Continuous Compounding

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What Is Continuous Compounding?

Continuous compounding is the mathematical limit that compound interest can reach if it's calculated and reinvested into an account's balance over a theoretically infinite number of periods. While this is not possible in practice, the concept of continuously compounded interest is important in finance. It is an extreme case of compounding, as most interest is compounded on a monthly, quarterly, or semiannual basis.

Calculating the limit of this formula as n approaches infinity (per the definition of continuous compounding) results in the formula for continuously compounded interest:

$FV = PV \times e^{(i \times t)}$, where e is the mathematical constant approximated as 2.7183.

Source : Time value of money part : 1

Pamela Peterson

Example 9: Solving for a present value

Problem

Which of the following requires the least amount of a deposit today?

a. A balance of $10,000 four years from today that has grown from a sum deposited in an account that pays 8 percent interest, compounded quarterly.

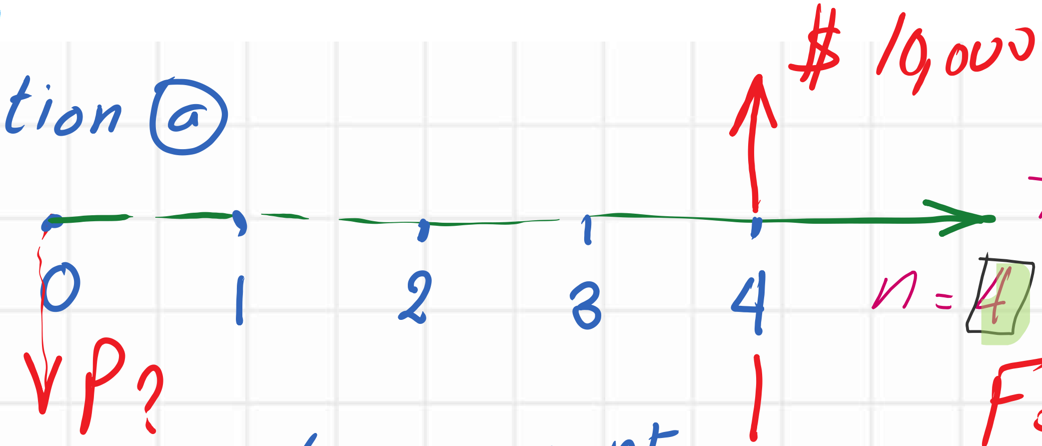
b. A balance of $10,000 five years from today that has grown from a sum deposited in an account that pays 7 percent interest, compounded annually.

c. A balance of $10,000 ten years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded continuously.

d. A balance of $10,000 eight years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded semi-annually.

Solution

Option a



$t = 4$ Time in years

$i = 8\%$ Compounded quarterly

$n = \boxed{4}$ Four quarters/years

$F = P\left(1 + \frac{i}{n}\right)^{nt}$

$nt = 4(4)$

$$P = F/\left(1 + \frac{i}{n}\right)^{nt} = 10,000/\left(1 + \frac{0.08}{4}\right)^{4(4)} = \$7284.45$$

Source : Time value of money part : 1

Example 9: Solving for a present value

Problem

Which of the following requires the least amount of a deposit today?

a. A balance of $10,000 four years from today that has grown from a sum deposited in an account that pays 8 percent interest, compounded quarterly.

b. A balance of $10,000 five years from today that has grown from a sum deposited in an account that pays 7 percent interest, compounded annually.

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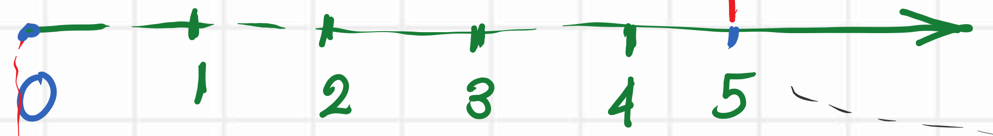
Solution

option (b)

$ 10,000



Time in years

$i = 7\%$

Compounded annually

$n = 1$, $t = 5$

P?

$F = P\left(1 + \frac{i}{n}\right)^{nt}$

$P = F/\left(1 + \frac{i}{n}\right)^{nt} = 10{,}000/\left(1 + \frac{0.07}{1}\right)^{(1)(5)} = 7129.86$ $

Source : Time value of money Part : 1

Example 9: Solving for a present value

Problem

Which of the following requires the least amount of a deposit today?

a. A balance of \$10,000 four years from today that has grown from a sum deposited in an account that pays 8 percent interest, compounded quarterly.

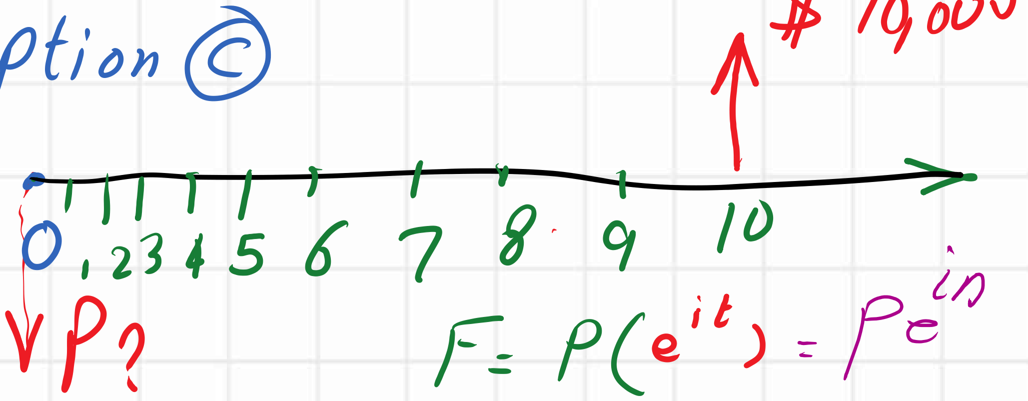
b. A balance of \$10,000 five years from today that has grown from a sum deposited in an account that pays 7 percent interest, compounded annually.

c. A balance of \$10,000 ten years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded continuously.

d. A balance of \$10,000 eight years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded semi-annually.

Solution

Option (c)



Time in years

$i = 4\%$

Compound-Continuously

$t = n = 10$

$in = \frac{4}{100}(10) = 0.4$

$F = P(e^{it}) = Pe^{in}$

$P = \frac{F}{e^{it}} = 10{,}000 / (2.7183)^{0.4}$

$= 6703.1825$

The **exponential** constant is an important mathematical constant and is given the symbol **e**. Its **value** is approximately 2.7183.

Source : Time value of money part : 1

Example 9: Solving for a present value

Problem

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c. A balance of $10,000 ten years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded continuously.

d. A balance of $10,000 eight years from today that has grown from a sum deposited in an account that pays 4 percent interest, compounded semi-annually.

semi-annually

Solution

option d

$10,000

$i = 4\%$

Time in years

0 1 2 3 4 5 6 7 8

P?

$n = 2$ number of compounds/year

$t = 8$

$$F = P\left(1 + \frac{i}{n}\right)^{nt}$$

$$P = 10{,}000 \Big/ \left(1 + \frac{0.04}{2}\right)^{2(8)} = \$7284.458$$

Example 9: Solving for a present value

Problem

Which of the following requires the least amount of a deposit today?

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b. A balance of $10,000 five years from today that has grown from a sum deposited in an account that pays 7 percent interest, compounded annually.

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Author's solution.

Solution

C

Least PV value

$PV_a = \$10,000 / (1 + 0.02)^{16} = \$7,284.46$

$PV_b = \$10,000 / (1 + 0.07)^{5} = \$7,129.86$

$PV_c = \$10,000 / e^{(0.04)(10)} = \$6,703.20$ ⇒ Least deposit value ⟹ option C

$PV_d = \$10,000 / (1 + 0.02)^{16} = \$7,284.46$