1. The terms E & E_T
   Young Modulus of Elasticity & Tangent moment of Elasticity → From Prof. SEGUi

2. Effective Length (KL) - Cases of End Connections For Columns
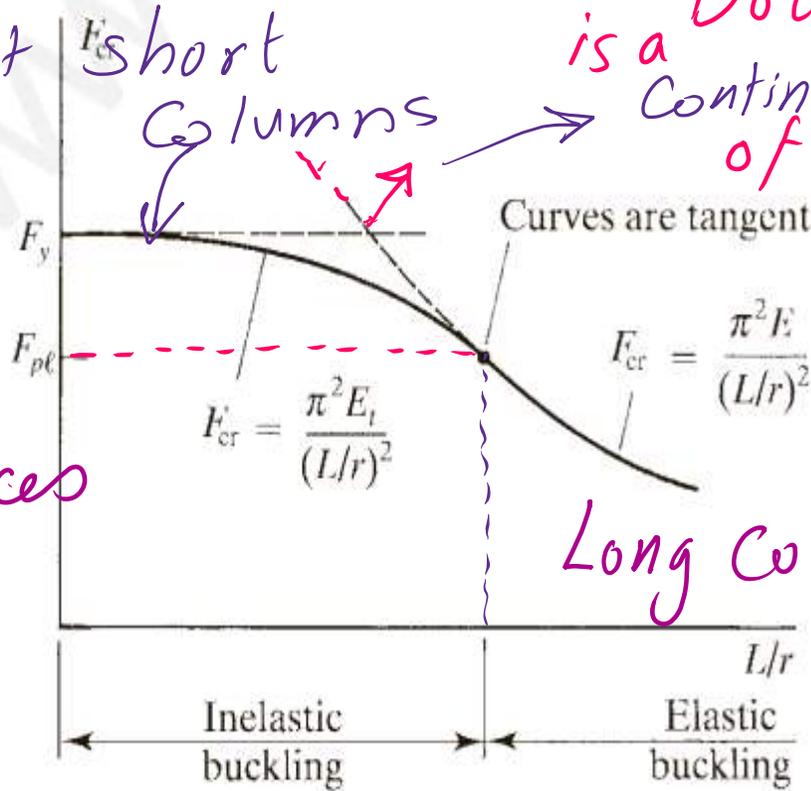
3. AISC Provision For Compresion members

4. Solved Problem 4.2 Prof. SEGUi's book

Prepared by Eng. Maged Kamel.

This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, E is not a constant for stresses greater than the proportional limit $F_{p\ell}$. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of f between $F_{p\ell}$ and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that
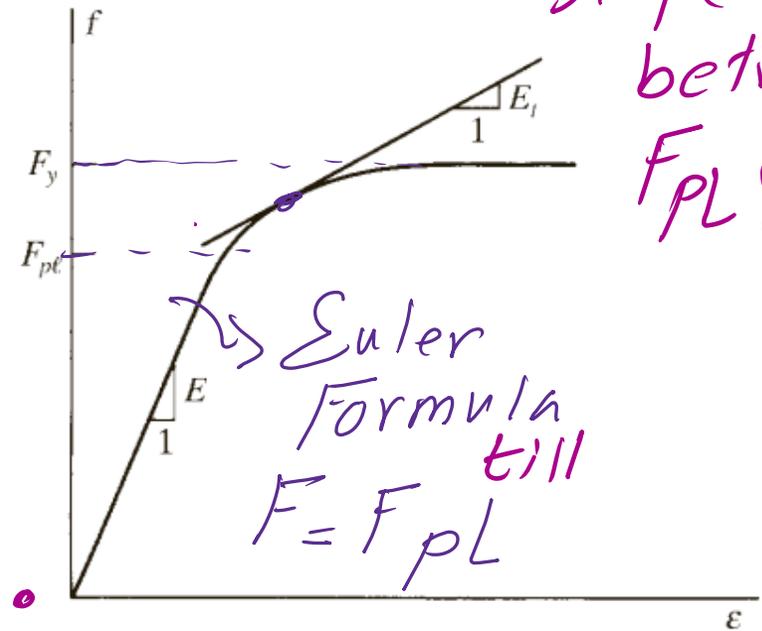
$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \tag{4.5}$$

4.5

[Handwritten: slope between F_{PL} & F_y]

[Handwritten: Euler Formula till $F = F_{PL}$]

FIGURE 4.6

[Handwritten: represent / short Columns]

[Handwritten: is a Dotted Line is a Continuation of Euler Formula]

[Handwritten: E_t replaces E]

Curves are tangent

$$F_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

$$F_{cr} = \frac{\pi^2 E_t}{(L/r)^2}$$

[Handwritten: Long Columns]

Inelastic buckling | Elastic buckling

L/r

Effective Length kL

For a Column with Length L (Height)

$$F_{Cr} = \frac{\pi^2 E}{(KL)^2}$$

$K = 1$

Reducing Column height will increase F_{Cr}
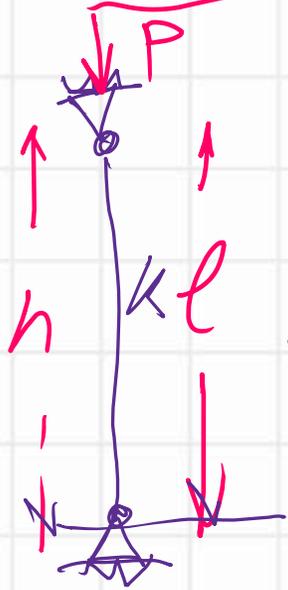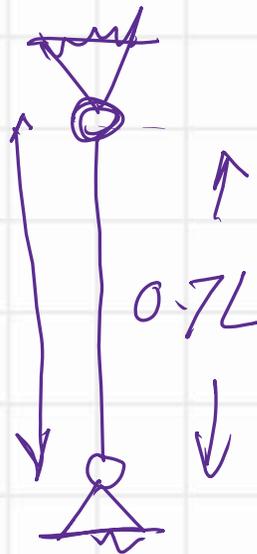
IF the Same Column has

$$h = 0.70 L$$

then

$$F_{Cr} = \frac{\pi^2 E}{(0.7L)^2} = 2.05 \frac{\pi^2 E}{(L)^2}$$
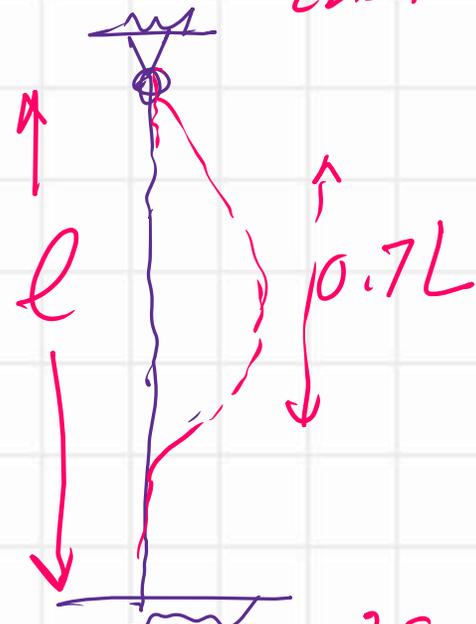
it is found that

$$F_{Cr} = \frac{\pi^2 E}{(0.7L)^2}$$

$K = 0.70$

$$F_{Cr} = 2.04 \frac{\pi^2 E}{L^2}$$

same

Then it is Equivalent to $0.70 L$
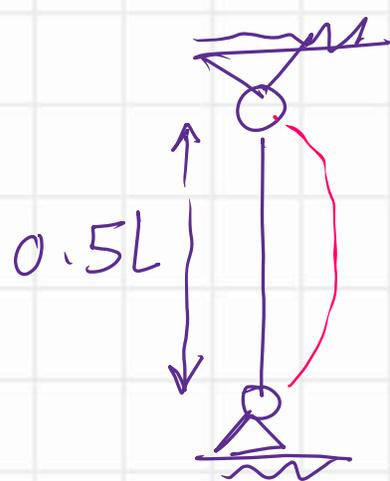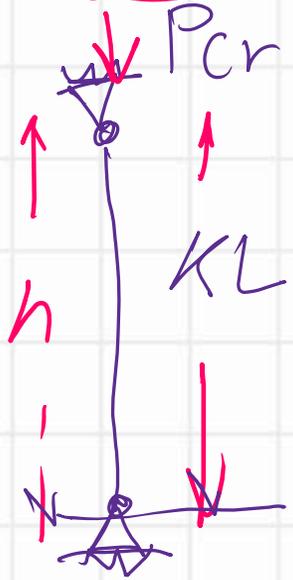
$K = 1$

Effective Length kL

P_{cr}

h

kL

$0.5L$

Reducing Column height will increase F_{cr}

$\Rightarrow$

ℓ

$0.5L$

$$F_{cr} = \frac{\pi^2 E}{(0.5L)^2}$$

$k = 0.5$

For a Column with Length L (Height)

$$F_{cr} = \frac{\pi^2 E}{(kL)^2}$$

$k = 1$

IF the Same Column has

$h = 0$

then

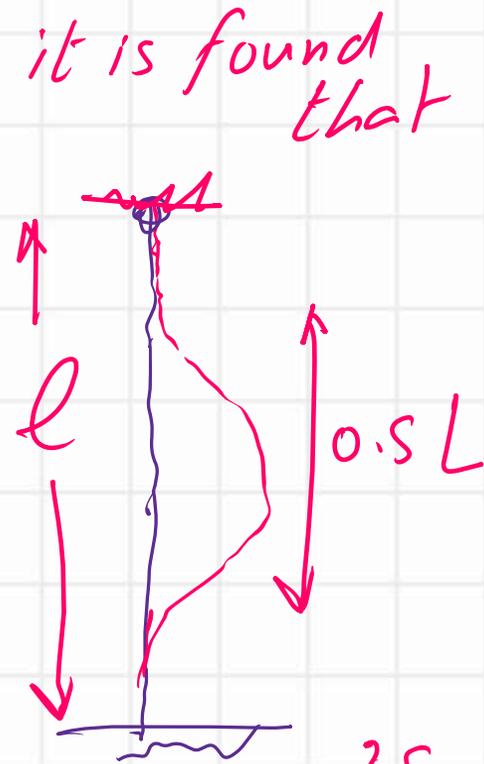$$F_{cr} = \frac{\pi^2 E}{(0.5L)^2} = (4) \frac{\pi^2 E}{(L)^2}$$

$k = 0.5$

it is found that

$$F_{cr} = (4) \frac{\pi^2 E}{L^2}$$

same

Then it is Equivalent to $0.70L$

(a) $F_{cr} = \dfrac{4\pi^2 E}{L^2} = \dfrac{\pi^2 E}{(0.5L)^2}$

$K = 0.50$

(b) $F_{cr} = \dfrac{2.04\,\pi^2 E}{L^2} = \dfrac{\pi^2 E}{(0.7L)^2}$

(c) $F_{cr} = \dfrac{\pi^2 E}{L^2} = \dfrac{\pi^2 E}{(1.0)(L)^2}$

(d) F_{cr} $\Downarrow$ same

(e) $F_{cr} = \dfrac{0.25\,\pi^2 E}{L^2} = \dfrac{\pi^2 E}{(2L)^2}$

original Expression

TABLE C-A-7.1

APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, K

BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(a)	(b)	(c)	(d)	(e)	(f)
THEORETICAL K VALUE	0.5	0.7	1.0	1.0	2.0	2.0
RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0
END CONDITION CODE		ROTATION FIXED AND TRANSLATION FIXED				
		ROTATION FREE AND TRANSLATION FIXED				
		ROTATION FIXED AND TRANSLATION FREE				
		ROTATION FREE AND TRANSLATION FREE				

0.5L

0.7L

2L
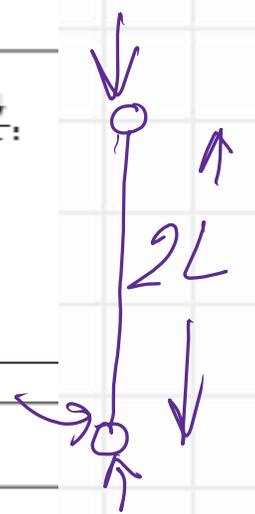
CHAPTER E

DESIGN OF MEMBERS FOR COMPRESSION

This chapter addresses members subject to axial compression.

The chapter is organized as follows:

E1. General Provisions
E2. Effective Length
E3. Flexural Buckling of Members without Slender Elements
E4. Torsional and Flexural-Torsional Buckling of Single Angles and Members without Slender Elements
E5. Single-Angle Compression Members
E6. Built-Up Members
E7. Members with Slender Elements

GENERAL PROVISIONS

The *design compressive strength*, $\phi_c P_n$, and the *allowable compressive strength*, $P_n \Omega_c$, are determined as follows.

The *nominal compressive strength*, P_n, shall be the lowest value obtained based on the applicable *limit states* of *flexural buckling*, *torsional buckling*, and *flexural-torsional buckling*.

$$\phi_c = 0.90 \text{ (LRFD)} \qquad \Omega_c = 1.67 \text{ (ASD)}$$

AISC   E-1 $\Rightarrow$ AISC-360
10 & 16

$F_{cr} = 0.877 F_e$

Global buckling

$$F_e = \frac{P_e}{A_g} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad min.$$

2010

$$F_{cr} = \left(0.658^{F_y/F_e}\right) F_y$$

Elastic Buckling (Long columns)

$$F_{cr} = 0.877 F_e$$

F_{cr}/F_y

0.39

Inelastic Buckling (Short columns)
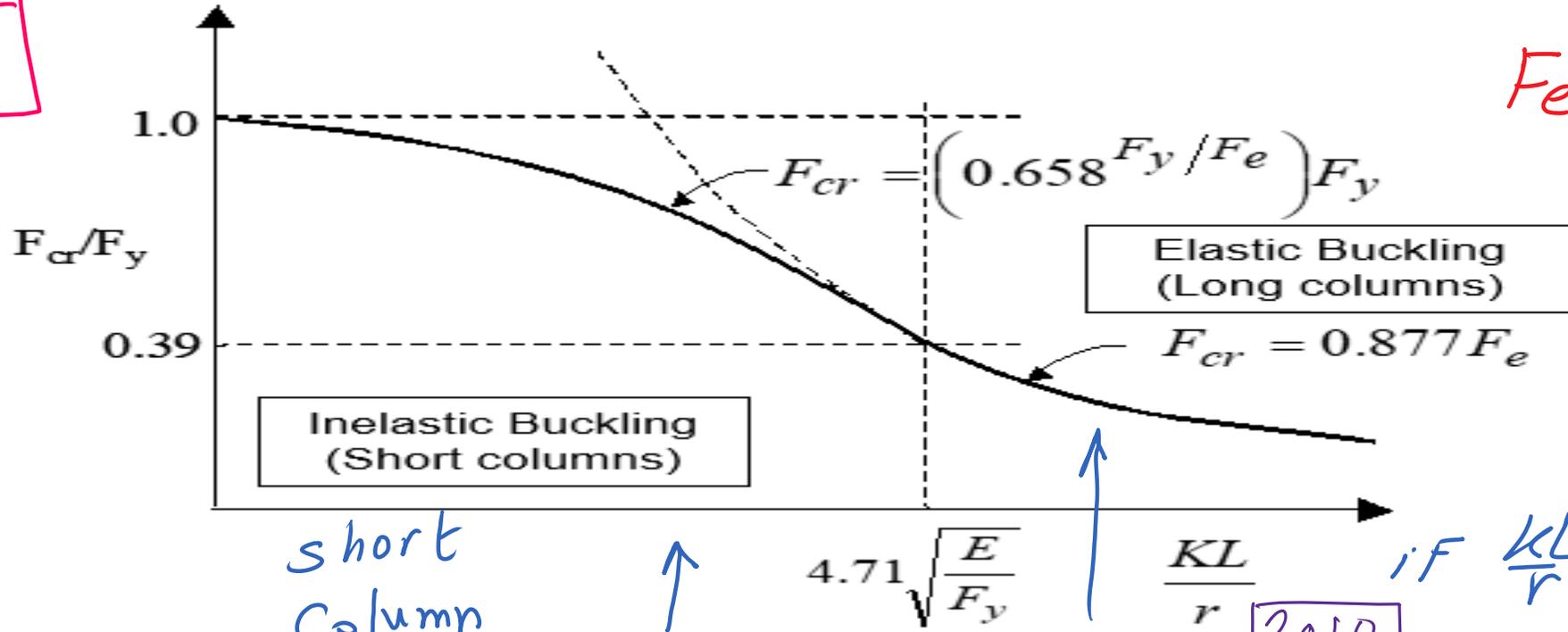
short Column

IF Less

$$4.71\sqrt{\frac{E}{F_y}}$$

$$\frac{KL}{r}$$

2010

if $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$

Long Column

Prepared by Eng.Maged Kamel.

AISC-360-2010   *KL Expression*

E2. EFFECTIVE LENGTH

The *effective length factor*, K, for calculation of member slenderness, KL/r, shall be determined in accordance with Chapter C or Appendix 7,

where

L = laterally *unbraced length* of the member, in. (mm)
r = radius of gyration, in. (mm)

$KL/r \ngtr 200$

> **User Note:** For members designed on the basis of compression, the effective slenderness ratio KL/r preferably should not exceed 200.

AISC-360–16   *16.1-35*

E2. EFFECTIVE LENGTH

The effective length, L_c, for calculation of member slenderness, L_c/r, shall be determined in accordance with Chapter C or Appendix 7,

where

K = effective length factor
$L_c = KL$ = effective length of member, in. (mm)
L = laterally unbraced length of the member, in. (mm)
r = radius of gyration, in. (mm)

$L_c = KL$

> **User Note:** For members designed on the basis of compression, the effective slenderness ratio, L_c/r, preferably should not exceed 200.

> **User Note:** The effective length, L_c, can be determined through methods other than those using the effective length factor, K.

$L_c/r \ngtr 200$

Prepared by Eng.Maged Kamel.

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to nonslender element compression members as defined in Section B4.1 for elements in uniform compression.

User Note: When the torsional *unbraced length* is larger than the lateral unbraced length, Section E4 may control the design of wide flange and similarly shaped *columns*.

The *nominal compressive strength, P_n,* shall be determined based on the *limit state* of *flexural buckling.*

$$P_n = F_{cr} A_g \qquad \text{(E3-1)}$$

The *critical stress, F_{cr},* is determined as follows:

(a) When $\dfrac{KL}{r} \leq 4.71 \sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} \leq 2.25$)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}} \right] F_y \qquad \text{(E3-2)}$$

(b) When $\dfrac{KL}{r} > 4.71 \sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} > 2.25$)

$$F_{cr} = 0.877 F_e \qquad \text{(E3-3)}$$

2016

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to nonslender-element compression members, as defined in Section B4.1, for elements in axial compression.

> **User Note:** When the torsional effective length is larger than the lateral effective length, Section E4 may control the design of wide-flange and similarly shaped columns.

The nominal compressive strength, P_n, shall be determined based on the limit state of flexural buckling:

$$P_n = F_{cr} A_g \tag{E3-1}$$

The critical stress, F_{cr}, is determined as follows:

Lc instead of KL

(a) When $\dfrac{L_c}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} \leq 2.25$)

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \tag{E3-2}$$

(b) When $\dfrac{L_c}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} > 2.25$)

$$F_{cr} = 0.877 F_e \tag{E3-3}$$

where

F_e = elastic *buckling* stress determined according to Equation E3-4, as specified in Appendix 7, Section 7.2.3(b), or through an elastic buckling analysis, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad \text{(E3-4)}$$

16.1-36   FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS   [Sect. E3.

F_e = elastic buckling stress determined according to Equation E3-4, as specified in Appendix 7, Section 7.2.3(b), or through an elastic buckling analysis, as applicable, ksi (MPa)

$$= \frac{\pi^2 E}{\left(\dfrac{L_c}{r}\right)^2} \qquad \text{(E3-4)}$$

← $Lc = KL$

F_y = specified minimum yield stress of the type of steel being used, ksi (MPa)
r = radius of gyration, in. (mm)

Prepared by Eng.Maged Kamel.