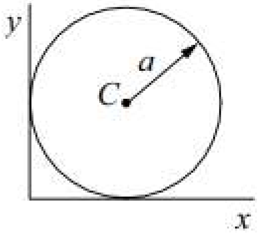
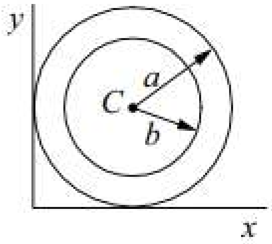
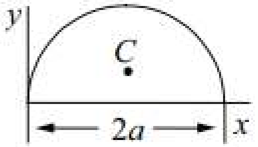
Area and CG.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi(a^4 - b^4)/2$	$r_{x_c}^2 = r_{y_c}^2 = (a^2 + b^2)/4$ $r_x^2 = r_y^2 = (5a^2 + b^2)/4$ $r_p^2 = (a^2 + b^2)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2(a^2 - b^2)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4(9\pi^2 - 64)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2(9\pi^2 - 64)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$

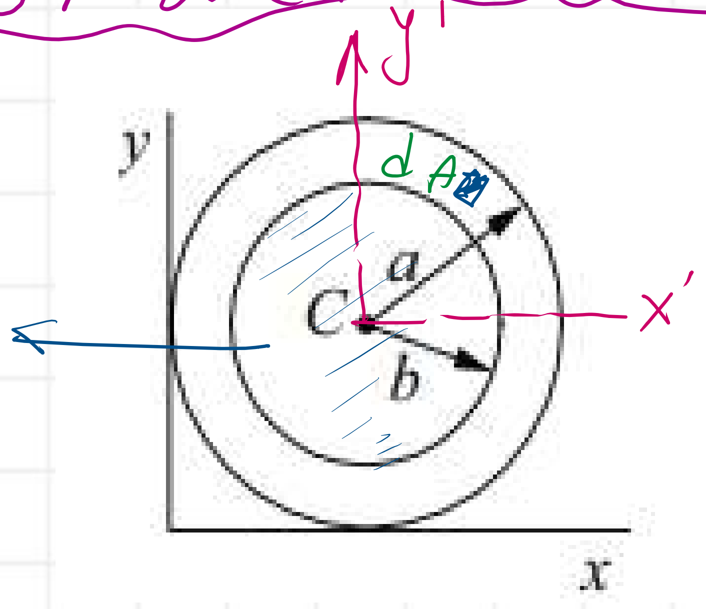
Circular tube

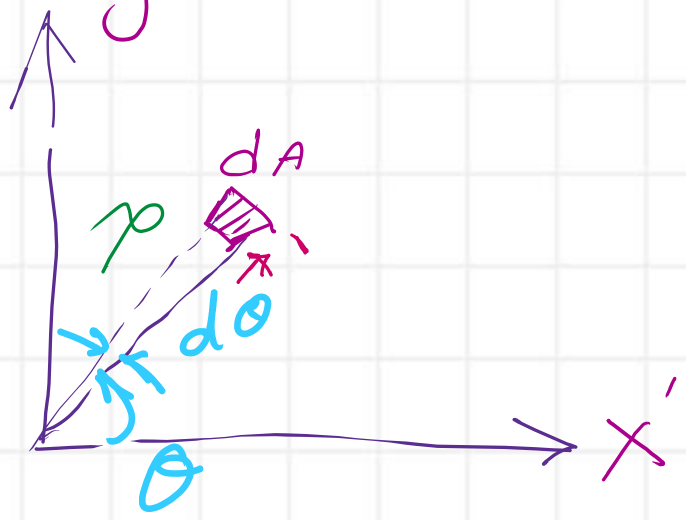
Prepared by Eng.Maged Kamel.

Area and CG For a circular tube

dA is Located in the solid part.

hollow Part





a: is the outer radius.
b: is the inner radius.

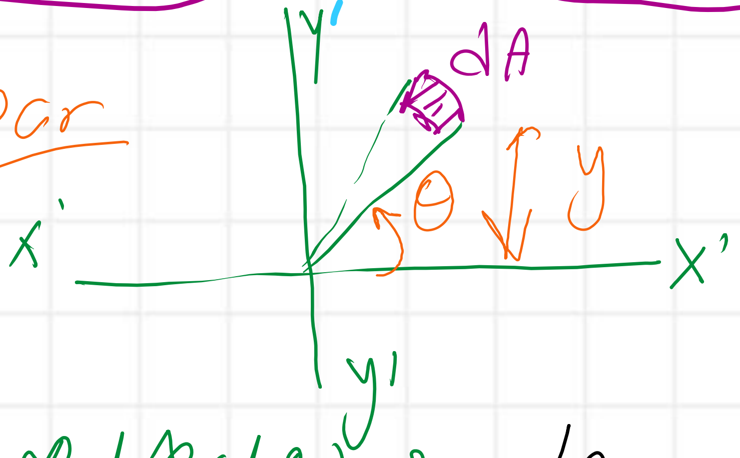
Due to symmetry - CG will be at the point of intersection of x' and y'-axis

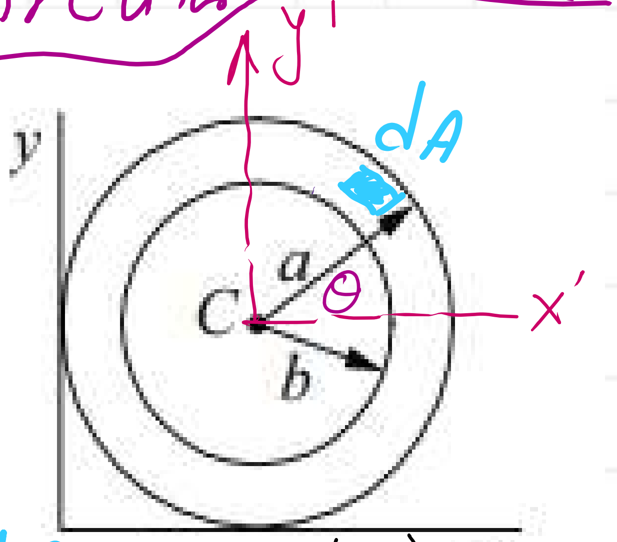
To estimate the area and first moment of area - we select small area dA

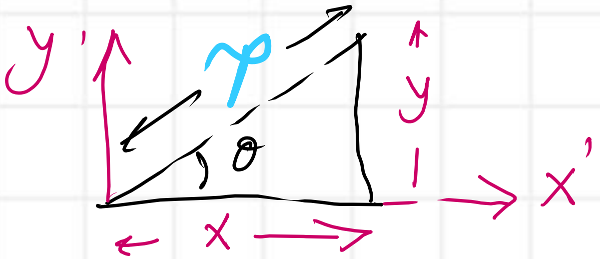
Prepared by Eng.Maged Kamel.

Find $\bar{y}$ bar for a circular tube

$\bar{y}$ bar







$dA = r\,dr\,(d\theta)$ & $dA \cdot y = r^2\,dr(\sin\theta\;d\theta)$

$$\int dA\cdot y = \int_{b}^{a}\int_{\theta=0}^{\theta=2\pi} r^2\,dr\,(\sin\theta)\,d\theta$$

$$A\cdot y = \left(\frac{r^3}{3}\Bigg|_{b}^{a}\right)(-\cos\theta)\Bigg|_{0}^{2\pi} = (a^3-b^3)(-)(1-1)=0$$

y distance $=0$

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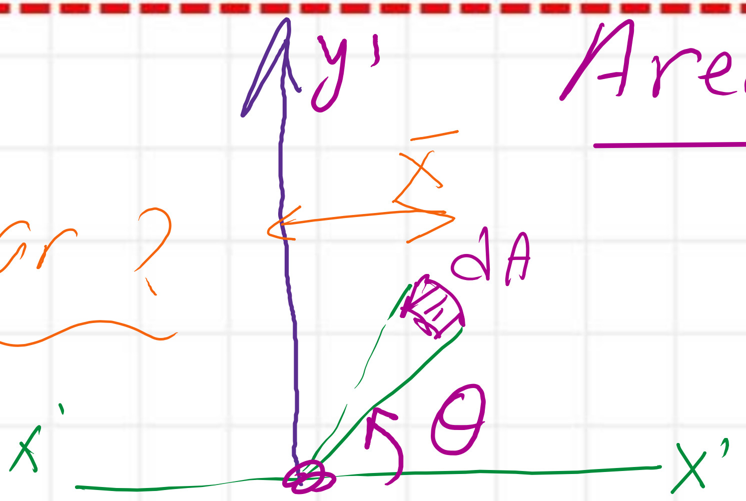
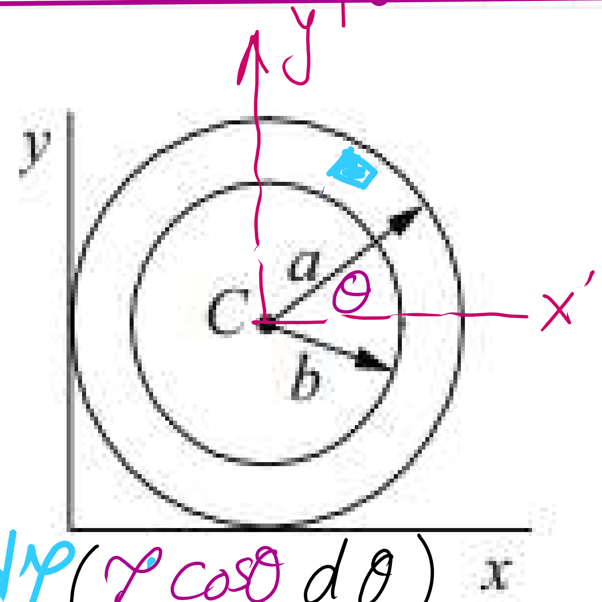
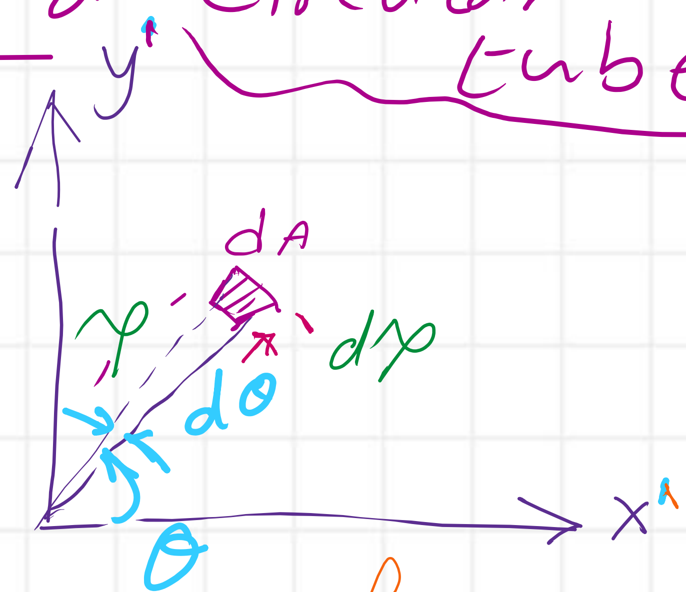
Area and Cg for a Circular tube

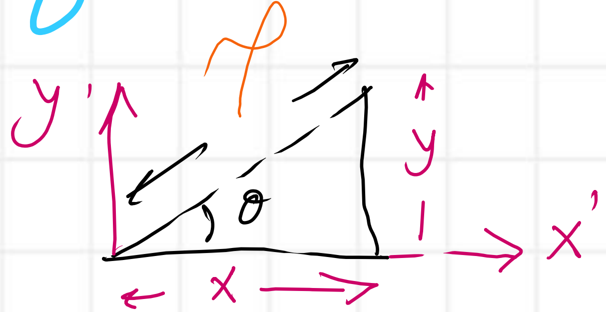
$\bar{x}$ bar ?







$$dA = r(dr)(d\theta) \quad \& \quad dA \cdot x = r^2 dr(r\cos\theta\, d\theta)$$



$$\int dA \cdot x = \int_a^b \int_{\theta=0}^{\theta=2\pi} r^2 dr(\cos\theta)\, d\theta$$

$$A \cdot x = \left(\frac{r^3}{3}\Big|_b^a\right)\left(\sin\theta\right)\Big|_0^{2\pi} = (a^3 - b^3)(0-0) = 0$$

X- distance = 0

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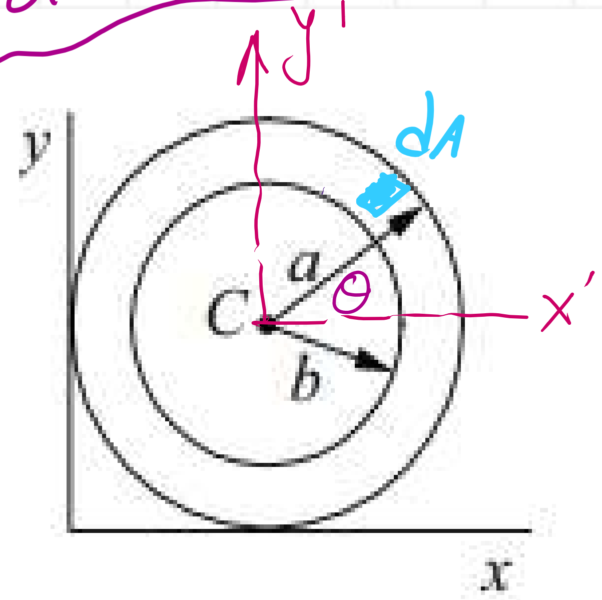
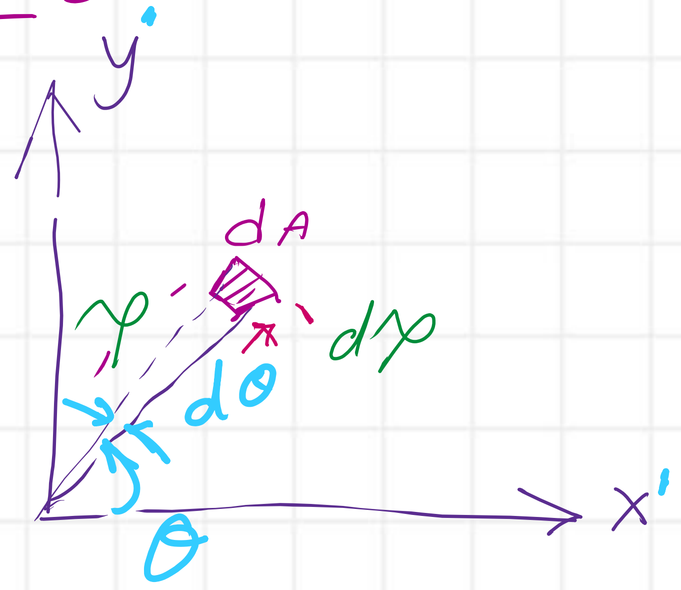
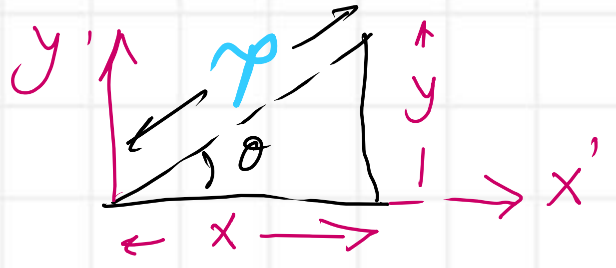
Find the area for a circular tube.





$dA = r\,(dr)(d\theta)$

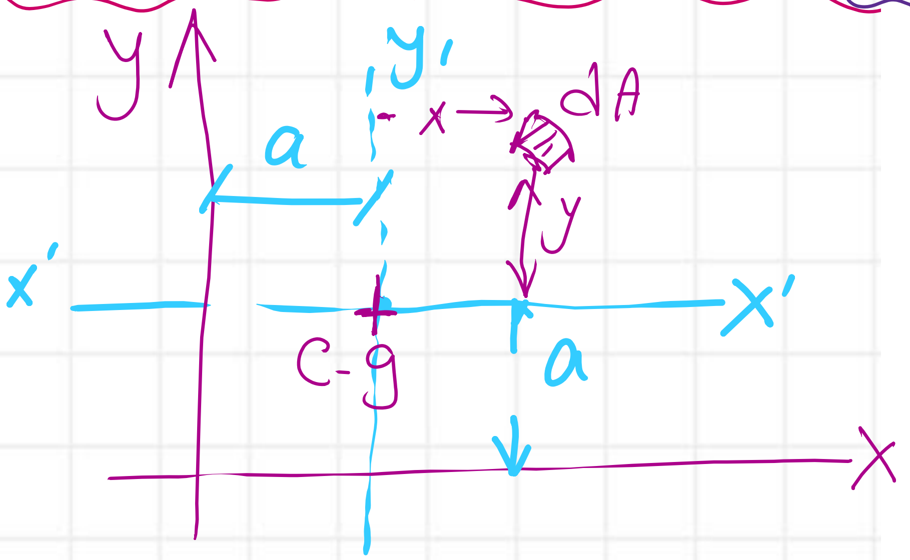


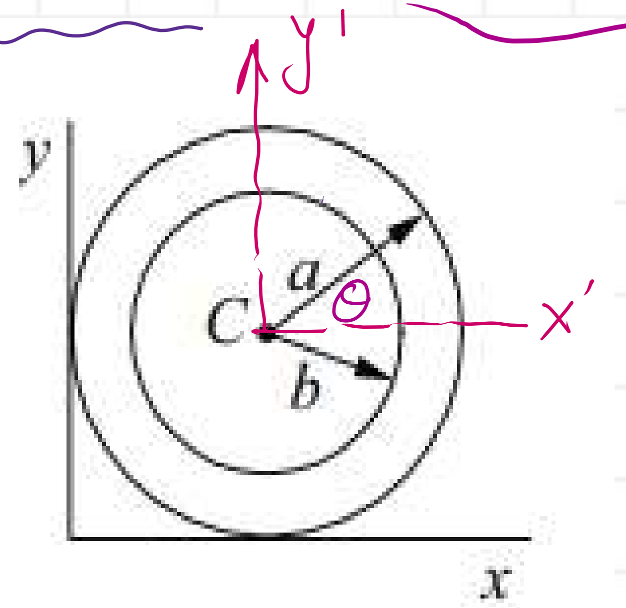
$$\int dA = \int_b^a \int_0^{2\pi} (r\,dr)\,d\theta$$

$$= \frac{r^2}{2}\Big|_b^a \left(\theta\Big|_0^{2\pi}\right) = \frac{1}{2}(a^2 - b^2)(2\pi) = \pi(a^2 - b^2)$$

Prepared by Eng.Maged Kamel.

Area and CG For Circular tube





Cg Lies in the point of intersection of x', y' axes.

Cg From x-axis = a = Xc
From y-axis = a = Yc } Same as the given table.

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