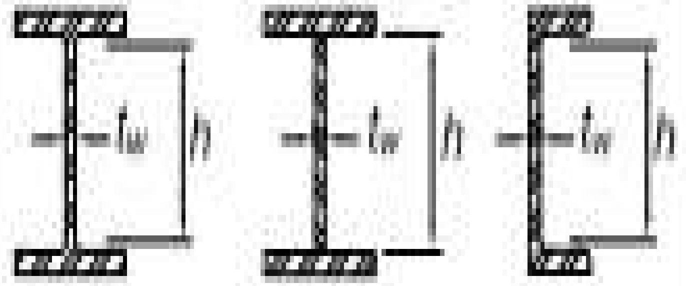
Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender/slender)	Examples
1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	b/t	$0.56\sqrt{\frac{E}{F_y}}$	

$\lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29000}{50}} = 13.49$



$\frac{b}{2t_f} = 7.97 < 13.49$ non slender Flange

STiFFened

	5	Webs of doubly symmetric rolled and built-up I-shaped sections and channels	h/t_w	$1.49\sqrt{\frac{E}{F_y}}$	

Case-5 stiffened

$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\,(29000/50)^{1/2} = 35.88$

Compare to $\frac{h}{t_w} = 56.8$

$\frac{h}{t_w} > \lambda_{r\,web}$

$56.8 > 35.88$

web is slender

Refer to chapter E → E7

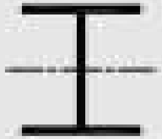
CHAPTER E

DESIGN OF MEMBERS FOR COMPRESSION

This chapter addresses members subject to axial compression.

The chapter is organized as follows:

TABLE USER NOTE E1.1
Selection Table for the Application of Chapter E Sections

Cross Section	Without Slender Elements		With Slender Elements	
	Sections in Chapter E	Limit States	Sections in Chapter E	Limit States
	E3 E4	FB TB	E7	LB FB TB

Check section E7

E7. MEMBERS WITH SLENDER ELEMENTS

This section applies to slender-element compression members, as defined in Section B4.1 for elements in axial compression.

The nominal compressive strength, P_n, shall be the lowest value based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling in interaction with local buckling.

$$P_n = F_{cr} A_e \tag{E7-1}$$

where

A_e = summation of the effective areas of the cross section based on reduced effective widths, b_e, d_e or h_e, or the area as given by Equations E7-6 or E7-7, in.^{2} (mm^2).

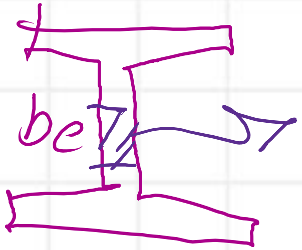
F_{cr} = critical stress determined in accordance with Section E3 or E4, ksi (MPa). For single angles, determine F_{cr} in accordance with Section E3 only.

User Note: The effective area, A_e, may be determined by deducting from the gross area, A_g, the reduction in area of each slender element determined as $(b - b_e)t$.

$P_n = F_{cr} \cdot A_e \Rightarrow A_e$ effective area

E7-1

reduction



$A_e = A_g - \sum(b - b_e)t$

reduction in web → our case

1. Slender Element Members Excluding Round HSS

The effective width, b_e, (for tees, this is d_e; for webs, this is h_e) for slender elements is determined as follows:

Need to Estimate Fcr

(a) When $\lambda \le \lambda_r \sqrt{\dfrac{F_y}{F_{cr}}}$

or $\frac{b}{tf} \leftrightarrow h/tw <$

$$b_e = b \tag{E7-2}$$

(b) When $\lambda > \lambda_r \sqrt{\dfrac{F_y}{F_{cr}}}$

or $\frac{b}{tf} \leftrightarrow \frac{h}{tw}$

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \tag{E7-3}$$

TABLE E7.1
Effective Width Imperfection Adjustment Factors, c_1 and c_2

Case	Slender Element	c_1	c_2
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

where

b = width of the element (for tees this is d; for webs this is h), in. (mm)

c_1 = effective width imperfection adjustment factor determined from Table E7.1

$$c_2 = \frac{1-\sqrt{1-4c_1}}{2c_1} \quad \text{(E7-4)}$$

λ = width-to-thickness ratio for the element as defined in Section B4.1

λ_r = limiting width-to-thickness ratio as defined in Table B4.1a

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \quad \text{(E7-5)}$$

= elastic local buckling stress determined according to Equation E7-5 or an elastic local buckling analysis, ksi (MPa)

Check whether section → Elastic
inelastic

$$\lambda = 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.43$$

$$\simeq 113$$

$$\frac{L_{cy}}{r_y} = 64.286 < \lambda$$

From page -1

Column is inelastic

$$F_{cr} = 0.658^{\lambda_c^2} F_y$$

$$\lambda^2 = \frac{F_y}{F_E} = \frac{F_y}{\dfrac{\pi^2 E}{\left(\dfrac{L_{cy}}{r_y}\right)^2}} = \frac{50\,(64.286)^2}{\pi^2 (29000)}$$

$$= 0.72$$

$$F_{cr} = 0.658^{(0.72)} (50) = 36.99 \text{ ksi}$$

$$F_E = \frac{\pi^2 (29000)}{(64.286)^2} = 69.30 \text{ ksi}$$

1. Slender Element Members Excluding Round HSS

The effective width method is employed for determining the reduction in capacity due to local buckling. The effective width method was developed by von Kármán et al. (1932), empirically modified by Winter (1947), and generalized for local-global buckling interaction by Peköz (1987); see Ziemian (2010) for a complete summary. The point at which the slender element begins to influence column strength, $\lambda_r\sqrt{F_y/F_{cr}}$, is a function of element slenderness from Table B4.1a and column slenderness as reflected through F_{cr}. This reflects the unified effective width approach where the maximum stress in the effective width formulation is the column stress, F_{cr} (as opposed to F_y). This implies that columns designated as having slender elements by Table B4.1a may not necessarily see any reduction in strength due to local buckling, depending on the column stress, F_{cr}.

$$\lambda = C_3\sqrt{\frac{E}{F_{cr}}} \longrightarrow \text{in lieu of } F_y \rightarrow \text{Table B4.1}$$

C_1 & C_2 For be Estimation

TABLE C-E7.1
Constants for Use in
Equations C-E7-12 and C-E7-13

Table B4.1a Case	Table E7.1 Case	k_c	c_1	c_2	c_3	c_4	c_5
1	(c)	1.0	0.22	1.49	0.56	0.834	0.184
2	(c)	k_c	0.22	1.49	0.64	0.954	0.210
3	(c)	1.0	0.22	1.49	0.45	0.671	0.148
4	(c)	1.0	0.22	1.49	0.75	1.12	0.246
web 5	(a)	1.0	0.18	1.31	1.49	1.95	0.351
6	(b)	1.0	0.20	1.38	1.40	1.93	0.386
7	(a)	1.0	0.18	1.31	1.40	1.83	0.330
8	(a)	1.0	0.18	1.31	1.49	1.95	0.351

where c_3 is the constant associated with slenderness limits given in Table B4.1 (Geschwindner and Troemner, 2016). Combining the constants in Equation C-E7-1 with $c_4 = c_2c_3$ and $c_5 = c_1c_2c_3$ yields

$\lambda = 56.8 = \frac{h_w}{t_w}$

$F_{cr} = 36.99$ ksi

$F_y = 50$ ksi

Check λ_{rw}

(a) When $\lambda \le \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

$$b_e = b \qquad \text{(E7-2)}$$

$\lambda_r = 1.49 \sqrt{\frac{29000}{50}} = 35.88$

ref to $\Rightarrow 1.49 \sqrt{\frac{E}{F_y}}$ Case - 5

modify $1.49 \sqrt{\frac{E}{F_{cr}}}$

$\lambda_{rw} = 1.49 \sqrt{\frac{E}{F_{cr}}} = 1.49 \sqrt{\frac{29000}{36.99}} = 41.72$

$\lambda = 56.8 > \lambda_{rw} \Longrightarrow$ Use E 7-3

get Be value

We have $56.80 > 41.72$

$\lambda < \lambda_{rw} \rightarrow$ Use hweb Ewb

Our Case

Find C_2, F_{eL}

$C_1 \rightarrow$ From Table

(b) When $\lambda > \lambda_r \sqrt{\dfrac{F_y}{F_{cr}}}$

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \tag{E7-3}$$

$C_1 = 0.18$, $C_2 = 1.31 \rightarrow$ Table

$\lambda = 56.80$ h/t_w

$\lambda_r = 35.88$ Table

$F_{eL} = \left(C_2 \dfrac{\lambda_r}{\lambda}\right)^2 F_y$

$F_{eL} = \left(1.31 \cdot \dfrac{35.88}{56.80}\right)^2 50$

$F_{eL} = 34.24$ ksi

where

b = width of the element (for tees this is d; for webs this is h), in. (mm)

c_1 = effective width imperfection adjustment factor determined from Table E7.1

$$c_2 = \frac{1 - \sqrt{1 - 4c_1}}{2c_1} \tag{E7-4}$$

λ = width-to-thickness ratio for the element as defined in Section B4.1

λ_r = limiting width-to-thickness ratio as defined in Table B4.1a

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \tag{E7-5}$$

= elastic local buckling stress determined according to Equation E7-5 or an elastic local buckling analysis, ksi (MPa)

$$b_e = b\left(1 - C_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}}$$

$$\left(\frac{h}{t_w}\right)t_w$$

$$b = h_w = 14.20''$$

$$C_1 = 0.18$$

$$F_{el} = 34.24 \text{ ksi}$$

$$F_{cr} = 36.99 \text{ ksi}$$

$$b_e = 14.20\left(1 - 0.18\sqrt{\frac{34.24}{36.99}}\right)\sqrt{\frac{34.24}{36.99}}$$

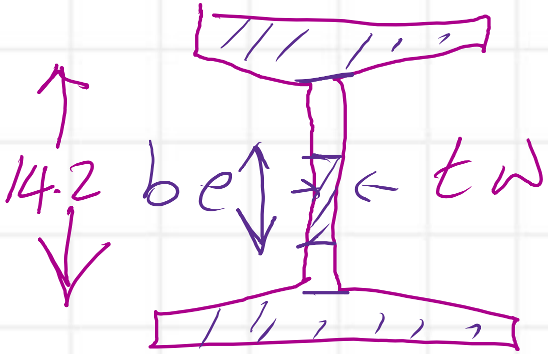
$$= 14.2\,(0.8268)\,(0.962)$$

$$b_e = 11.29'' \approx 11.30''$$

$A_g = 7.68 \text{ inch}^2$

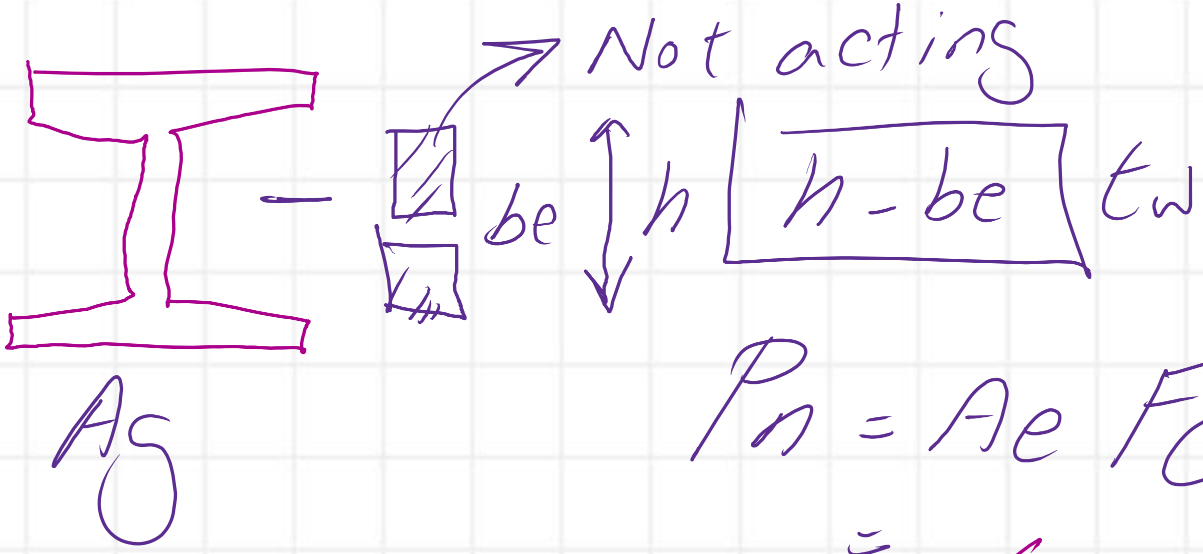
$h = 14.20''$

$t_w = 0.25''$



$A_e = A_g - (h - b_e)\,t_w$

$A_e = 7.68 - ((14.2 - 11.3)\,0.25'')$



$A_e = 6.955 \approx 6.96 \text{ inch}^2$

$F_{cr} = 36.99 \text{ ksi}$

$P_n = A_e F_{cr}$

$= 6.96(36.99) = 257.5$ kips

$$\phi_c P_n = 0.90(257.5) = 231.75 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{1}{1.67}(257.5) = 154 \text{ kips}$$

Check Table 6-2

$F_y = 50$ ksi
$F_u = 65$ ksi

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

W16–W14

W16×		W14×				Shape	W16×		W14×			
26[c]		873[h]		808[h]		lb/ft	26[v]		873[h]		808[h]	
P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	Design	M_{nx}/Ω_b	$\phi_b M_{nx}$	M_{nx}/Ω_b	$\phi_b M_{nx}$	M_{nx}/Ω_b	$\phi_b M_{nx}$
Available Compressive Strength, kips							Available Flexural Strength, kip-ft					
ASD	LRFD	ASD	LRFD	ASD	LRFD	…ect to least radius of gyration, r_y, … ft, for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD
198	298	7690	11600	7130	10700	0	110	166	5060	7610	4570	6860
154	231	7570	11400	7010	10500	6	98.0	147	5060	7610	4570	6860
140	211	7530	11300	6970	10500	7	92.0	138	5060	7610	4570	6860
126	190	7480	11200	6920	10400	8	86.0	129	5060	7610	4570	6860
112	168	7430	11200	6870	10300	9	80.1	120	5060	7610	4570	6860
98.1	147	7360	11100	6810	10200	10	74.1	111	5060	7610	4570	6860
83.1	125	7300	11000	6750	10100	11	68.1	102	5060	7610	4570	6860
69.8	105	7220	10900	6680	10000	12	59.1	88.8	5060	7610	4570	6860
59.5	89.4	7140	10700	6600	9920	13	51.5	77.5	5060	7610	4570	6860
51.3	77.1	7060	10600	6520	9800	14	45.5	68.4	5060	7610	4570	6860
44.7	67.2	6970	10500	6440	9680	15	40.6	61.1	5060	7610	4570	6860
39.3	59.0	6880	10300	6350	9540	16	36.6	55.0	5060	7610	4570	6860
34.8	52.3	6780	10200	6250	9400	17	33.2	50.0	5060	7610	4570	6860
31.0	46.6	6680	10000	6160	9250	18	30.4	45.7	5060	7600	4560	6850
		6570	9870	6050	9100	19	28.0	42.1	5050	7590	4550	6840
		6460	9700	5950	8940	20	25.9	39.0	5040	7580	4540	6830

$\phi_c P_n \quad L_{cy} = 6' \Rightarrow 231$ kips $\rightarrow 231.75$ kips

$\frac{P_n}{\Omega_c} \rightarrow 154$ kips $\rightarrow 154$ kips