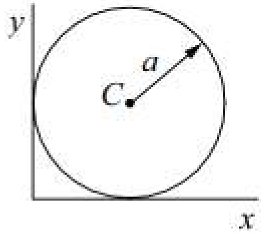
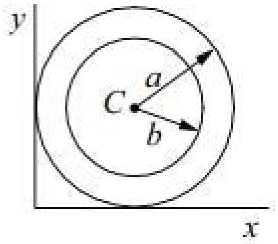
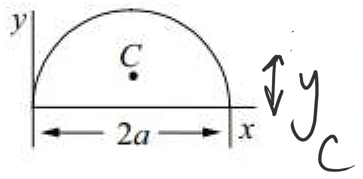
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi(a^4 - b^4)/2$	$r_{x_c}^2 = r_{y_c}^2 = (a^2 + b^2)/4$ $r_x^2 = r_y^2 = (5a^2 + b^2)/4$ $r_p^2 = (a^2 + b^2)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2(a^2 - b^2)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4(9\pi^2 - 64)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2(9\pi^2 - 64)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$

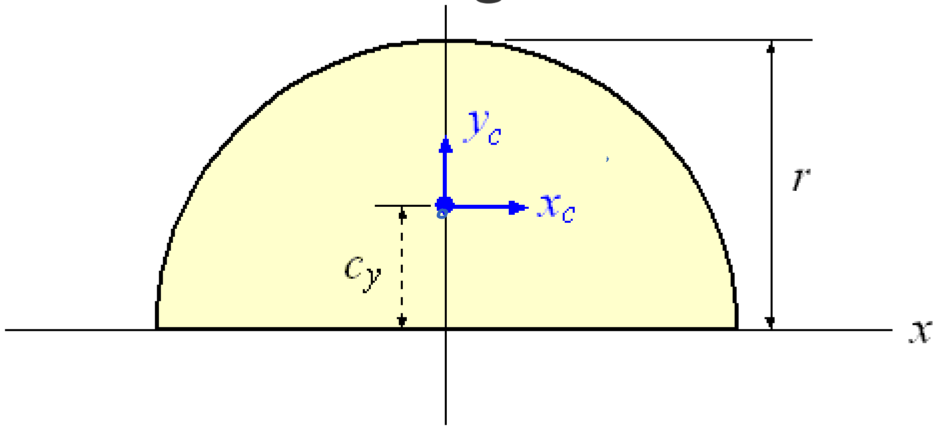
$$A = \frac{\pi a^2}{2}$$

if radius = a $\Rightarrow y_c = \frac{4a}{3\pi}$

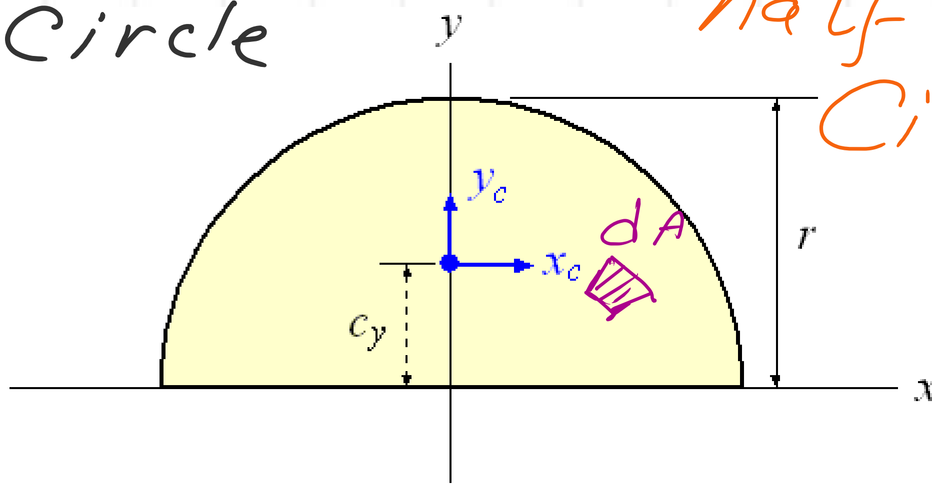
Prepared by Eng.Maged Kamel.

Area and Cg for Semi-Circle

half Circle



C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$



C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

Due to symmetry - CG will be along y-axis

X bar = 0

To estimate the area and first moment of area - we select small area dA

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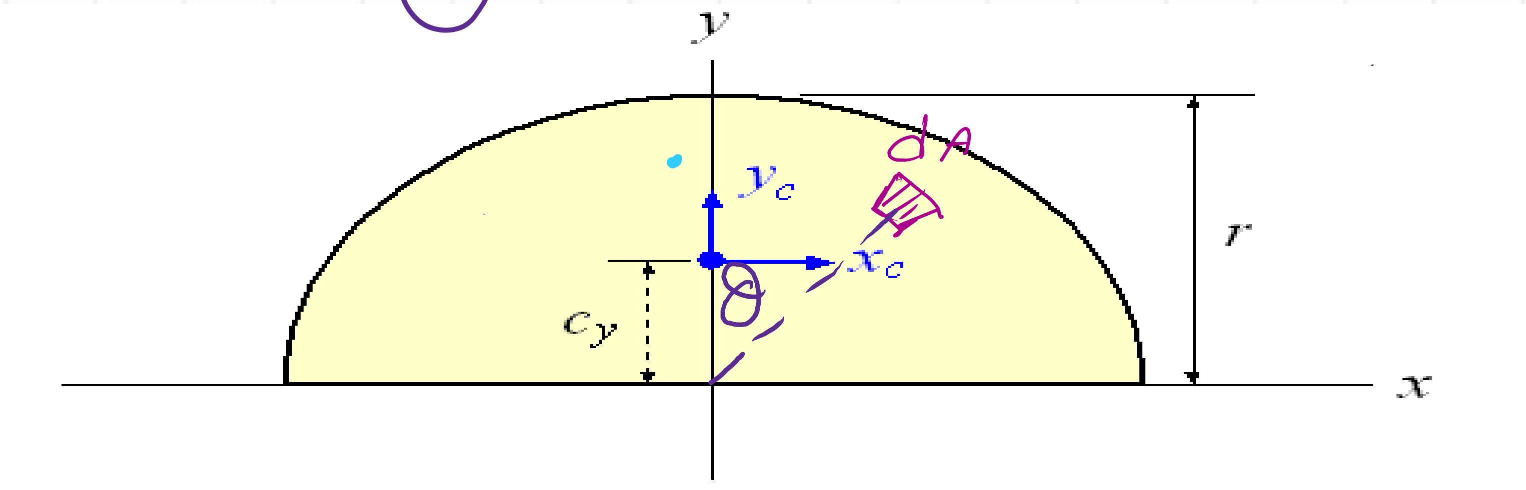
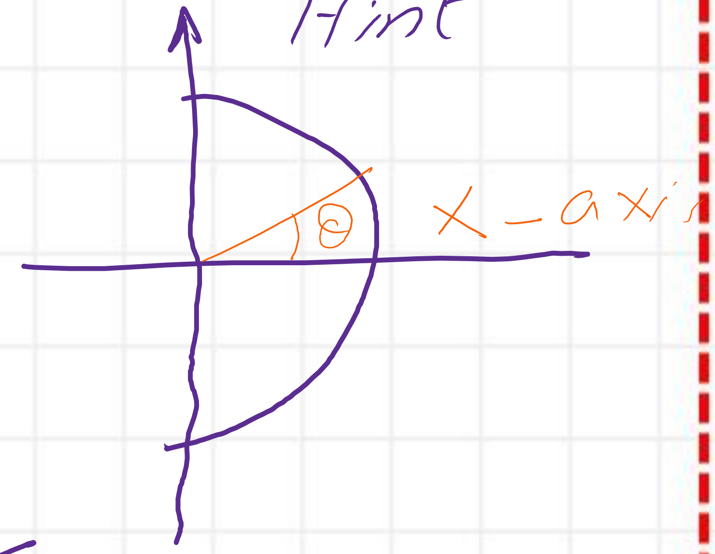
Area and Cg For Semi-Circle



C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

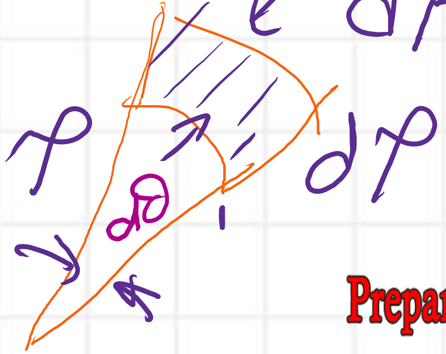
Hint



For X-axis as symmetry Use θ from $\frac{\pi}{2} \rightarrow -\frac{\pi}{2}$

But for this Case use θ from y-axis

$dA = \rho \, d\theta \, d\rho$



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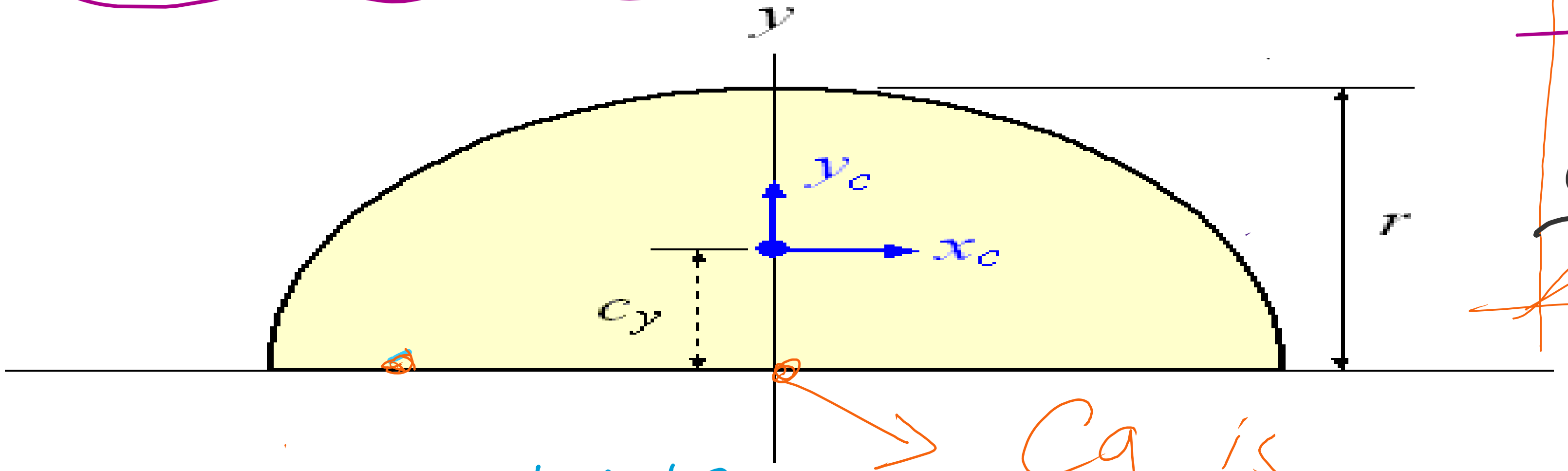
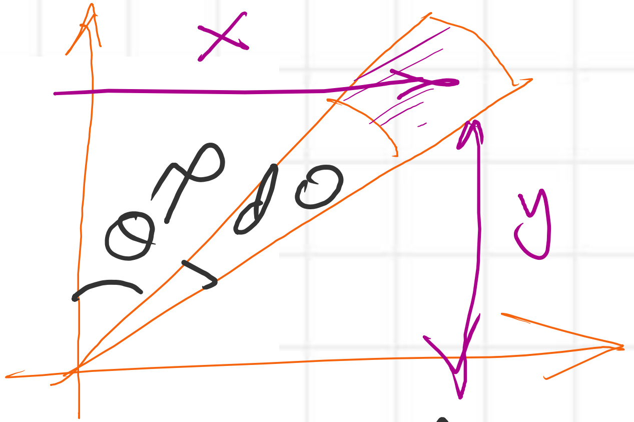
Area and $\bar{x}$ for Semi-circle





$dA = r\,dr\,d\theta$

C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$x = r\sin\theta$

$y = r\cos\theta$

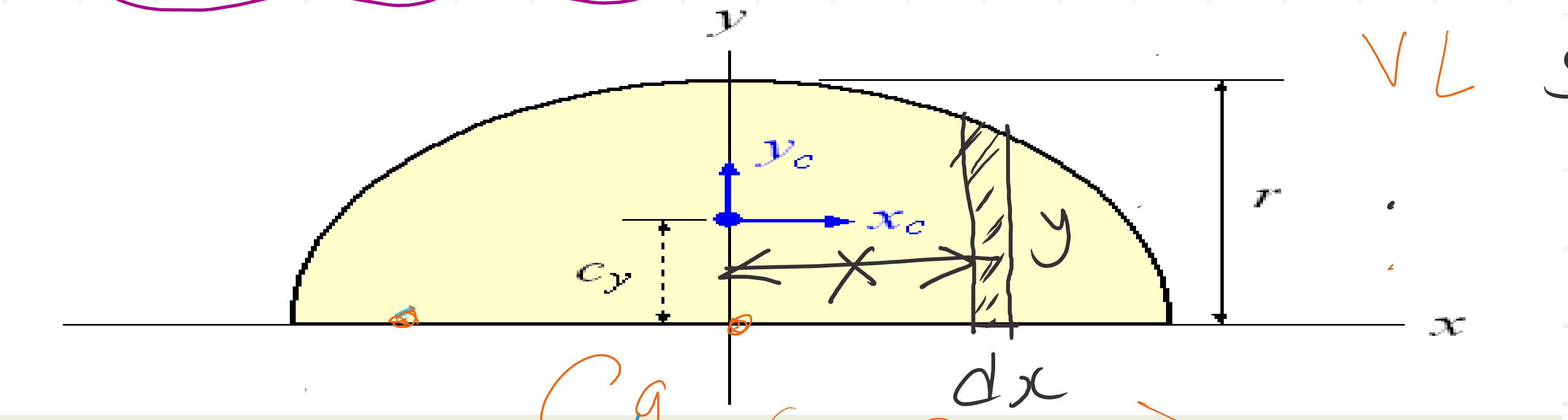
$$dM_x = dA \cdot x = (r\,dr)(d\theta)(r\sin\theta)$$

$$A\bar{x} = \int_{-\pi/2}^{\pi/2}\int_0^r r^2\,dr\,(\sin\theta\,d\theta) = -\cos\theta\Big|_{-\pi/2}^{\pi/2}\left[\frac{r^3}{3}\right]_0^r$$

$$-\left[\cos\left(-\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)\right]\frac{r^3}{3} = 0$$

Area and $\bar{x}$ for Semi-circle alternative

VL strip



Cg

$\longleftarrow r \longrightarrow$

Cx	Cy	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$$dA = dx\, y$$

$$x^2 + y^2 = r^2$$

$$dA \cdot x = x\, dx\, y$$

$$y = (r^2 - x^2)^{1/2}$$

$$U = r^2 - x^2$$

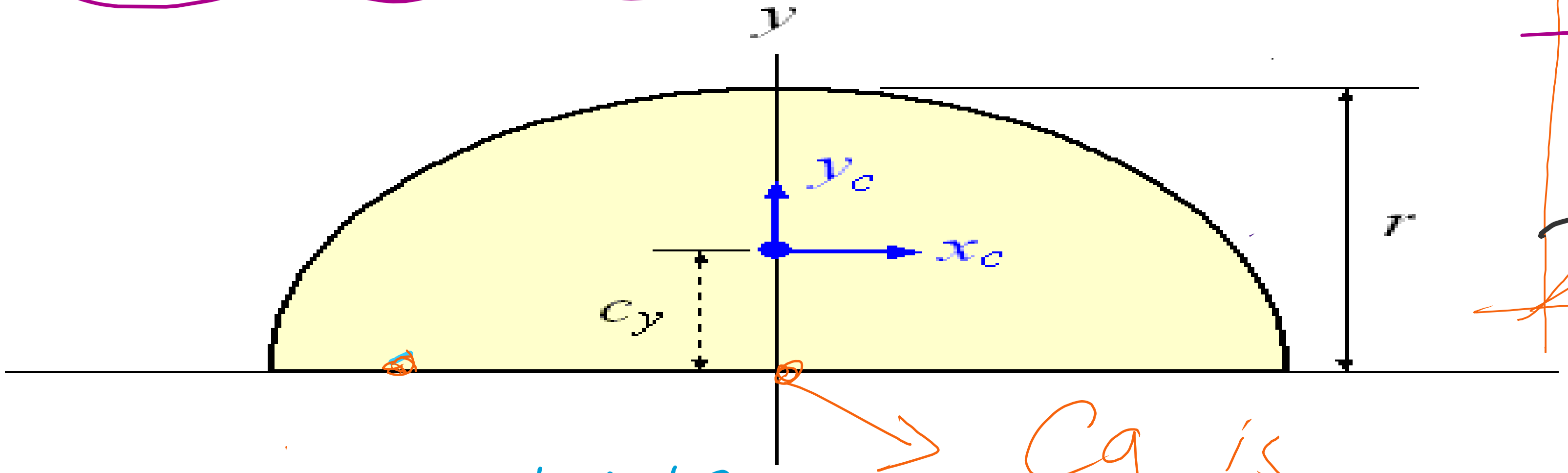
Use Substitution

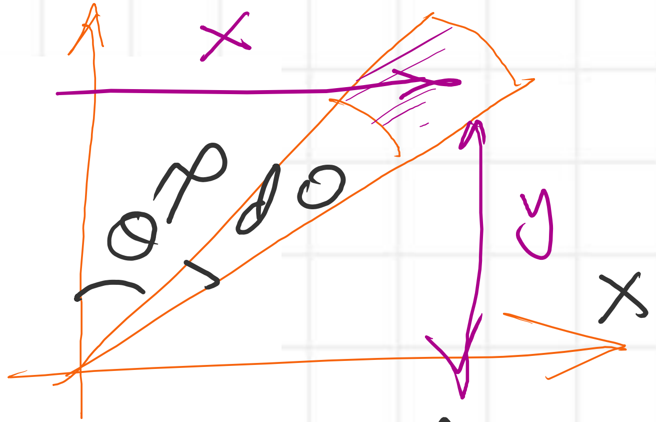
$$\int_{x=-r}^{x=r} dA \cdot x = \int -\frac{du}{2}\sqrt{U} = 0$$

$$y = \sqrt{U}$$

$$du = -2x$$

Area and $\bar{y}$ for Semi-circle





$$x = \rho \sin\theta$$

$$y = \rho \cos\theta$$

$$dA = \rho\, d\rho\, d\theta$$

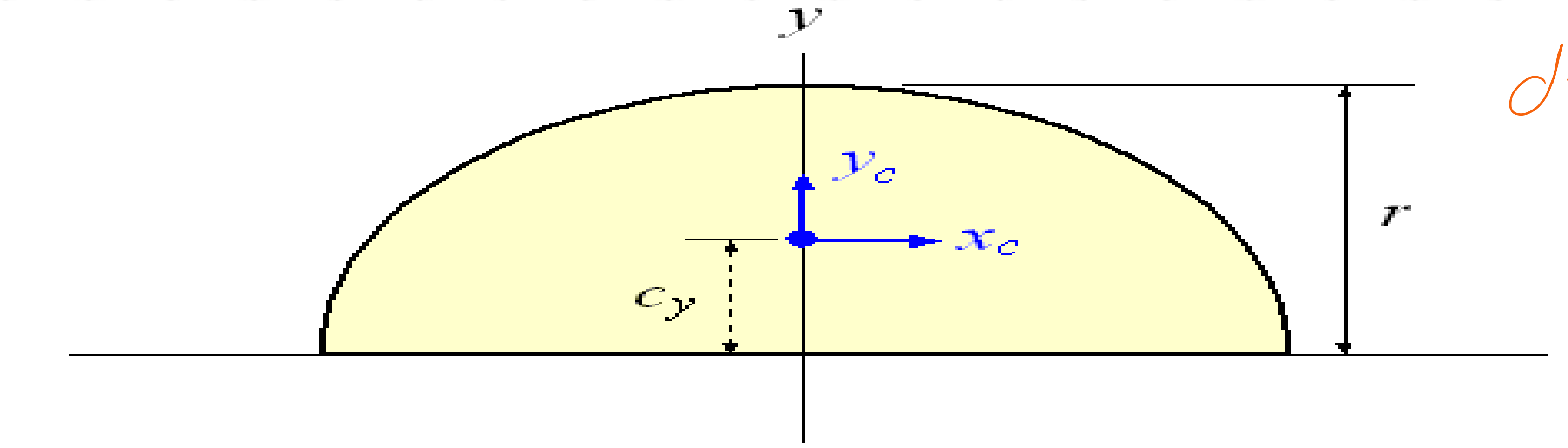
Cx	Cy	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

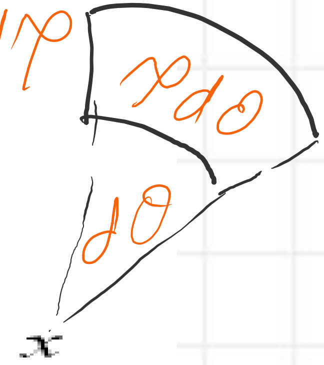
$$dM_x = dA \cdot y = (\rho\, d\rho)(d\theta)(\rho \cos\theta)$$

$$A\bar{x} = \int_{-\pi/2}^{\pi/2}\int_{0}^{r} \rho^2\, d\rho\, (\cos\theta\, d\theta) = \frac{\rho^3}{3}\Big|_0^r \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{r^3}{3}\left[1-(-1)\right] = \frac{2r^3}{3}$$

$\bar{y}$ - Continue



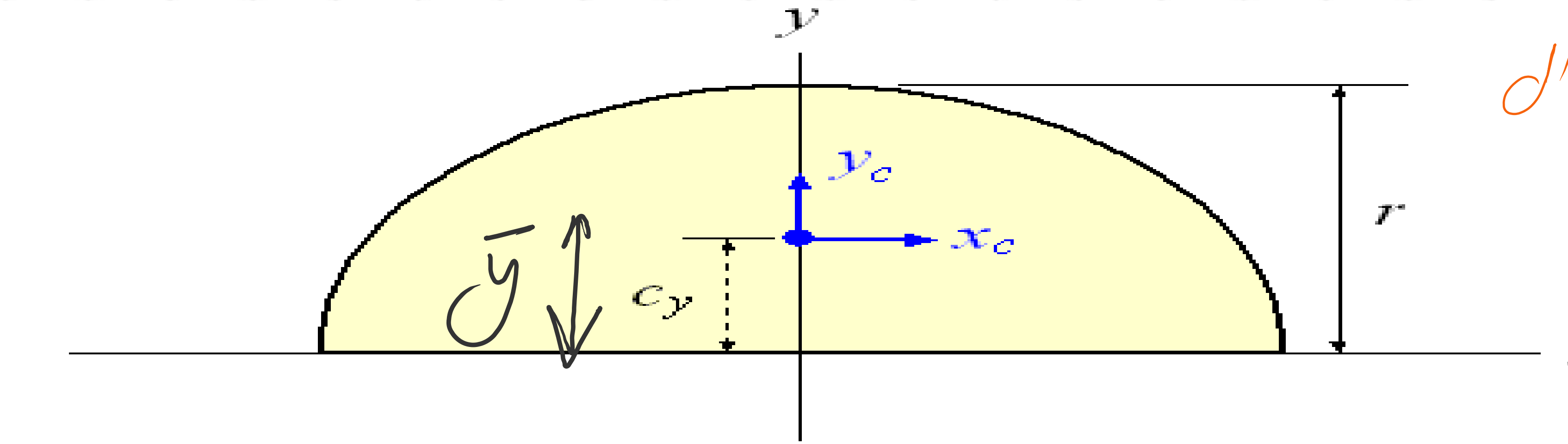


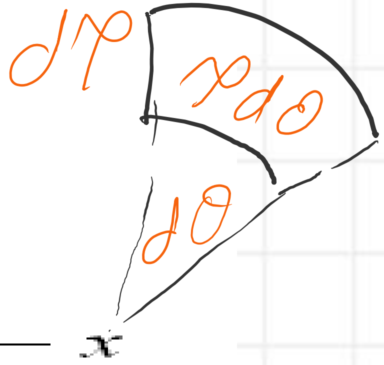
C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$$dA\text{-Area} = r\,d\theta\,dr$$

$$A = \int_0^r \int_{-\pi/2}^{\pi/2} r\,dr\,d\theta = \frac{r^2}{2}\Big|_0^r \;\theta\Big|_{-\pi/2}^{\pi/2} = \frac{r^2}{2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \pi r^2/2$$

$\bar{y}$ - Continue



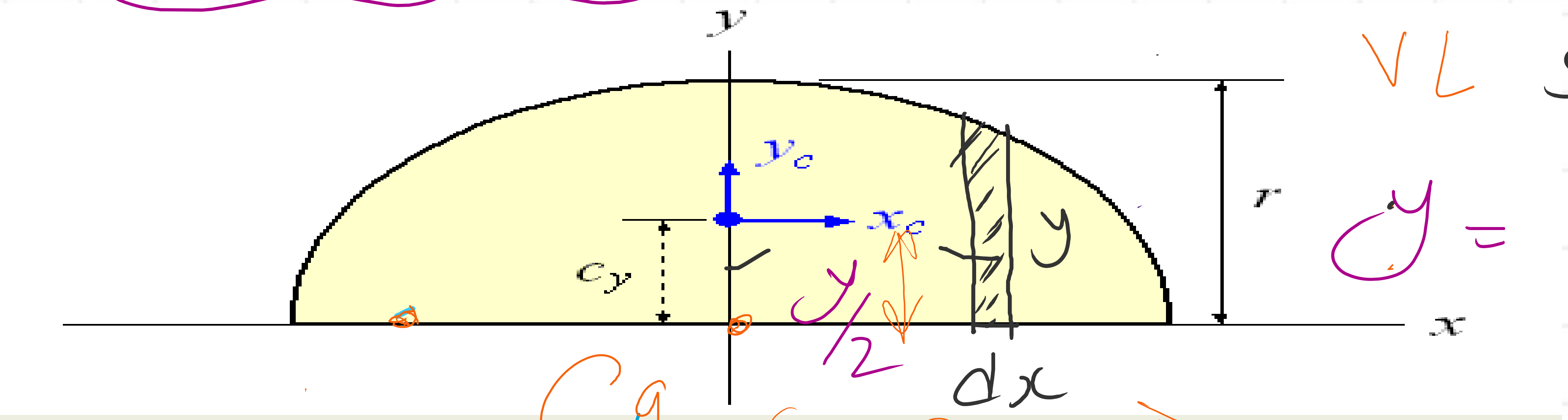


C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$$A\bar{y} = \frac{2r^3}{3} \quad \text{while } A = \frac{\pi r^2}{2}$$

$$\bar{y} = \frac{2r^3}{3}\left(\frac{2}{\pi r^2}\right) = \frac{4r}{3\pi}$$

Area and $\bar{y}$ for Semi-circle   alternative



VL strip

$y = y/2$ Cg strip

C_x	C_y	Area
0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

$dA = dx . y$

$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$$A\bar{y} = \int_{x=-r}^{x=r} dx \cdot \frac{y^2}{2}$$

$$\int_{x=-r}^{r} \frac{(r^2 - x^2)}{2}\, dx = \frac{1}{2}\left[r^2x \Big|_{-r}^{r} - \frac{x^3}{3}\Big|_{-r}^{r} \right]$$

$$A\bar{y} = \frac{1}{2}\left[r^2\left[r-(-r)\right] - \frac{1}{3}\left[+r^3-(-r)^3\right]\right]$$

$$= \frac{1}{2}\left[2r^3 - \frac{1}{3}(2r^3)\right] = \frac{4r^3}{6} = \frac{2}{3}r^3$$

Same value

$$\bar{y} = \frac{A\bar{y}}{A} = \frac{2}{3}r^3\left(\frac{2}{\pi r^2}\right) = \frac{4r}{3\pi}$$