Objective of lecture.

To understand the following points

1- Uniform series of compound interest formula.

2- Uniform series sinking funds.

Solved examples.

BASICS of Engineering Economy

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2.2 UNIFORM SERIES FORMULAS (*P/A, A/P, A/F, F/A*)

There are four *uniform series* formulas that involve A, where A means that:

1. The cash flow occurs in *consecutive interest periods*, and
2. The cash flow *amount* is the *same* in each period.

The formulas relate a present worth P or a future worth F to a uniform series amount A. The two equations that relate P and A are as follows. (See Figure 2.5 for cash flow diagrams.)

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

A: UniForm Series

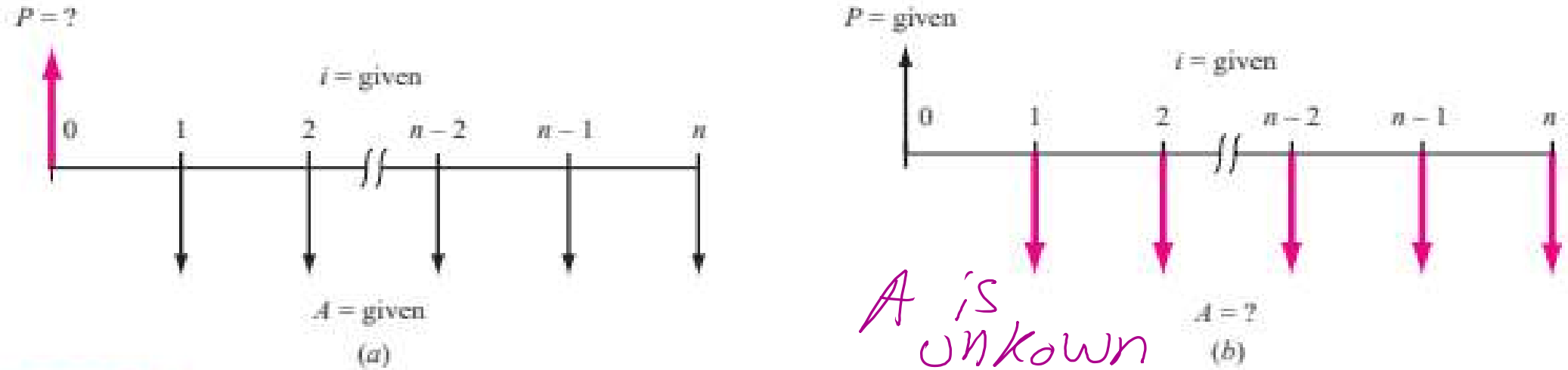
First A is seperated From P by one interest Period

In standard factor notation, the equations are $P = A(P/A, i, n)$ and $A = P(A/P, i, n)$, respectively. It is important to remember that in these equations, the P and the first A value are separated by one interest period. That is, the present worth P is always located one interest period prior to the first A value. It is also important to remember that the n is always equal to the number of A values.

The factors and their use to find P and A are summarized in Table 2.2. The spreadsheet functions shown in Table 2.2 are capable of determining both

P is Unkown

Relation between P & A



A is unkown

FIGURE 2.5 Cash flow diagrams used to determine (a) P of a uniform series and (b) A for a present worth.

TABLE 2.2 *P*/*A* and *A*/*P* Factors: Notation, Equation and Spreadsheet Function

Factor Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Function
$(P/A,i,n)$	Uniform-series present worth	P/A	$\frac{(1+i)^n-1}{i(1+i)^n}$	$P = A(P/A,i,n)$	$= PV(i\%,n,A,F)$
$(A/P,i,n)$	Capital recovery	A/P	$\frac{i(1+i)^n}{(1+i)^n-1}$	$A = P(A/P,i,n)$	$= PMT(i\%,n,P,F)$

Excel Functions

P and A values in lieu of applying the P/A and A/P factors. The PV function calculates the P value for a given A over n years, and a separate F value in year n, if present. The format is

To get Present value $= PV(i\%,n,A,F)$

Similarly, the A value is determined using the PMT function for a given P value in year 0 and a separate F, if present. The format is

$= PMT(i\%,n,P,F)$

To get A value

F Future value

P

i% interest

n number of periods

Present value

2nd Case relation between A & F

F is given ?

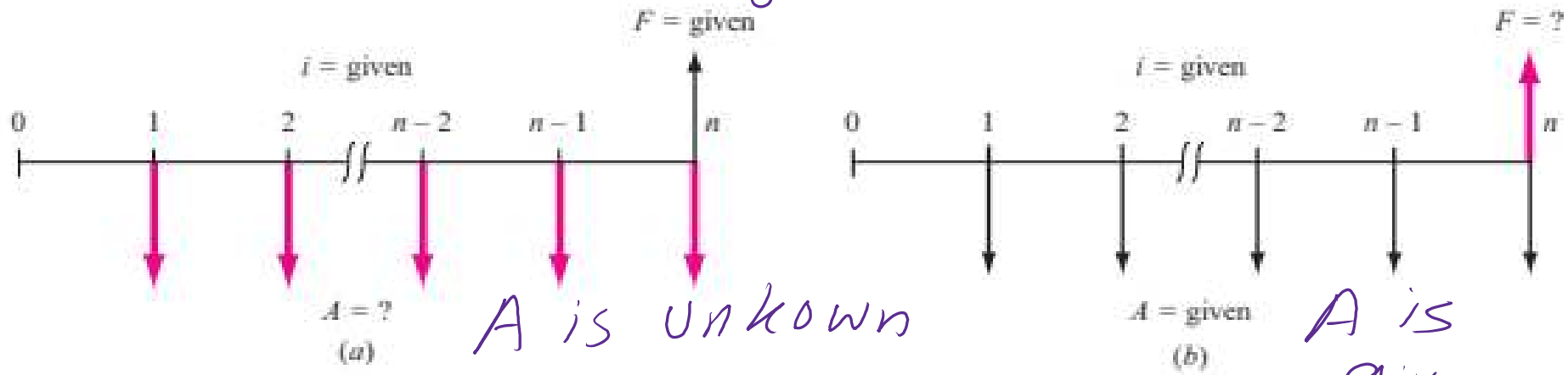


FIGURE 2.7 Cash flow diagrams to (a) find *A*, given *F*, and (b) find *F*, given *A*.

TABLE 2.3 ***F/A* and *A/F* Factors: Notation, Equation and Spreadsheet Function**

Factor			Factor	Standard	Excel
Notation	Name	Find/Given	Formula	Notation Equation	Function
$(F/A,i,n)$	Uniform-series compound amount	F/A	$\frac{(1+i)^n-1}{i}$	$F = A(F/A,i,n)$	$= \text{FV}(i\%,n,A,P)$
$(A/F,i,n)$	Sinking fund	A/F	$\frac{i}{(1+i)^n-1}$	$A = F(A/F,i,n)$	$= \text{PMT}(i\%,n,P,F)$

Last A value occurs in the same time Period as the Future worth F

The uniform series formulas that relate A and F follow. See Figure 2.7 for cash flow diagrams.

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \Rightarrow \text{Find } A \rightarrow \text{given } i\%, F \& n$$

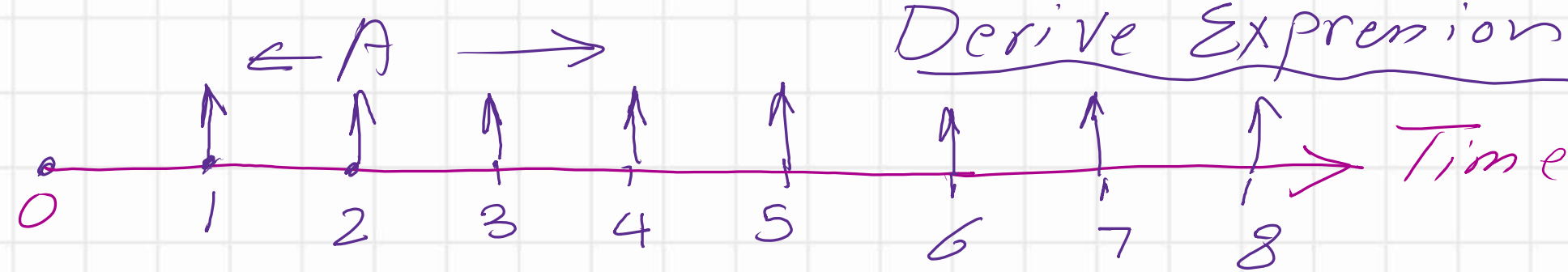
$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow \text{Find } F \rightarrow \text{given } i\%, n, A$$

It is important to remember that these equations are derived such that the last A value occurs in the *same* time period as the future worth F, and n is always equal to the number of A values.

Standard notation follows the same form as that of other factors. They are $(F/A,i,n)$ and $(A/F,i,n)$. Table 2.3 summarizes the notations and equations.

If P is not present for the PMT function, the comma must be entered to indicate that the last entry is an F value.

Derive Expression



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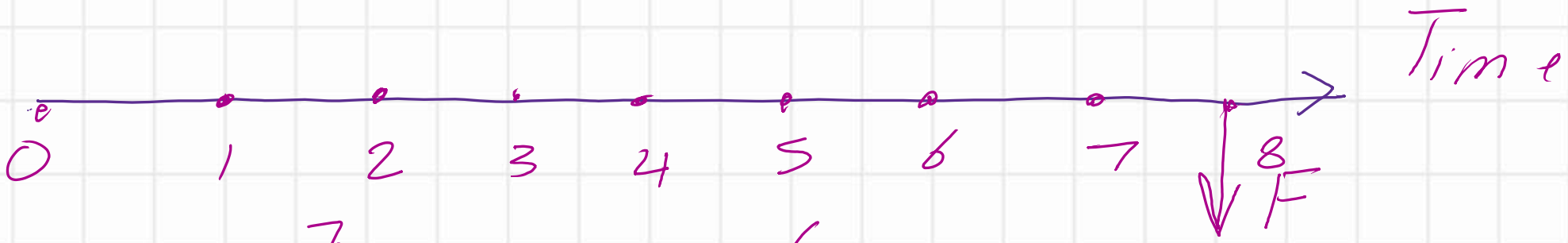

$$F = A(1+i)^7 + A(1+i)^6 + A(1+i)^5 + A(1+i)^4 + A(1+i)^3 + A(1+i)^2 + A(1+i)^1 + A$$

Multiply by $(1+i)$

$$F(1+i) = A\left[(1+i)^8 + (1+i)^7 + (1+i)^6 + (1+i)^5 + (1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i)\right]$$

Deduct F From $F(1+i)$

$F + Fi - F = Fi = L.H.S$

$A(1+i)^8 - A = R.H.S$

$Fi = A(1+i)^n - A$ put $n = 8$

$Fi = A[(1+i)^n - 1] \Rightarrow F = \frac{A}{i}[(1+i)^n - 1]$

$F = A[F/A, i\%, n]$

$A = F[A/F, i\%, n]$

$A = \frac{Fi}{[(1+i)^n - 1]} \Rightarrow$ Sinking Fund

TABLE 2.3 *F/A* and *A/F* Factors: Notation, Equation and Spreadsheet Function

Factor Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Function
$(F/A,i,n)$	Uniform-series compound amount	F/A	$\frac{(1+i)^n-1}{i}$	$F = A(F/A,i,n)$	$= \text{FV}(i\%,n,A,P)$
$(A/F,i,n)$	Sinking fund	A/F	$\frac{i}{(1+i)^n-1}$	$A = F(A/F,i,n)$	$= \text{PMT}(i\%,n,P,F)$

We can derive the relation between P & A

Since $F = P(1+i)^n$

while $F = A\frac{(1+i)^n-1}{i} = P(1+i)^n$

$A = \frac{P(1+i)^n i}{(1+i)^n-1}$

$A = P\left(\frac{A}{P}, i\%, n\right)$

$P = A\left(\frac{P}{A}, i\%, n\right)$