

Engineering economic analysis by Eschenbach, Ted G. Newnan

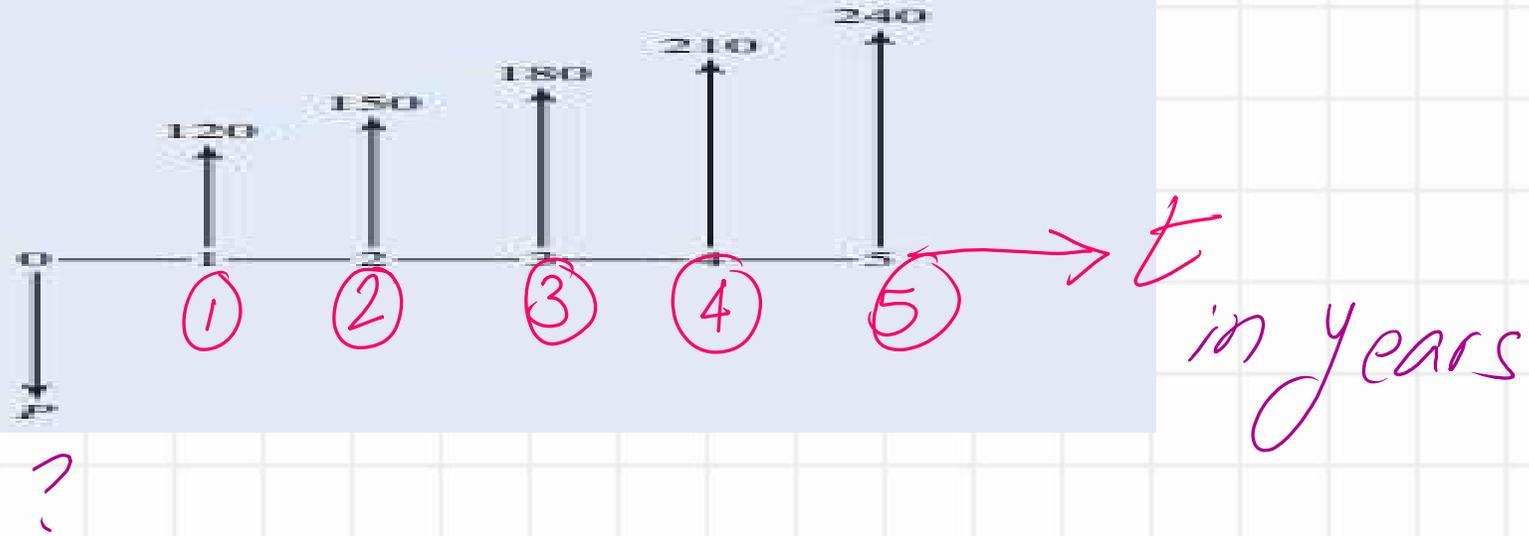
EXAMPLE 4-9

Andrew has purchased a new car. He wishes to set aside enough money in a bank account to pay the maintenance for the first 5 years. It has been estimated that the maintenance cost of a car is as follows:

Year	Maintenance Cost
1	\$120
2	150
3	180
4	210
5	240

Assume the maintenance costs occur at the end of each year and that the bank pays 5% interest. How much should Andrew deposit in the bank now?

SOLUTION



P is unknown

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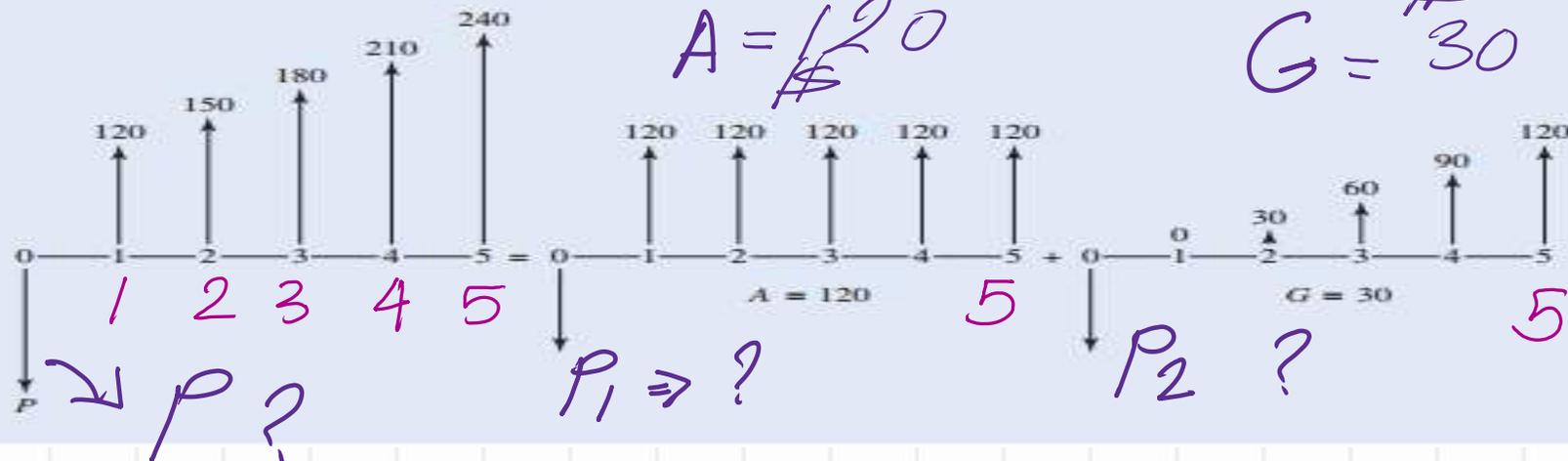
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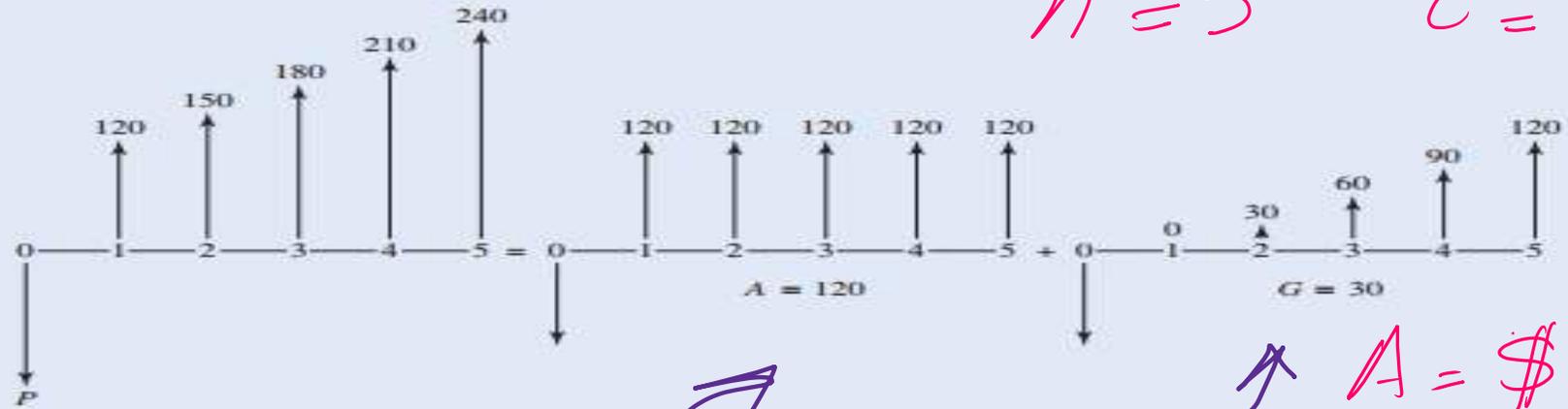
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$$n = 5$$

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$A = \$120$
 $G = \$30$

Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n - 1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\frac{(1+i)^n - 1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G ✓	$(P/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i} - \frac{n}{(1+i)^n - 1}$

$$i = 5\% \quad n = 5, \quad A = 120, \quad B = 30$$

Using Formula

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{0.05(1+0.05)^5}{(1.05)^5 - 1} = \frac{0.063814}{0.27628} = 0.2309$$

$$\frac{P}{A} = \frac{1}{0.2309} = 4.329$$

$$(P/G, i, n) = \left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n} \right]$$

$$\frac{P}{G} = \frac{(1+0.05)^5 - 0.05(5) - 1}{(0.05)^2(1+0.05)^5} = \frac{1.27628 - 0.25 - 1}{3.1907} = 8.236$$

The Present Value is composed of $(P_1 + P_2)$

P_1 is due to Uniform deposit A : $A = 120, i = 5\%$
 $n = 5$ years

P_2 is due to Gradient deposit $\Rightarrow G : 30, i = 5\%$

$$P_1 = A \left(\frac{P}{A}, i\%, n \right) = A \left(\frac{P}{A} \right) = 120 (4.329) = 519.48$$

$$P_2 = G \left(\frac{P}{G}, i\%, n \right) = G \left(\frac{P}{G} \right) = 30 (8.237) = 247.11$$

Adding $P_1 + P_2 \rightarrow$ get P

$$P = 519.48 + 247.11 = \$ 766.59 \approx 766$$

$$A = 120$$

$$G = 30$$

$$n = 5$$

5%

Compound Interest Factors

5%

$i = 5\%$

n	Single Payment		Uniform Payment Series				Arithmetic Gradient		n
	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F	Sinking Fund Factor Find A Given F A/F	Capital Recovery Factor Find A Given P A/P	Compound Amount Factor Find F Given A F/A	Present Worth Factor Find P Given A P/A	Gradient Uniform Series Find A Given G A/G	Gradient Present Worth Find P Given G P/G	
1	1.050	.9524	1.0000	1.0500	1.000	0.952	0	0	1
2	1.102	.9070	.4878	.5378	2.050	1.859	0.488	0.907	2
3	1.158	.8638	.3172	.3672	3.152	2.723	0.967	2.635	3
4	1.216	.8227	.2320	.2820	4.310	3.546	1.439	5.103	4
5	1.276	.7835	.1810	.2310	5.526	4.329	1.902	8.237	5

$$\frac{P}{A} = 4.329, \quad \frac{P}{G} = 8.237 \implies P = \frac{P}{A}(A, i, n) + \frac{P}{G}(G, i, n)$$

$$P = 4.329(120) + 8.237(30)$$

$$P = 519.48 + 247.11 \approx \$766.59 \implies 766^\#$$

Solving By Excel

No dedicated Function For G, but
we can use

$$P = 120 \$$$
$$G = 30 \$$$

PV($i\%$, n , A , F) To get P with
given

$$\text{Plus } \frac{P}{G} = \frac{(1+i)^n - in - 1}{i^2 (1+i)^n} \times G \quad i\%, n, A$$

PV (\$519.54)

$$\times \frac{P}{G} = 8.2369 \rightarrow \frac{P}{G} = 8.2369(30)$$
$$= 247.1075$$

$$\text{Final } P = 519.54 + 247.11$$
$$= \$ 766$$