# Summary

(A) Review of the plate theory

get Expresion of Fcr For Plates under Lateral Compresion

(B) What are $\lambda_r$ values for stiffened and unstiffened parts?

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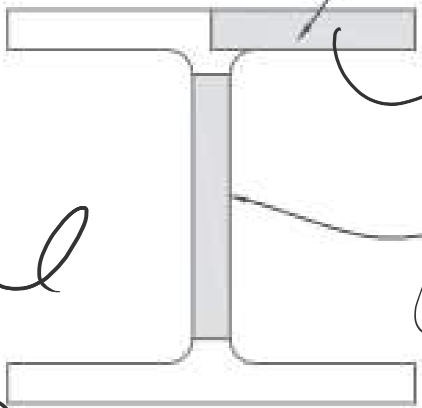
# Column is Composed of plates

When Column is Slender

⇓

Plates will have Local buckling

unstiffened element: flange is supported at one end and free at the other



stiffened element: web is supported at both ends

Figure 5-7 Stiffened and unstiffened elements.

localized buckling of column flange under compression stress

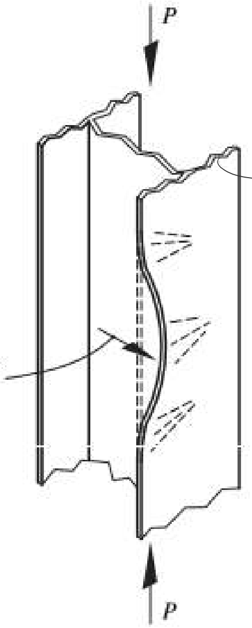



Figure 5-6 Local buckling of column under axial compression load.

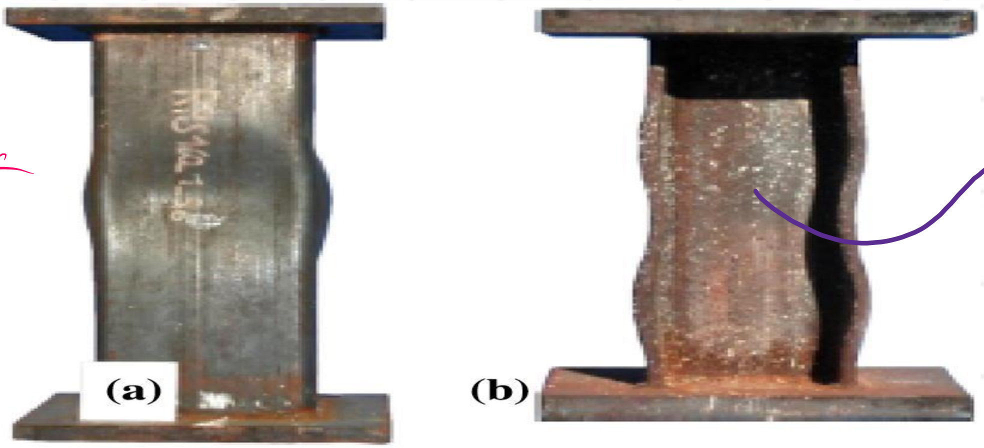
Flange

Local buckling

Local buckling For Column

Local buckling

Unstiffend Part




Local buckling for Web

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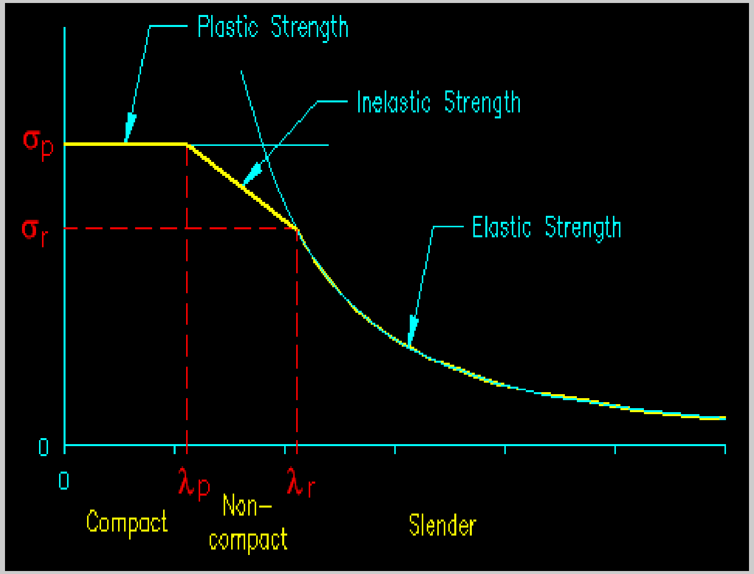
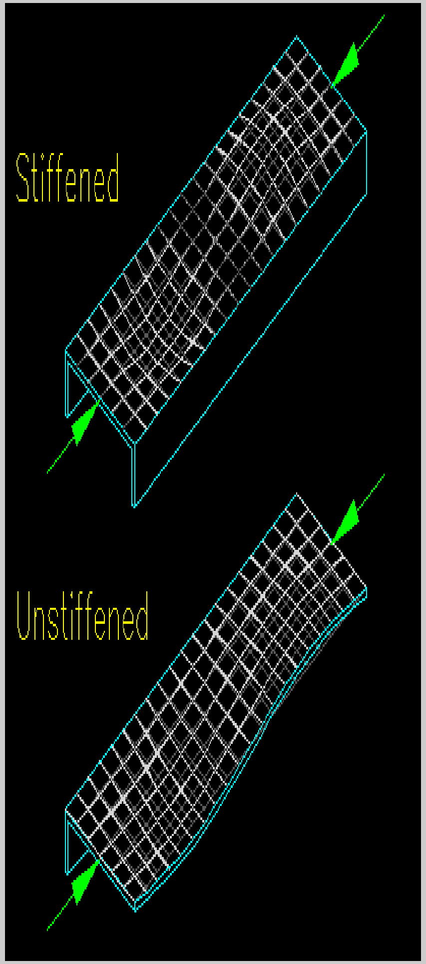
Figure 6.1.3
Theoretical Maximum Compressive Stress
Click on image for larger view




$\left(\frac{KL}{r}\right)$ # 2010

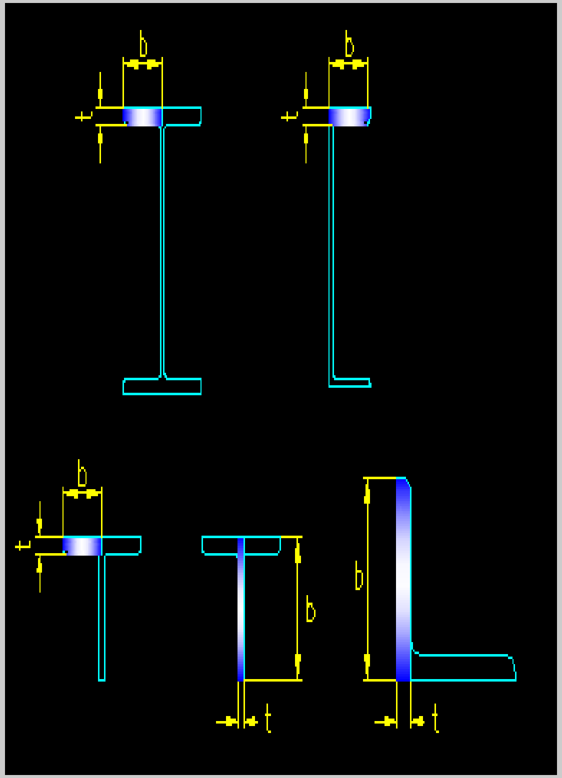
http://www.bgstructuralengineering.com/BGSCM14/BGSCM006/BGSCM00603.htm

Figure 6.3.3
Plate Buckling Modes
Click on image for larger view




Unstiffend elements

Figure 6.3.5
Unstiffened Elements
Click on image for larger view




Samples

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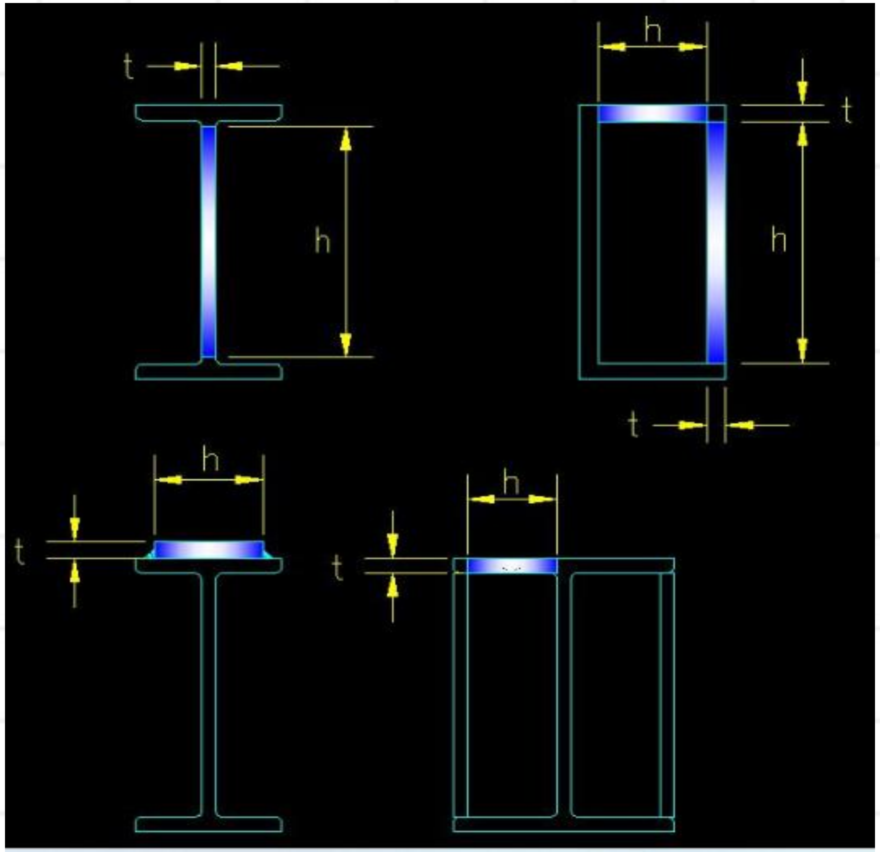
half sin waves

Figure 6.3.6 shows the *stiffened elements* on some typical steel sections and the measurement of the element width, h, and thickness, t. Note that "W" shapes and channels each have one stiffened plate element in their cross-section. A square or rectangular HSS has four stiffened elements in its cross-section. Normally, unstiffened plate elements can be stiffened with the addition of plate elements as seen in the figure.




https://www.bgstructuralengineering.com/BGSCM15/BGSCM006/BGSCM00603.htm

→ Sample of stiffened elements

https://www.bgstructuralengineering.com/BGSCM15/BGSCM006/Stiff.jpg

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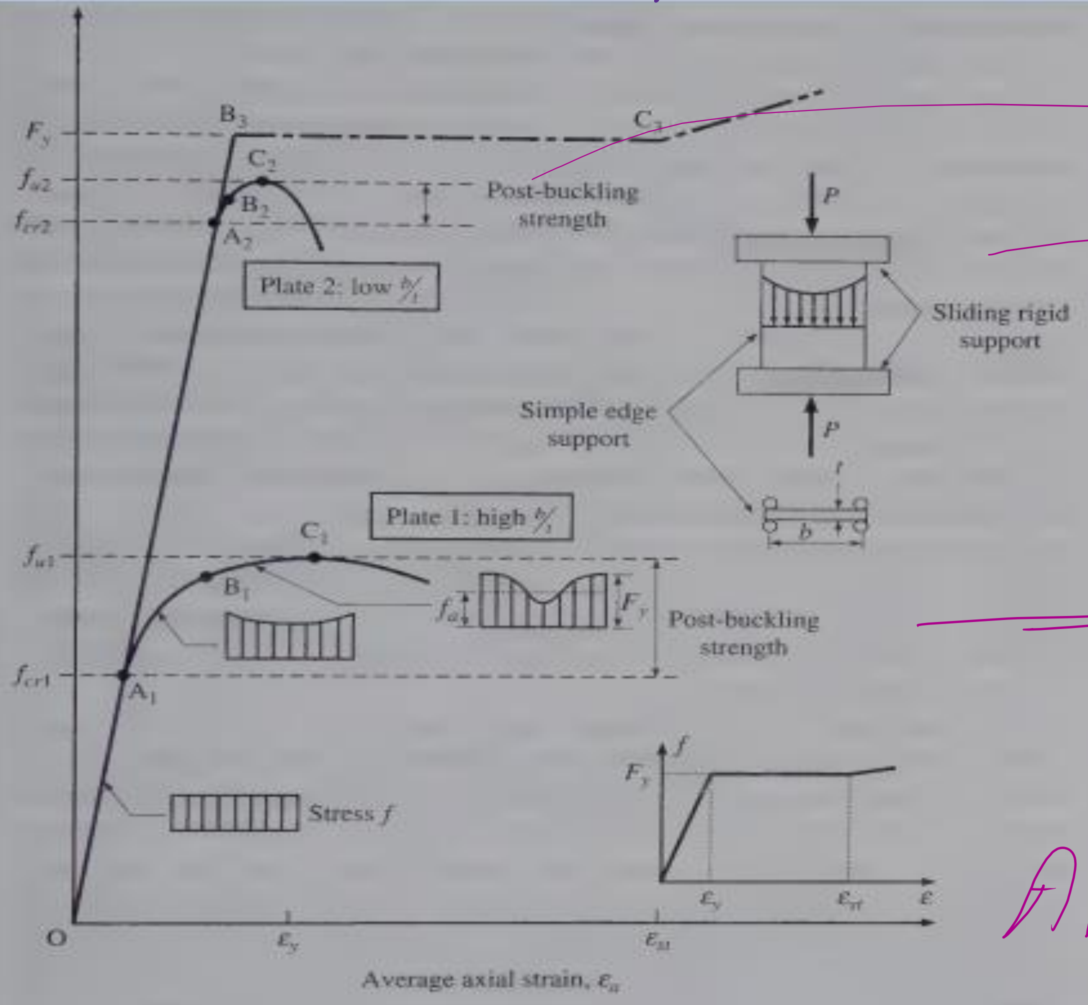
# Behavior of plates under Loads

$F_c$




Stress shape

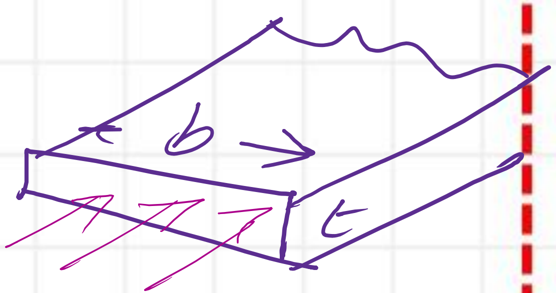
higher $\frac{b}{t}$

Average strain

**Figure 8.9.2:** Behavior of rectangular plates under edge compression.

Smaller $\frac{b}{t}$

higher fcr




Slender

Lower Fcr

Post buckling strength

the edges to remain straight in the process of loading. The plate is made of linearly elastic, plastic, strain-hardening material containing no residual stresses. A diagram of plate behavior is obtained by plotting the average compressive stress $f_a$ versus the average strain $\varepsilon_a$, where

Forces acting on a plate and their notations

**In-plane normal forces and bending moments, Fig. 6.1.7:**

$$N_x = \int_{-h/2}^{+h/2} \sigma_{xx} dz, \qquad N_y = \int_{-h/2}^{+h/2} \sigma_{yy} dz$$
$$M_x = -\int_{-h/2}^{+h/2} z\sigma_{xx} dz, \quad M_y = -\int_{-h/2}^{+h/2} z\sigma_{yy} dz \tag{6.1.1}$$

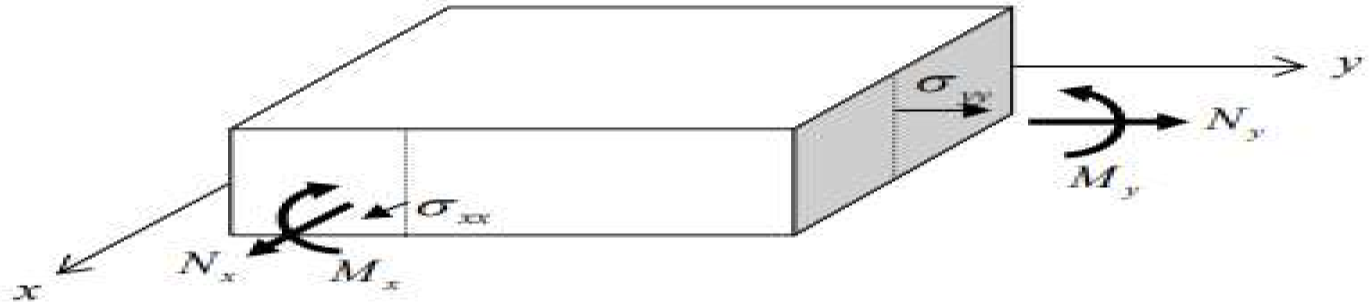



**Fig. 6.1.7: in-plane normal forces and bending moments**

We are dealing with Nx or Nxx

Solid Mechanics – II

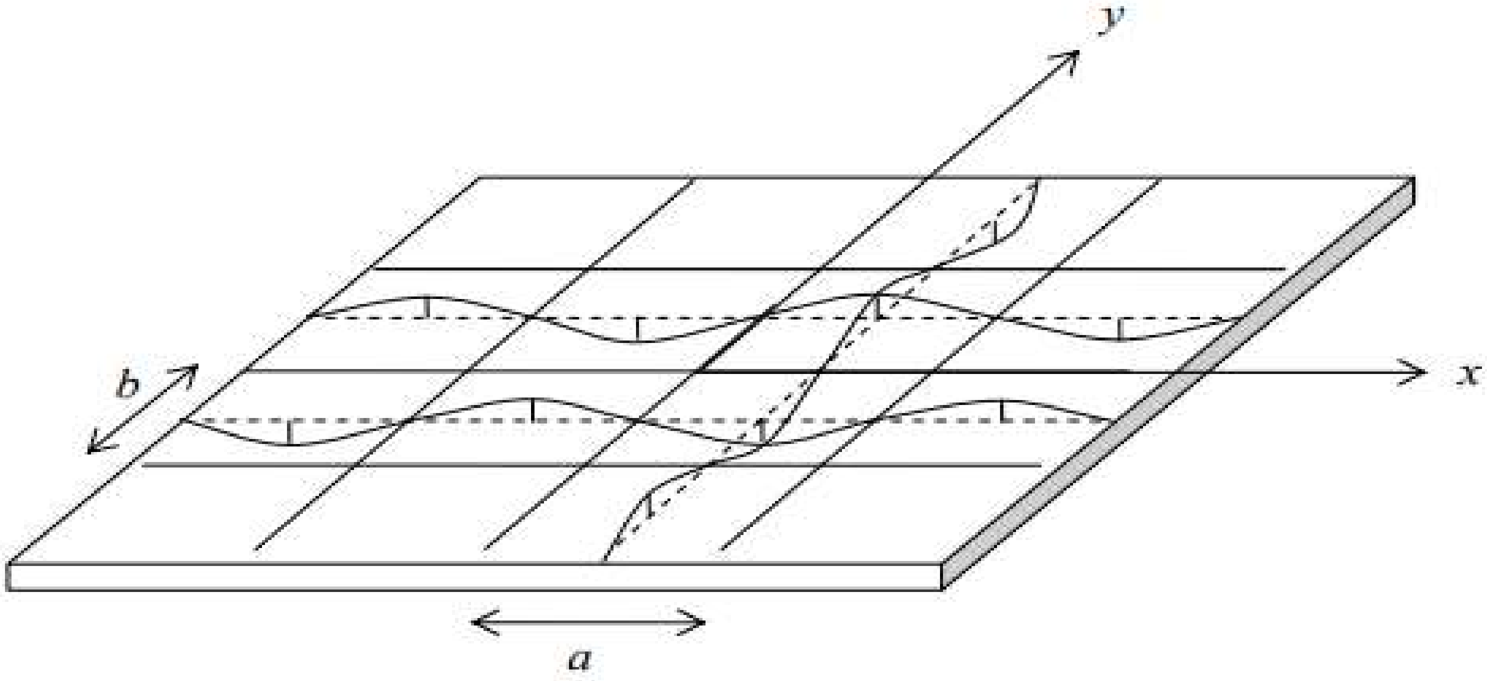
Piaras Kelly

### 6.5.3 An Infinite Plate with Sinusoidal Deflection

Consider next the classic plate problem addressed by Navier in 1820. It consists of an infinite plate with an undulating "up/down" sinusoidal deflection, Fig. 6.5.5,

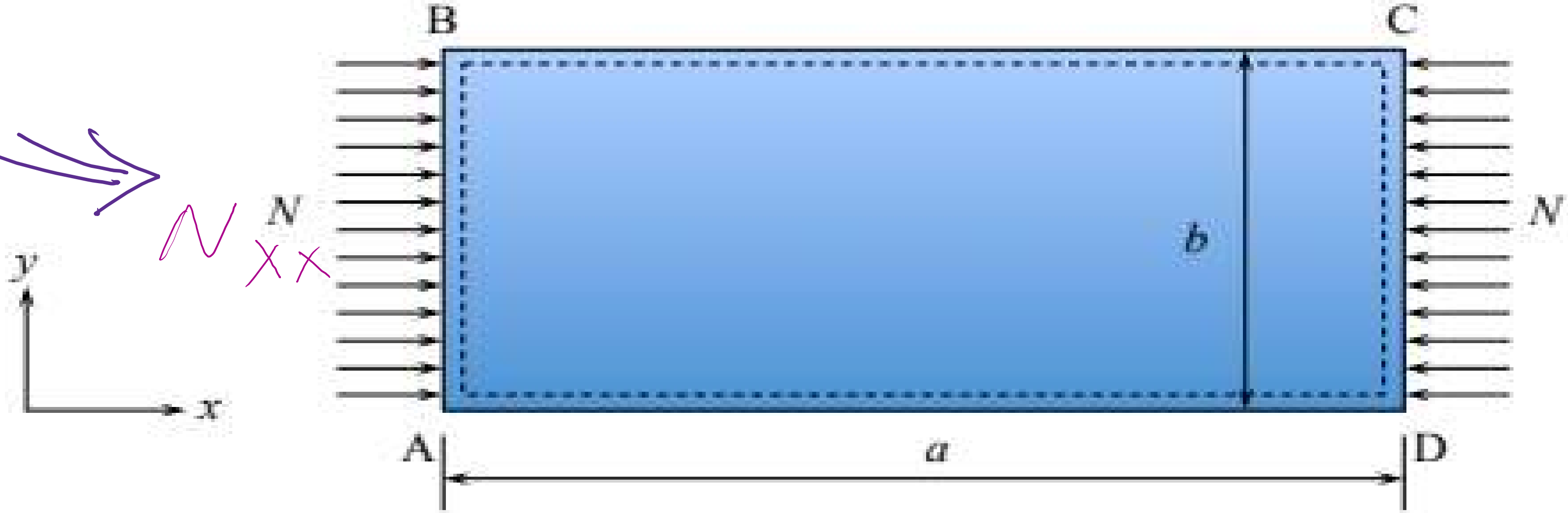
Deflection

$$w(x, y) = w_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \tag{6.5.24}$$



Column is Composed of Plates → Stiffened / Unstiffened

Load acts as shown → $N_{xx}$




$N_{xx}$ Load/m

Figure 11.1: Geometry and loading of the classical plate buckling problem.

$EI\omega^{IV} + N\omega^{II} = 0$

## 11.1 Governing Equations and Boundary Conditions

In the present notes the column buckling was extensively studied in Lecture 9. The governing equation for a geometrically perfect column is

$\omega$: Lateral deflection

$$EIw^{IV} + Nw'' = 0 \tag{11.1}$$

$M = -N\omega$

$EI\omega'' = -N\omega$

Column is Composed of Plates → Stiffened / Unstiffened

Load acts as shown

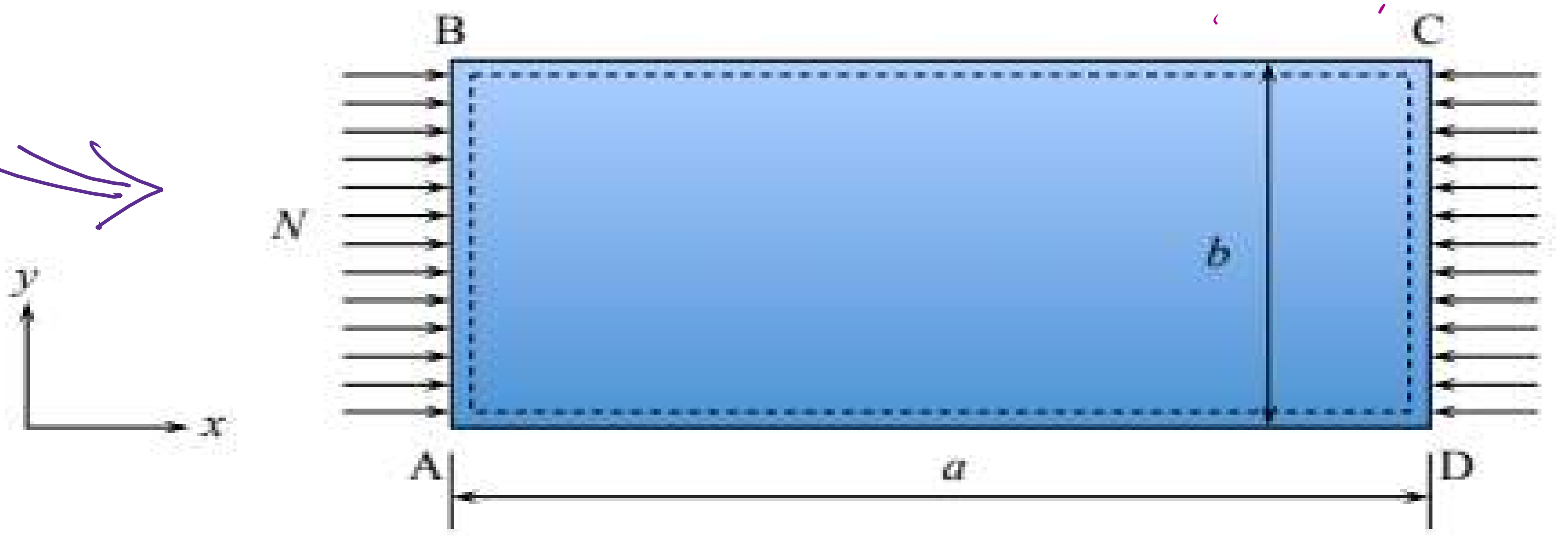
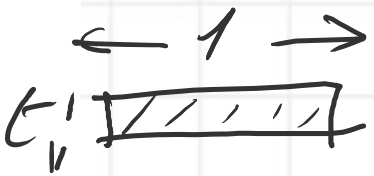



Figure 11.1: Geometry and loading of the classical plate buckling problem.

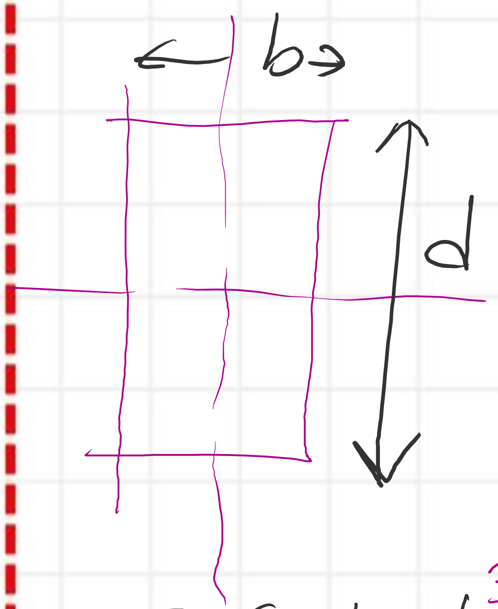



$$N_{xx} = \frac{K\pi^2 D}{b^2} \quad D \rightarrow \text{flexural strength} \rightarrow EI \qquad I = \frac{1}{12}(t)^3$$

$$F_{cr} = \frac{N_{xx}}{(1\,t)} = \frac{K\pi^2}{t\,b^2}\left[\frac{t^3 E}{12(1-\nu^2)}\right] = \frac{K\pi^2}{12}\left(\frac{t}{b}\right)^2\left(\frac{1}{1-\nu^2}\right)$$

Flexural rigidity beam

EI




$b \neq 1$

No $\nu$

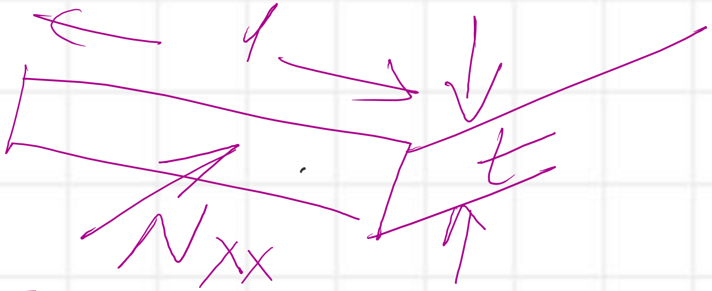
Possions

$$E\left(\frac{bd^3}{12}\right)$$

Flexural rigidity Plate

D

$b = 1$




EI

$$\frac{E \cdot t^3 (1)}{12(1-\nu^2)}$$

Comparison between EI For a beam / plate

$k_c$ Equation which is based on $\frac{b}{a}$ & $m$

$$\sigma = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2\left[\left(\frac{mb}{a}\right)^2 + 2n^2 + n^4\left(\frac{a}{mb}\right)^2\right] \tag{2.40}$$

We now desire the values of the coefficients $m$ and $n$ that result in the lowest characteristic value of stress and at which the buckled form first becomes stable. It is evident that the bracketed term in Eq. (2.30) increases as $n$ increases. Consequently, the lowest integer value of $n = 1$ will result in the critical compressive stress

$$\sigma_{cr} = \frac{\pi^2 k_c E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \tag{2.41}$$

where

$$k_c = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \tag{2.42}$$

The term $k_c$ is commonly referred to as the buckling coefficient. By inserting $n = 1$ into Eq. (2.37), we find that the buckle pattern of a simply supported plate is such that one half-wave forms across the width $b$ of the plate.

$K_c$ value for $m = 1$ For $\frac{a}{b} = 1$ $m=1$

$$K_c = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 = (1+1)^2 = 4 \qquad K_c = 4$$

square plate

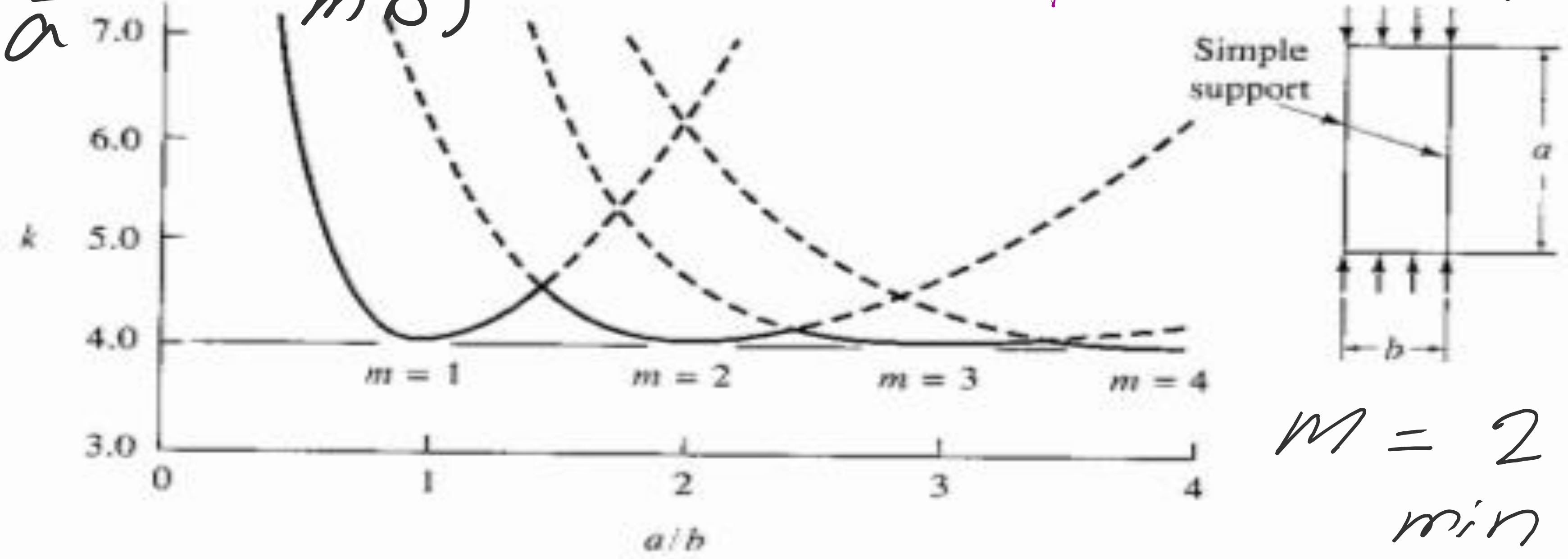



**Figure 6.14.8** Buckling coefficient for uniformly compressed plate—simple support longitudinal edges (Eq. 6.14.29).

$m = 2 \Rightarrow k = 4$ min. value

Pinned in all directions

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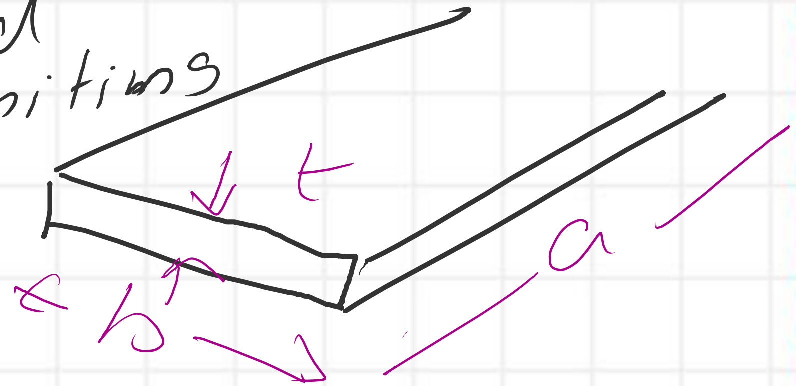
$$\frac{F_{cr}}{F_y} = \frac{1}{\lambda^2}$$

$$\lambda_c^2 < 1$$

Find Limiting $\frac{b}{t}$




$$F_{cr} = \frac{F_y}{\lambda^2} \geq \left(\frac{\pi \; k_c E}{12(1-\nu^2)}\right)\left(\frac{t}{b}\right)^2$$

$$\left(\frac{b}{t}\right)^2 \leq \frac{\lambda_c^2 \pi \; k_c E}{12(1-\nu^2)\lambda_c^2 F_y}$$

$$F_{cr} \geq F_y$$

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}}$$

$$\frac{b}{t} \leq \frac{\lambda_c (\pi) \sqrt{k_c} \sqrt{E}}{\sqrt{12(0.91)} \sqrt{F_y}}$$

$\nu = 0.30$ For steel $\Rightarrow \lambda_c = 1$

$$\frac{b}{t} \leq 0.9506 \, \lambda_c \sqrt{\frac{k_c}{F_y}} \sqrt{E}$$

reduce $\lambda_c$ to increase $F_{cr}$

But for $\lambda_c = 0.70$ multiply R.H.S

For design $F_{cr}$ plate $\geq F_{cr}$ Column

$F_{cr} \geq F_y$ $\quad \lambda_c^2 = F_y / F_{cr}$

$F_{cr} \geq F_{cr}$

component element, i.e., plate | overall column

$F_{cr} = F_y$

$\lambda_c^2 = 1$

actual behavior of plate

$\frac{F_{cr}}{F_y}$

$\frac{F_{cr}}{F_y} = \frac{1}{\lambda_c^2}$

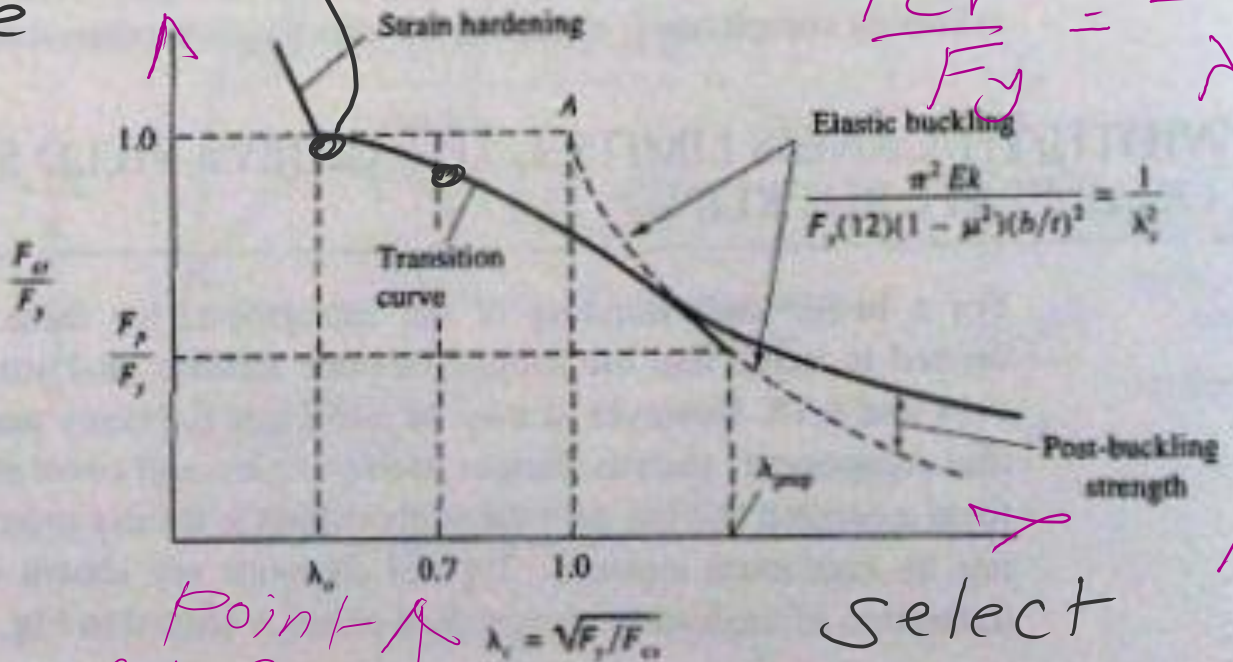



Figure 6.16.2
Dimensionless representation of plate strength in edge compression.

our point

select 0.70

$\lambda_c = \sqrt{\frac{F_y}{F_c}}$

Choose $\lambda_c = 0.70$ due to actual behavior

$\swarrow$ $b/t \Rightarrow$ increme

Back

$$\frac{b}{t} \leq 0.666\sqrt{K_c}\left(\sqrt{\frac{E}{F_y}}\right)$$

Use Transition Curve

dependend on stiffened or unstiffened

$\rightarrow$ $K_c$ values

$$\lambda_r \leq 0.666\sqrt{K_c}\sqrt{\frac{E}{F_y}}$$

# Five Types of End Conditions

Different End Conditions

A
B
C
D
E

ALL Loading Edges are Pinned

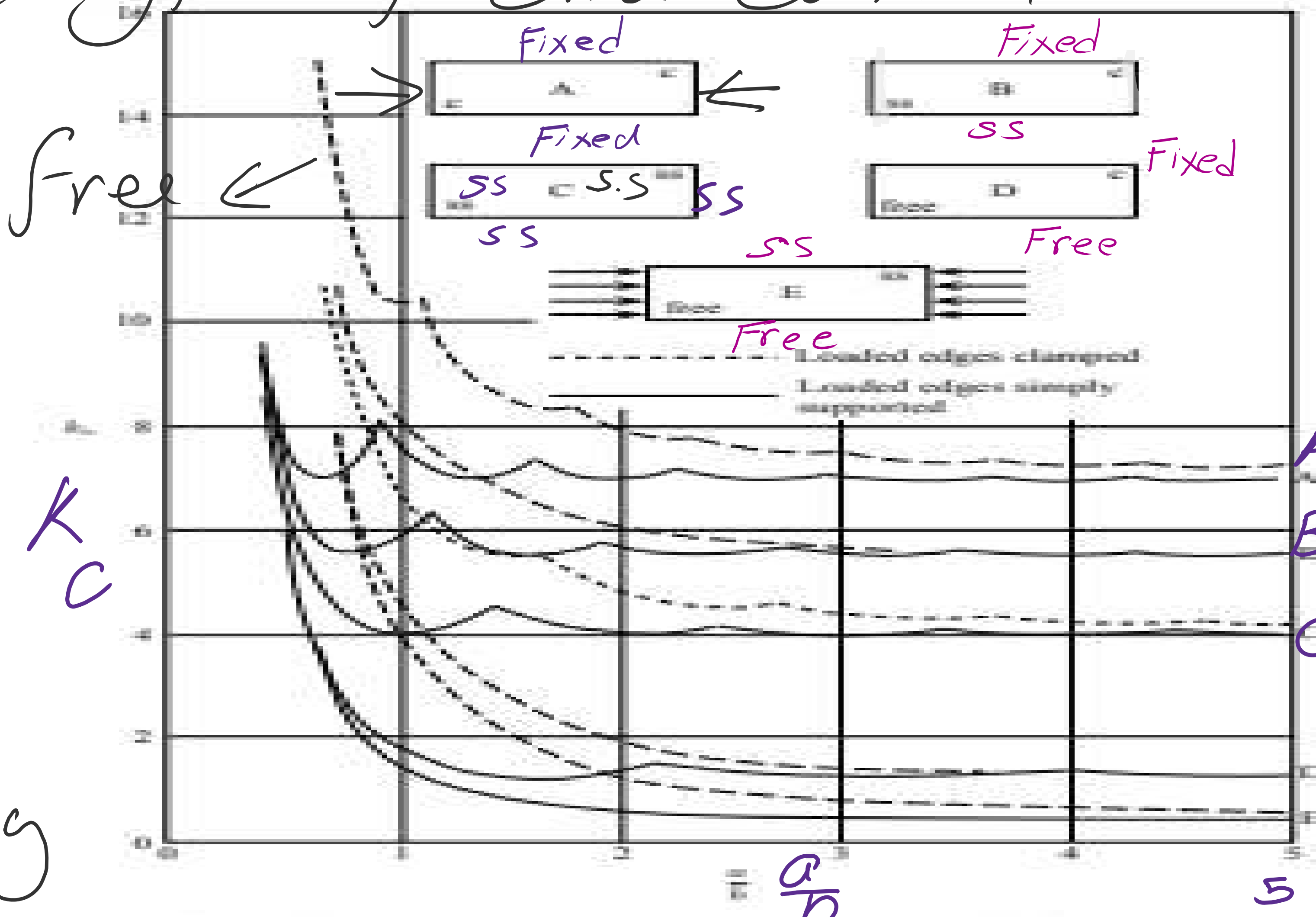



Figure 11.4: Effect of boundary conditions on the buckling coefficient of rectangular plates subjected to in-plane boundary conditions.

Unsported edge

| | | |
|---|---|---|
| A | 6.97 | Fixed - Fixed |
| B | 5.42 | Fixed - pinned |
| C | 4.00 | Pinned - pinned |
| D | 1.277 | → Fixed - Free |
| E | 0.425 | Pinned - Free |

$\frac{a}{b} \geq 5$

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K or Kc Values

| Axial Compression | Conditions at Nonloaded Edges | $k_c$ |
|---|---|---|
| All loaded edges simply supported<br>$a/b > 4.0$ | Case 1: Both fixed | 6.97 |
| | Case 2: One fixed, one simply supported | 5.42 |
| | Case 3: Both simply supported | 4.00 |
| | Case 4: One fixed, one free | 1.277 |
| | Case 5: One simply supported, one free | 0.425 |

Stiffened

unstiffened

From the previous Graph

## Local buckling-limits-report _aisc_adhoctg

Table 9 Assumptions underlying AISC 360-16 *w/t* limits - $\lambda_r$ Compression Only

| | | $k$ | $\lambda^* = \sqrt{\frac{F_y}{F_{cr}}}$ | Eq. (10) | AISC |
|---|---|---|---|---|---|
| | Unstiffened | | | $\lambda_r$ | $\lambda_r$ |
| 1 | Rolled Flange | $0.70^b$ | $0.70^a$ | $0.56\sqrt{\frac{E}{F_y}}$ | $0.56\sqrt{\frac{E}{F_y}}$ |
| 2 | Built-up Flange | 0.35~0.76 | $0.70^a$ | $0.39\sim0.58\sqrt{\frac{E}{F_y}}$ | $0.38\sim0.56\sqrt{\frac{E}{F_y}}$ |
| 3 | Angle leg, other | $0.425^e$ | $0.70^a$ | $0.43\sqrt{\frac{E}{F_y}}$ | $0.45\sqrt{\frac{E}{F_y}}$ |
| 4 | Stem of tee | $1.277^f$ | $0.70^a$ | $0.75\sqrt{\frac{E}{F_y}}$ | $0.75\sqrt{\frac{E}{F_y}}$ |

a. non-dimensional slenderness to achieve a plate strength approaching $F_y$
b. ~1/2 way between pinned and fixed k values
c. ~1/3 of the way between pinned and fixed k values
d. this k factor back-calculated from $\lambda^*$ and *w/t* limit
e. ideal case for simple-free longitudinal edge conditions
f. ideal case for fixed-free longitudinal edge condition

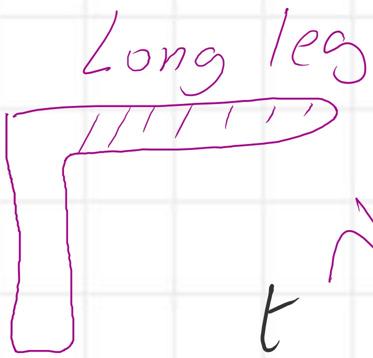
Unstiffened → (a) Rolled Flange

→ 1/3 distance $= 0.425 + \frac{1}{3}(1.277 - 0.425)$

Pinned - Fixed $= 0.70 \Rightarrow \lambda_r = 0.666\sqrt{0.70}\sqrt{\frac{E}{F_y}}$

$\lambda_r = 0.557\sqrt{\frac{E}{F_y}} \simeq 0.56\sqrt{\frac{E}{F_y}}$

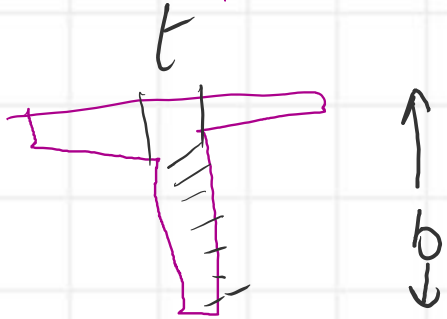
(b)




$K_c = 0.425$ → hinged - Free

$\lambda_r = 0.666\sqrt{0.425}\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{E}{F_y}}$

(c)




$K_c = 1.277$ Fixed - Free

$\lambda_r = 0.75\sqrt{E/F_y}$

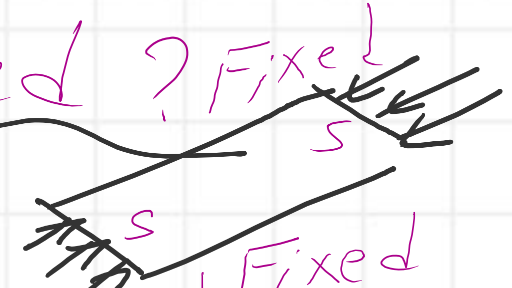
# Local buckling-limits-report _aisc_adhoctg

| | | | | | |
|---|---|---|---|---|---|
| | Stiffened | | | | |
| 5 | Rolled Web | 5.0[c] | 0.70[a] | $1.49\sqrt{\frac{E}{F_y}}$ | $1.49\sqrt{\frac{E}{F_y}}$ |
| 6 | HSS Wall | 4.4[d] | 0.70[a] | $1.40\sqrt{\frac{E}{F_y}}$ | $1.40\sqrt{\frac{E}{F_y}}$ |
| 7 | Cover plate | 4.4[d] | 0.70[a] | $1.40\sqrt{\frac{E}{F_y}}$ | $1.40\sqrt{\frac{E}{F_y}}$ |
| 8 | Other | 5.0[c] | 0.70[a] | $1.49\sqrt{\frac{E}{F_y}}$ | $1.49\sqrt{\frac{E}{F_y}}$ |

a. non-dimensional slenderness to achieve a plate strength approaching $F_y$
b. ~1/2 way between pinned and fixed k values
c. ~1/3 of the way between pinned and fixed k values
d. this k factor back-calculated from $\lambda^*$ and $w/t$ limit
e. ideal case for simple-free longitudinal edge conditions
f. ideal case for fixed-free longitudinal edge condition

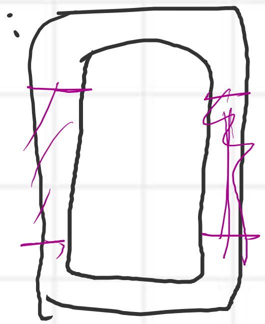
**Prepared by Eng.Maged Kamel.**

What are the values b/t selected?



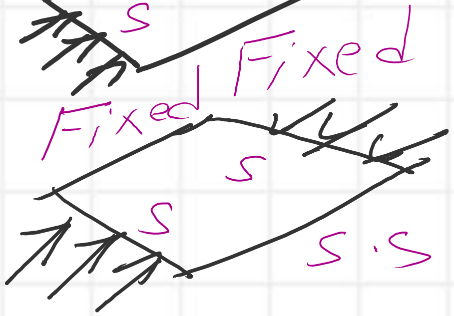

Stiffened

HSS



$K_c = 6.97$ →

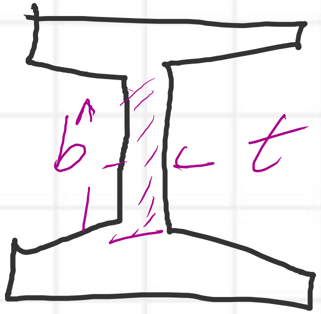
$= 5.42$




$K_c = 4.00$


$K_c = 4.40 \Rightarrow \lambda_r = 0.666\sqrt{4.40}\sqrt{E/F_y} \simeq 1.40\sqrt{\frac{E}{F_y}}$

(b)




Rolled web

$K_c = \frac{1}{3}$ Pinned → Fixed

$4 + \frac{1}{3}(6.97 - 4.00) \simeq 5.00$

$\lambda_r = 0.666\sqrt{5}\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{E}{F_y}}$

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