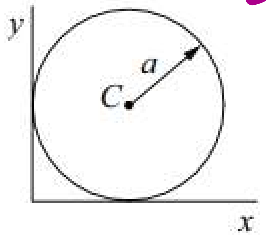
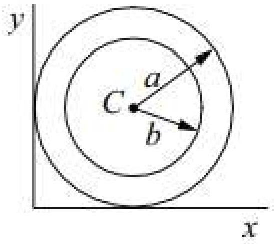
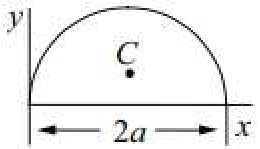
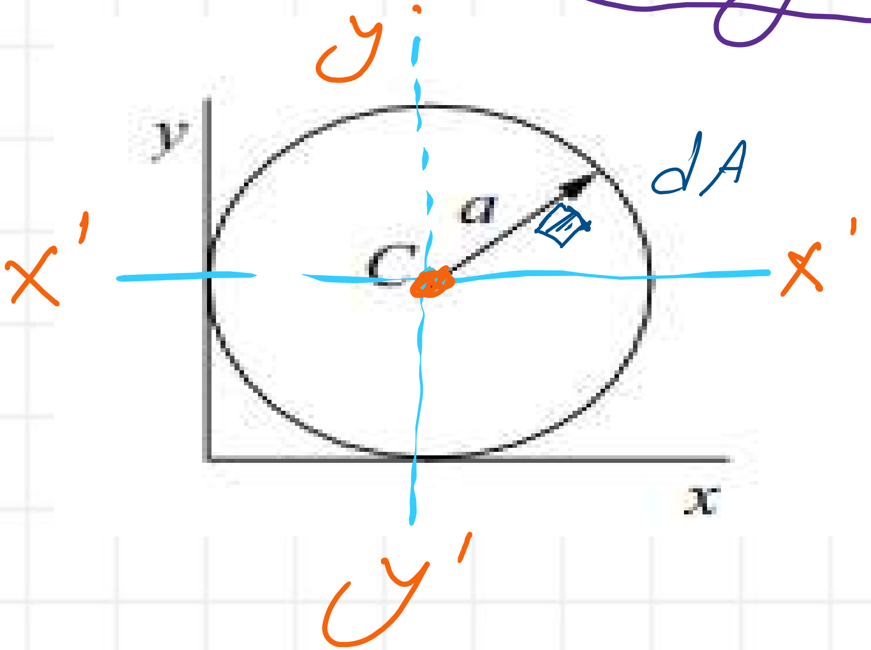
Our Case Area & Centroid

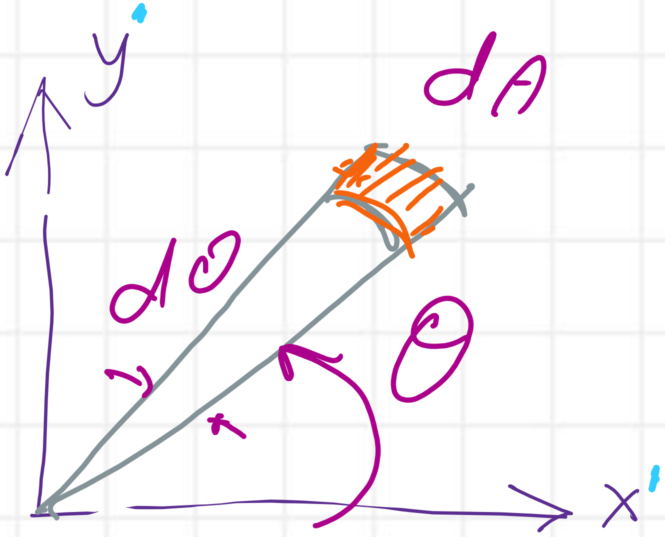
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi(a^4 - b^4)/2$	$r_{x_c}^2 = r_{y_c}^2 = (a^2 + b^2)/4$ $r_x^2 = r_y^2 = (5a^2 + b^2)/4$ $r_p^2 = (a^2 + b^2)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2(a^2 - b^2)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4(9\pi^2 - 64)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2(9\pi^2 - 64)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$

Prepared by Eng.Maged Kamel.

Area and CG for a Circle



x' – x'
y' – y'
Two symmetrical axes



Due to symmetry – CG will be at the point of intersection of x' and y' – axis

To estimate the area and first moment of area – we select small area dA

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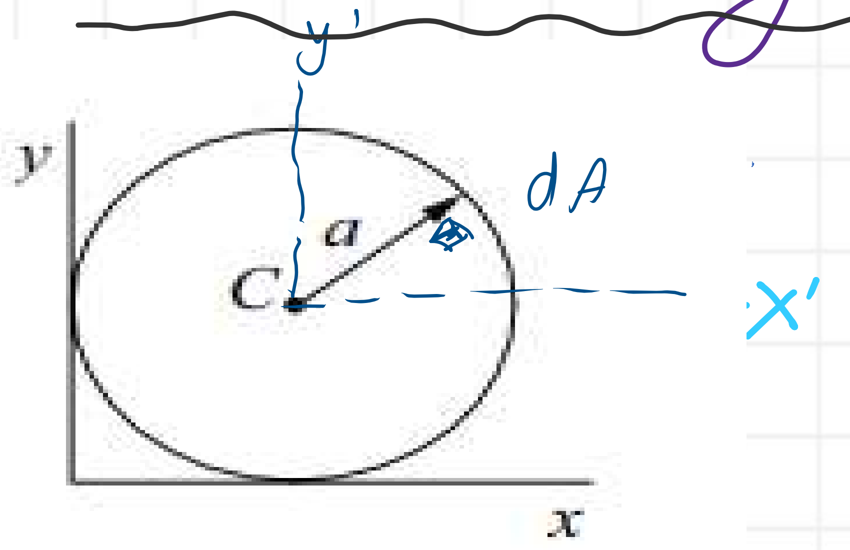
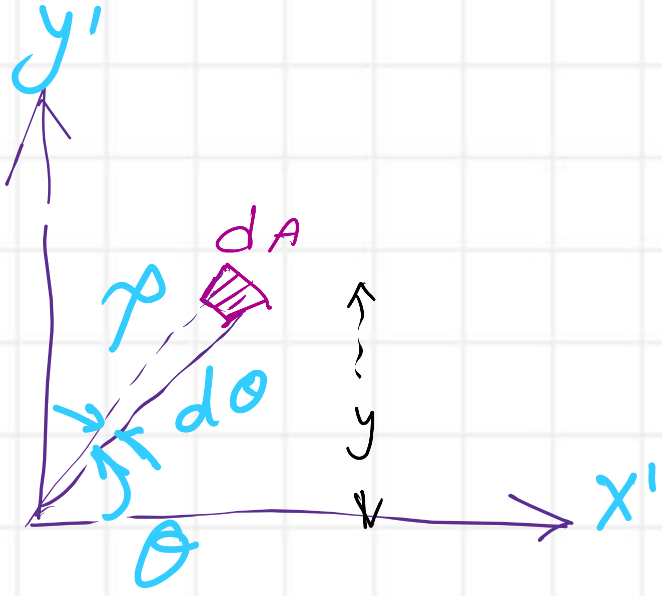
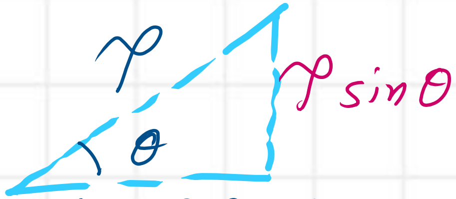
Area and CG For a Circle



$$dA = (r\,d\theta)\,dr$$





θ : enclosed angle between X-axis and strip Line

Let dM_x = First moment about x'-axis for strip dA. $= dA\,(y)$

$y = r\sin\theta$

while $dM_y = dA\,(x) \Rightarrow$ first moment about y'-axis for strip dA

$$dm_x = dA \cdot y = (r\,d\theta\,dr)(r\sin\theta) = (r^2\,dr)(\sin\theta)\,d\theta$$

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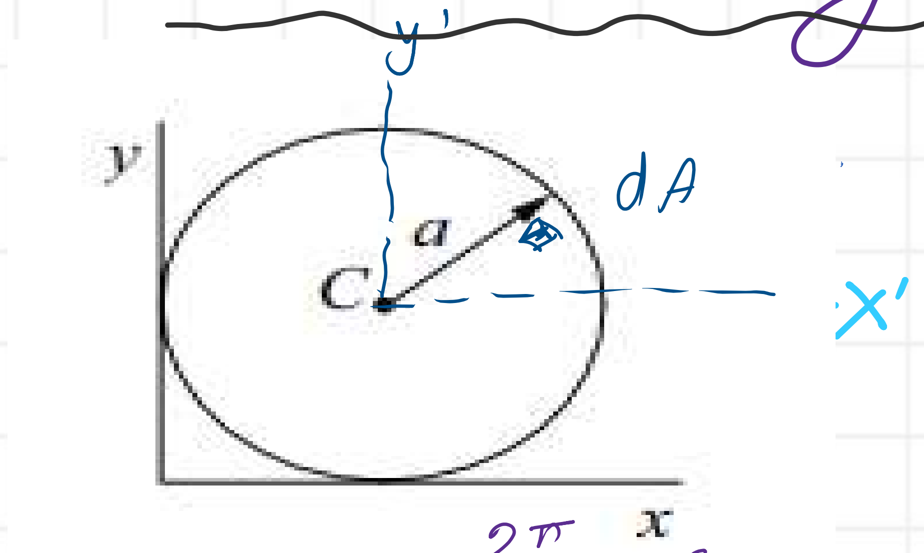
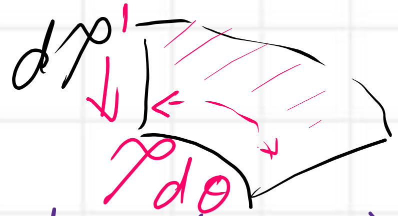
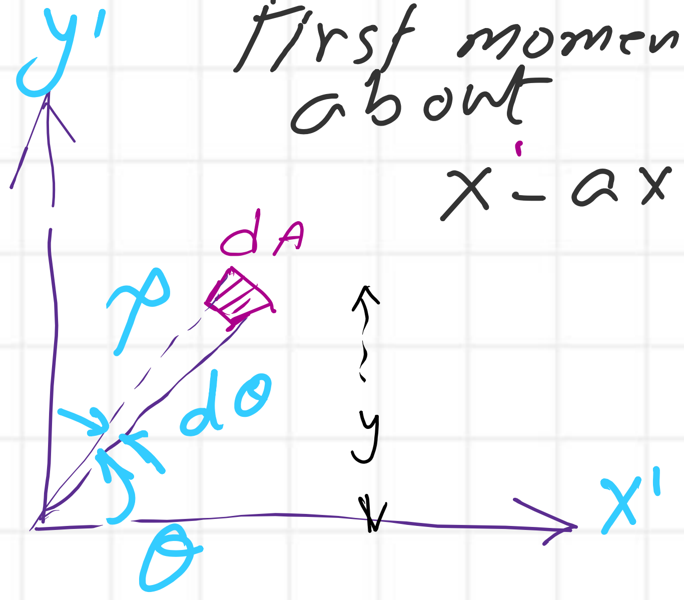
Area and Cg For a Circle

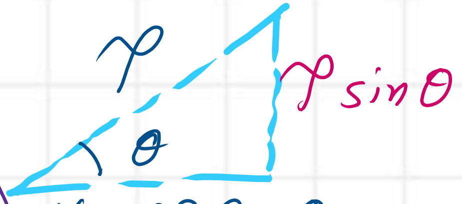
First moment about x'-axis





$$dA = (r\,d\theta)\,dr$$





$$\int dA\cdot y = \int_0^{2\pi}\int_0^{a} (r^2\,dr)(\sin\theta\,d\theta) \qquad x = r\cos\theta$$

θ: enclosed angle between X-axis and stri Line

$$= \left(\frac{r^3}{3}\Big|_0^a\right)(-\cos\theta)\Big|_0^{2\pi} = \frac{a^3}{3}(-)(1-1) = 0$$

C.g $\bar{y}$ bar = 0 → C.g is at x', y' intersection

$$A = \int dA = \int_0^{2\pi}\int_0^{a} r\,d\theta\,dr = \frac{r^2}{2}\Big|_0^a (\theta)\Big|_0^{2\pi} = \frac{a^2}{2}(2\pi) = \pi a^2$$

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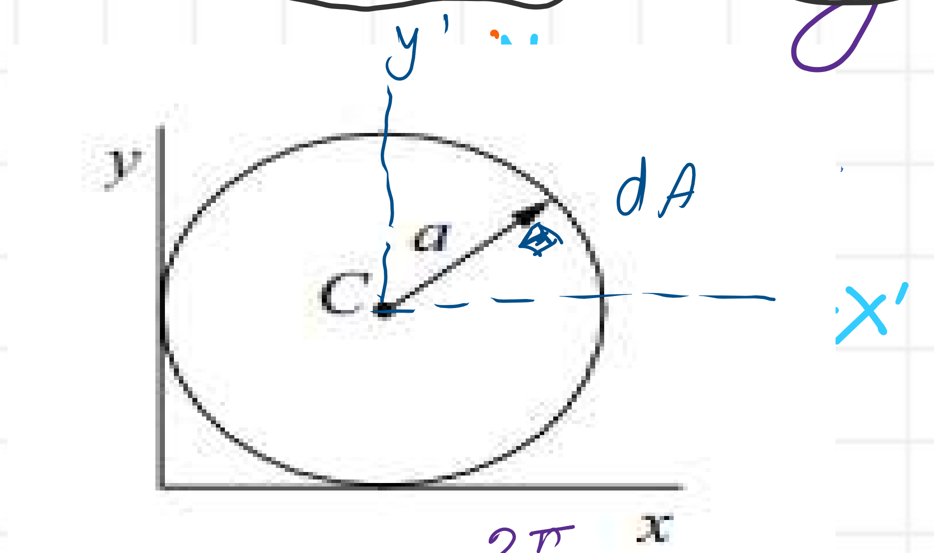
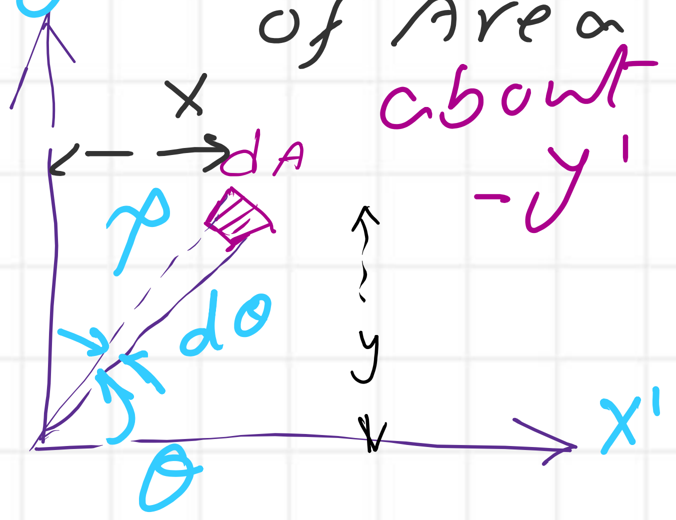
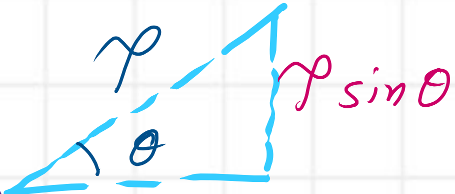
Area and Cg For a Circle



$dA = (r\,d\theta)\,dr$

First moment of Area about - y'





$$\int dA \cdot x = \int_0^{2\pi}\int_0^{a} (r^2\,dr)(r\cos d\theta)$$

$x = r\cos\theta$

θ : enclosed angle between X-axis and strip Line

$$\int dA \cdot x = \left(\frac{r^3}{3}\Big|_0^a\right)(-\sin\theta)\Big|_0^{2\pi} = \frac{a^3}{3}(-)(0) = 0$$

$\bar{X} = 0$, which → C.g is at x', y' intersection Point.

$$A = \int dA = \int_0^{2\pi}\int_0^{a} r\,d\theta\,dr = \frac{r^2}{2}\Big|_0^a(\theta)\Big|_0^{2\pi} = \frac{a^2}{2}(2\pi) = \pi a^2$$

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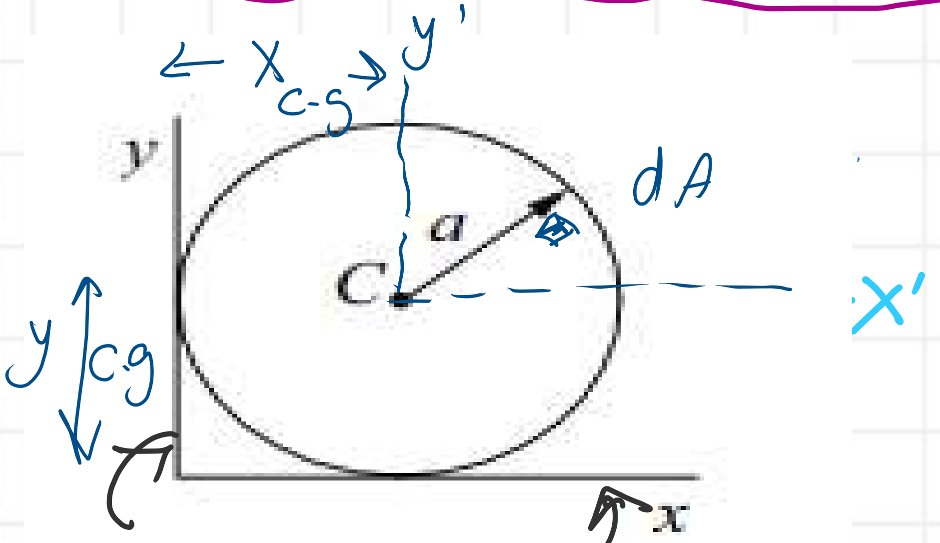
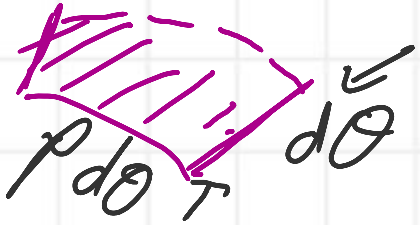
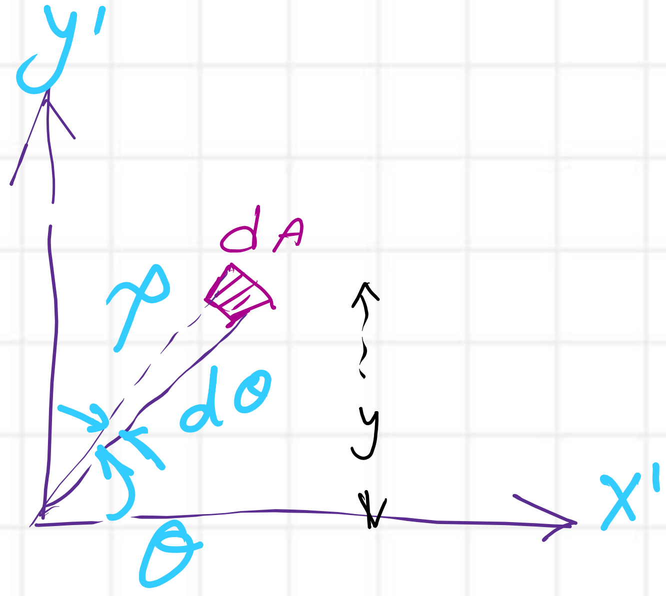
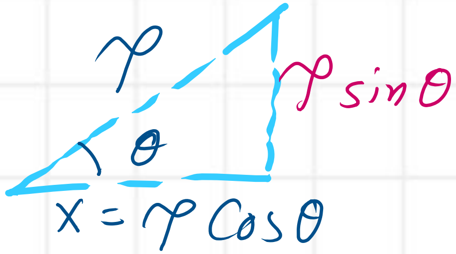
Area and C For a Circle





$$dA = (r\,d\theta)\,dr$$





θ : enclosed angle between x – ; and

For the Two axes x & y

$x_{c\cdot g} = a$ from y_axis $= x_c$ } at the table

$y_{c\cdot g} = a$ from x_axis $= y_c$ }

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