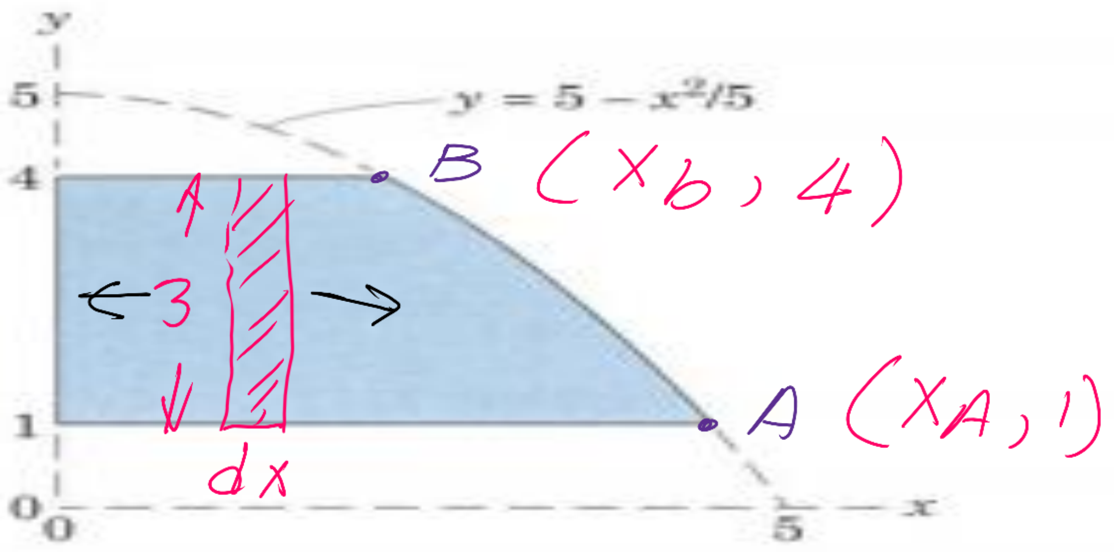
5/8 Locate the centroid of the shaded area shown.

William Kraige



For $\bar{x}$

Problem 5/8

Solution: get the x Coordinate For point A, B

We have $y = 5 - \frac{x^2}{5} \rightarrow \frac{x^2}{5} = 5 - y \Rightarrow x^2 = 25 - 5y$

$x_A^2 = 25 - 5(1) = 20 \longrightarrow x = \sqrt{20} = 4.4721$

$y = 1$

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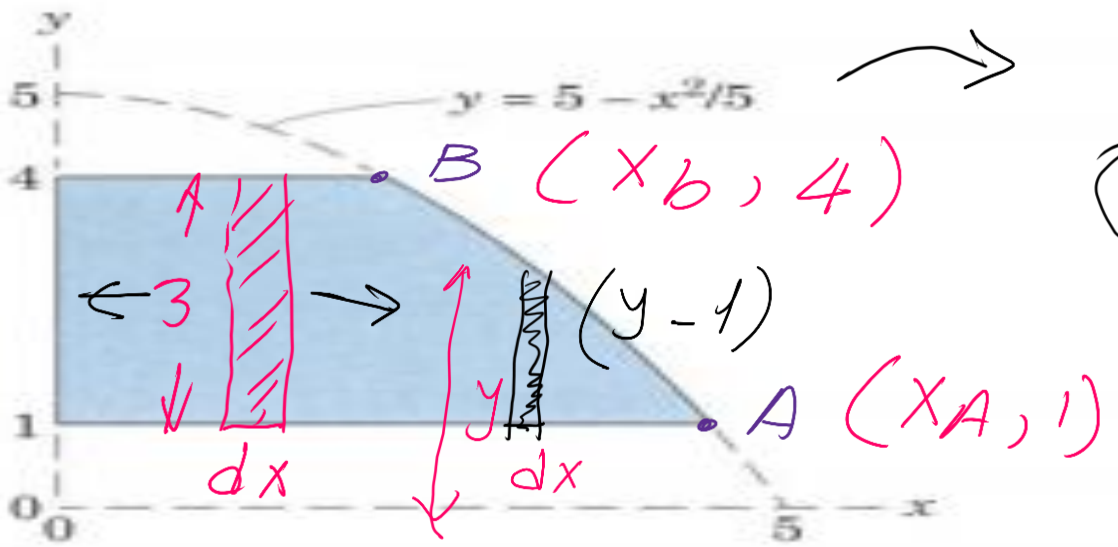
5/8 Locate the centroid of the shaded area shown.



Problem 5/8

For $\bar{x}$

$x^2 = (5-y)5$

$x^2 = 25 - 5y$

Use a small strip dx

Solution: get the x coordinate for point B

We have $x_b^2 = 25 - 5(4) = 5 \longrightarrow x_b = \sqrt{5} = 2.2361$

$\Sigma Ax = \int_0^{x_B} 3x\,dx + \int_{x_B}^{x_A} dx\,(y-1)\,x$

$\Sigma A = \int_0^{x_A} 3\,dx + \int_{x_B}^{x_A} dx\,(y-1)$

Prepared by Eng.Maged Kamel.

$$\sum A x = \int_{0}^{X_B} 3x\,dx + \int_{X_B}^{X_A} x\,dx\,(y-1) \qquad y = 5 - \frac{x^2}{5}$$

$$X_B = 4.4721$$

Re-write

$$\int_{0}^{2.2361} 3x\,dx + \int_{2.2361}^{4.4721} x\left(5 - \frac{x^2}{5} - 1\right) dx \;\Rightarrow\; \int x\left(4 - \frac{x^2}{5}\right) dx$$

$$\sum A x = 3\frac{x^2}{2}\Big|_{0}^{2.2361} + 4\frac{x^2}{2}\Big|_{2.2361}^{4.4721} - \frac{x^4}{20}\Big|_{2.2361}^{4.4721}$$

$$\sum A x = 1.5(2.2361)^2 + 2\left[\overline{4.4721}^{\,2} - \overline{2.2361}^{\,2}\right] - \frac{1}{20}\left[\overline{4.4721}^{\,4} - \overline{2.2361}^{\,4}\right]$$

$$= 7.50 + 2\left[20 - 5\right] - \frac{1}{20}\left[400 - 25\right]$$

$$= 37.50 - 18.75 = 18.75$$

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$$\sum A = \int_{0}^{X_B} 3\, dx + \int_{X_B}^{X_A} dx\,(y-1) \qquad y = 5 - \frac{x^2}{5}$$

$X_B = 4.4721$

Re-write

$$\int_{0}^{2.2361} 3\, dx + \int_{2.2361}^{4.4721} \left(5 - \frac{x^2}{5} - 1\right) dx \Rightarrow \int_{2.2361}^{4.4721} \left(4 - \frac{x^2}{5}\right) dx$$

$$\sum A = 3 \cdot X \Big|_{0}^{2.2361} + 4 \cdot X \Big|_{2.2361}^{4.4721} - \frac{X^3}{15} \Big|_{2.2361}^{4.4721}$$

$$\sum A = 3(2.2361) + 4(4.4721 - 2.2361) - \frac{1}{15}\left(\overline{4.4721}^3 - \overline{2.2361}^3\right)$$

$$\sum A = 6.7083 + 8.944 - \frac{1}{15}(89.4406 - 11.181)$$

$$\sum A = 15.652 - 5.2173$$

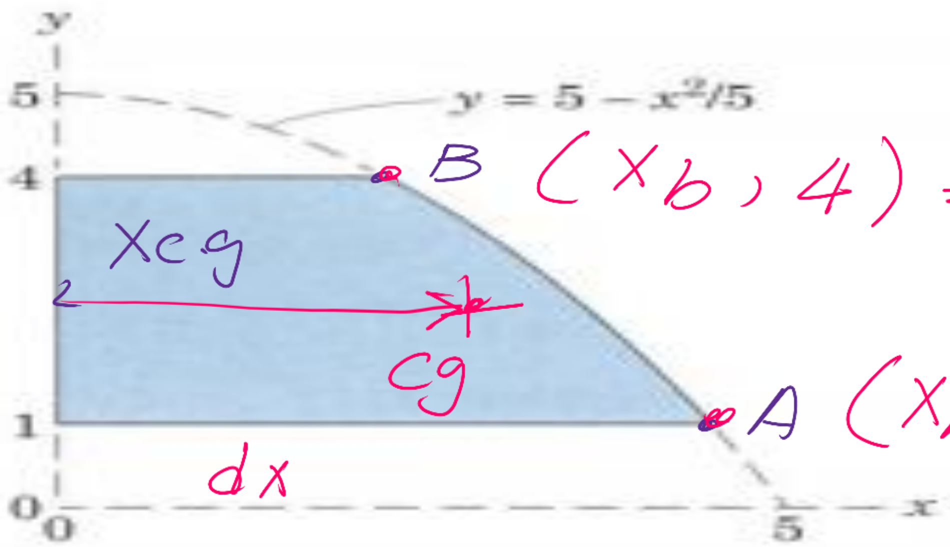
$$= 10.4347$$

Prepared by Eng.Maged Kamel.

5/8 Locate the centroid of the shaded area shown.

William Kraige

For X bar



B $(X_b , 4) = (2.2361, 4)$

A $(X_A , 1) = (4.471, 1)$

Problem 5/8

$\Sigma AX = 18.75$, $\Sigma A = 10.4347$

$\bar{X}_{c.g} = 18.75/10.4347 = 1.7969 \simeq 1.797$

X Coordinate $= \bar{X}_{cg} = 1.797$

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5/8 Locate the centroid of the shaded area shown.

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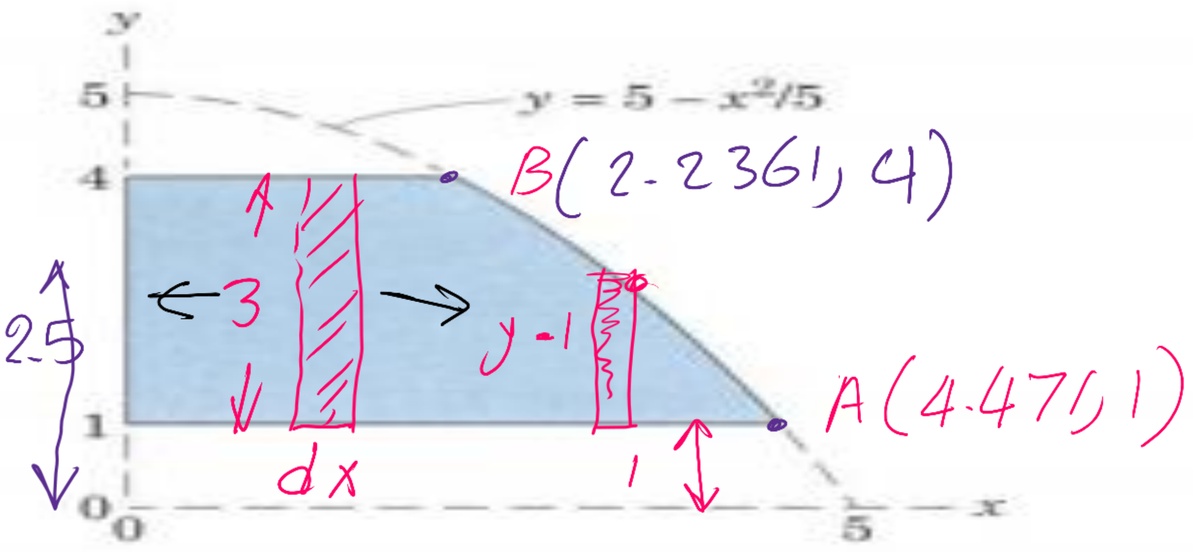
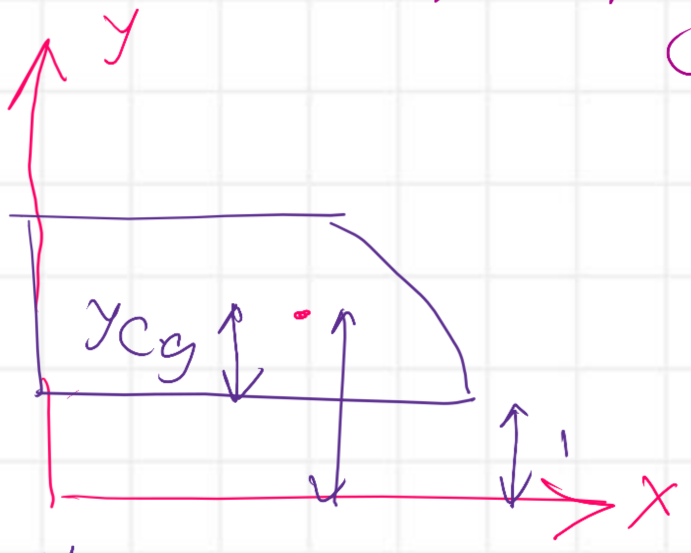
For y bar

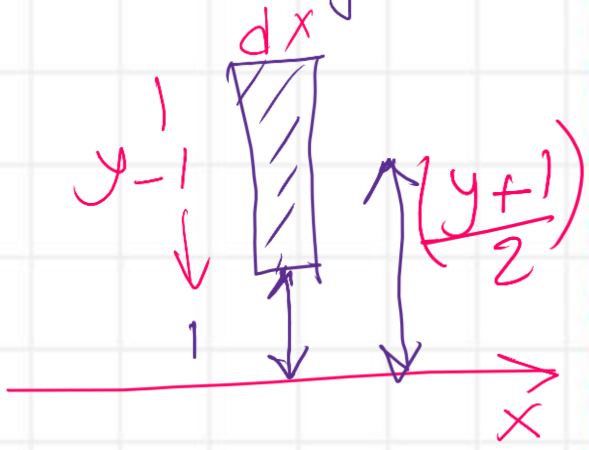


Problem 5/8



$y_{coord} = y_{c.g} + 1$

$$\sum A y = \int_0^{x_B} 3\,dx\,(2.5) + \int_{x_B}^{x_A} dx\,(y-1)\left(\frac{1}{2}\right)(y+1)$$



$$\sum A y = 7.5 \int_0^{x_B} dx + \frac{1}{2}\int_{x_B}^{x_A} (y^2-1)\,dx$$

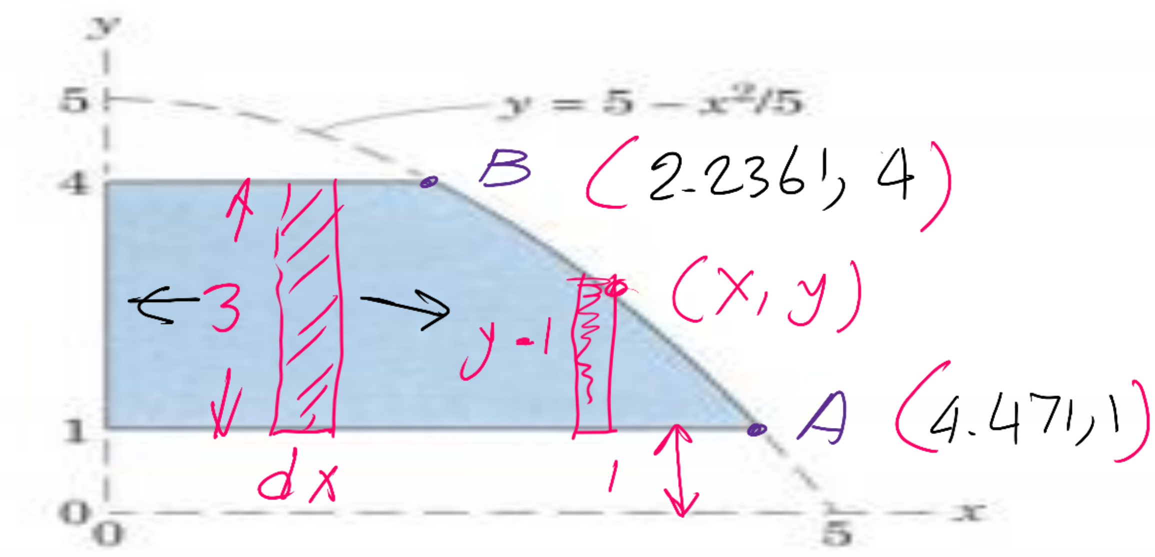
$$y^2 - 1 = \left(5 - \frac{x^2}{5}\right)^2 - 1 = 25 - 2x^2 + \frac{x^4}{25} - 1$$

Prepared by Eng.Maged Kamel.

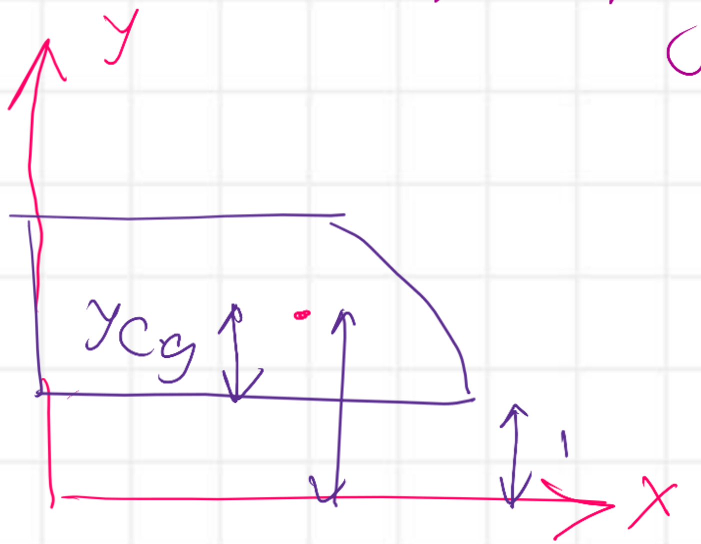
5/8 Locate the centroid of the shaded area shown.

William Kraige

For $\bar{y}$ bar



Problem 5/8



$$y_{coord} = y_{c.g} + 1$$

$$\sum Ay = 7.5\int_{0}^{2.2361} dx + \frac{1}{2}\int_{2.2361}^{4.471} 24 - 2x^2 + \frac{x^4}{25}\, dx$$

$$y^2 = \left(25 - 2x^2 + \frac{x^4}{25}\right)$$

$$-1 = -1$$

$$24 - 2x^2 + \frac{x^4}{25}$$

$$= 12x\Big|_{2.2361}^{4.471} - \frac{x^3}{3}\Big|_{2.2361}^{4.471} + \frac{1}{50}\frac{x^5}{5}\Big|_{2.2361}^{4.471}$$

Prepared by Eng.Maged Kamel.

5/8 Locate the centroid of the shaded area shown.

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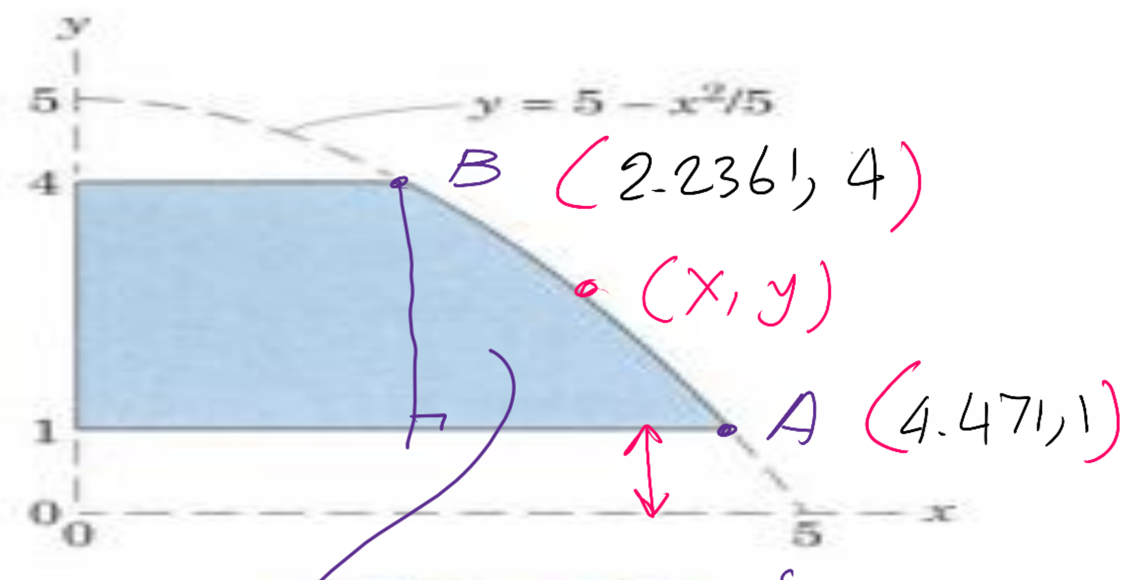
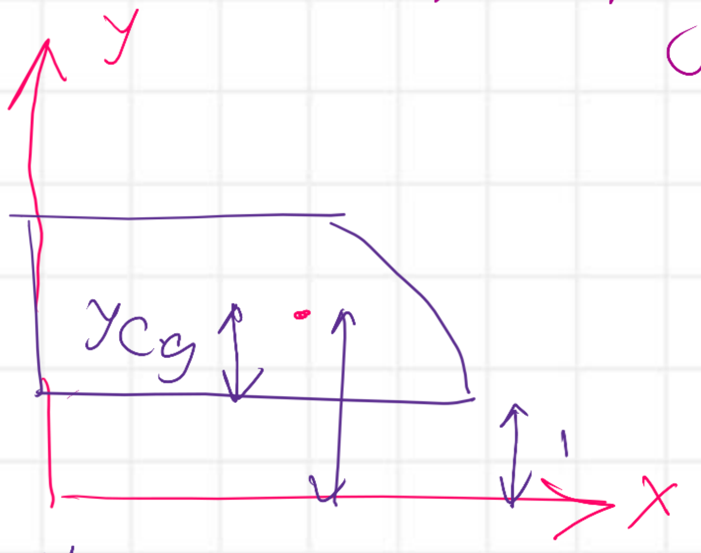


Problem 5/8



For y bar

2nd Int

$$= 12x\Big|_{2.2361}^{4.471} - \frac{x^3}{3}\Big|_{2.2361}^{4.471} + \frac{1}{50}\frac{x^5}{5}\Big|_{2.2361}^{4.471}$$

$$= 26.8188 - 26.0645 + 6.92271 = 7.6770$$

$y_{coord} = y_{c.g} + 1$

$$y^2 = \left(25 - 2x^2 + \frac{x^4}{25}\right)$$

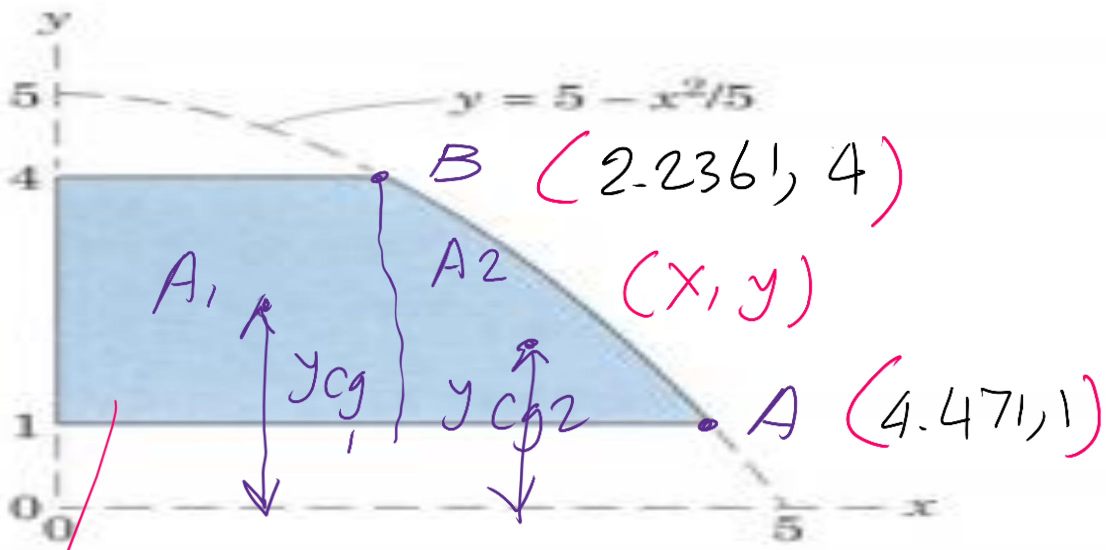
$$-1 = -1$$

$$24 - 2x^2 + \frac{x^4}{25}$$

Prepared by Eng.Maged Kamel.

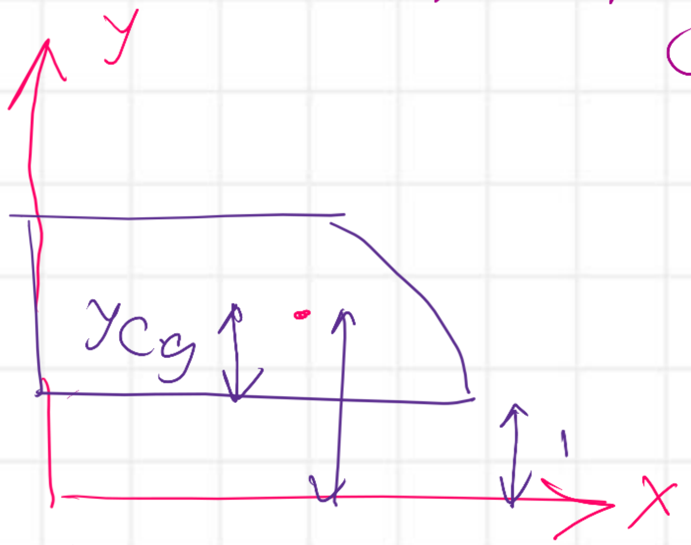
5/8 Locate the centroid of the shaded area shown.

William Kraige



Problem 5/8

For $\bar{y}$ bar



$y_{coord} = y_{c.g} + 1$

From First integration

$7.5\,x \Big|_0^{2.2361} = 7.5(2.2361) = 16.7708$

From 2nd integration = 7.6770 ⇒ $Ay = 7.6770 + 16.7708 = 24.4477$

$\Sigma A = 10.4347 \rightarrow \bar{y} = 24.4477/10.4347 = 2.3429$

y coordinate $= 2.3429 + 1.00 = 3.3429$

Prepared by Eng.Maged Kamel.