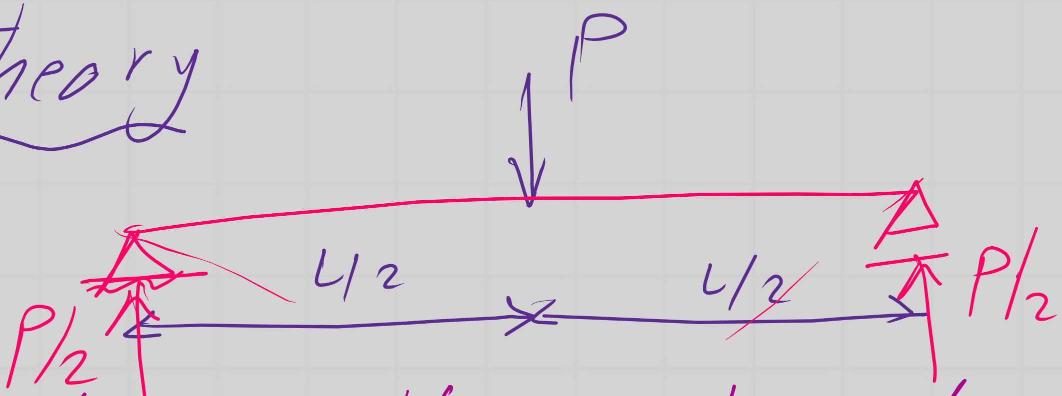
Topics

Ⓐ How Can we Find the plastic Nominal Load P_n value? Plastic hinge Length.
The Load is acting on the Middle of a simple beam → Two methods are Use : statical method

Ⓑ Determine P_n For a given W18×76 Beam A992 steel of 32 FT Long Simple → Verify the value by using MASTAN-2

Prepared by Eng.Maged Kamel.

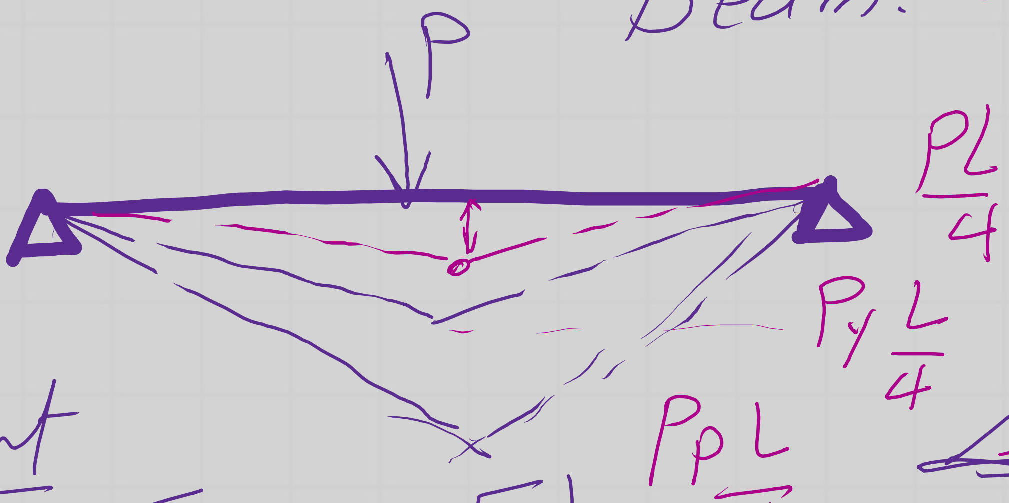
Plastic Theory

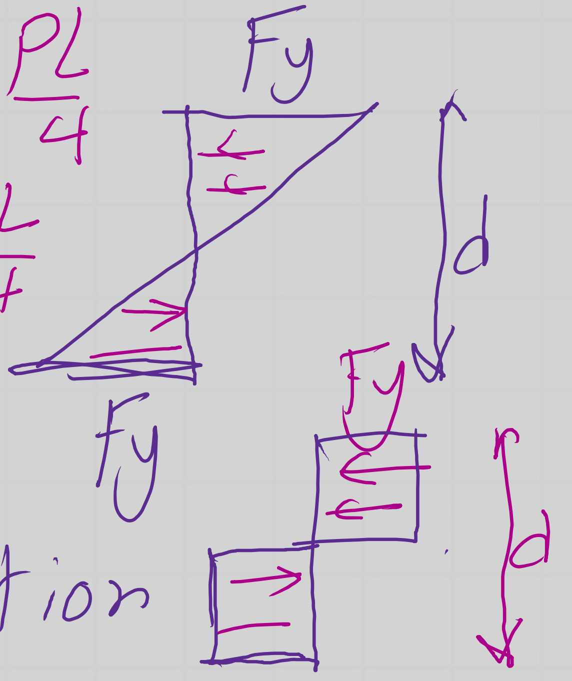


Beam of Length L

Estimate the Collapse Load For the given Beam.

Load is increasing From $P \rightarrow P_y \rightarrow P_p$

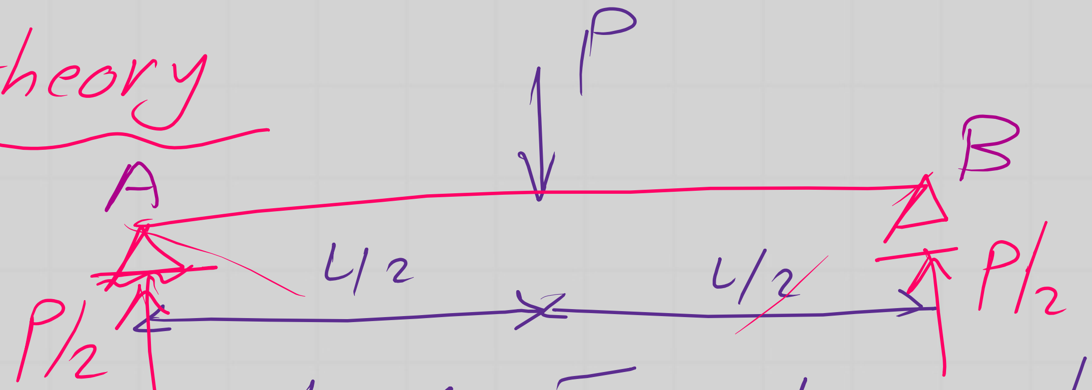




P_y : The Load that Causes stress $= F_y$ For Top Fibre

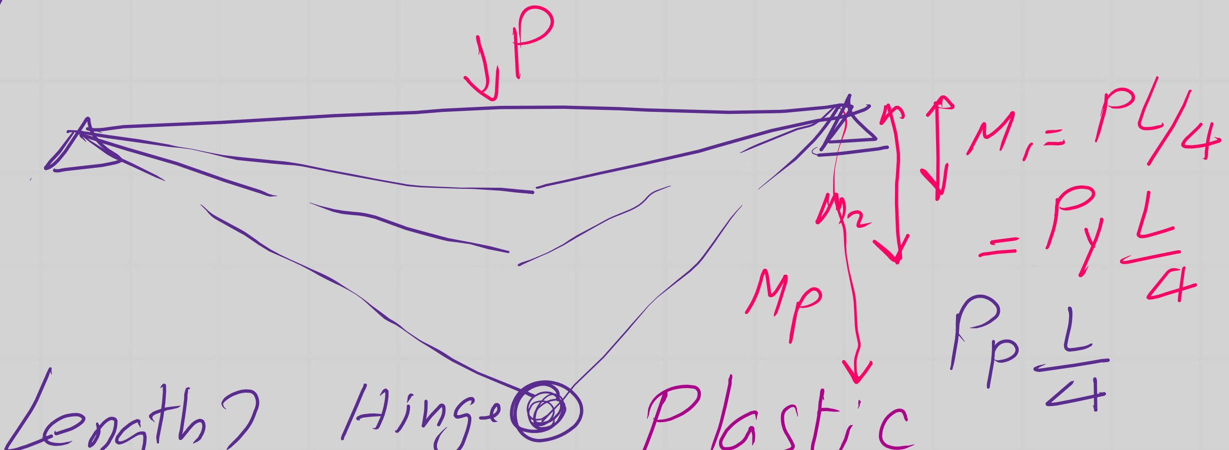
P_p : The Load that Causes stress $= F_y$ For Hole section

Plastic theory

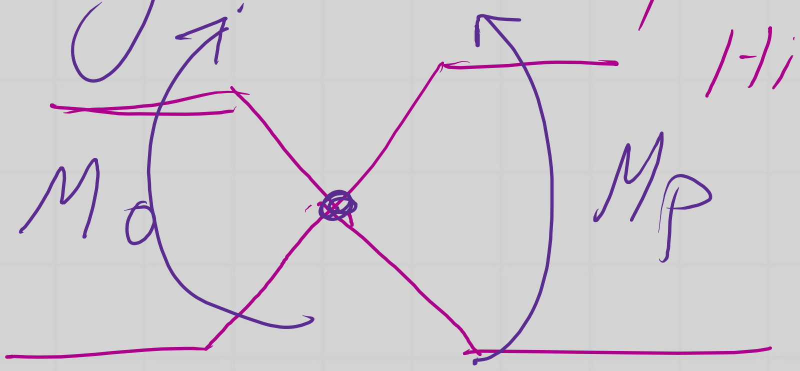


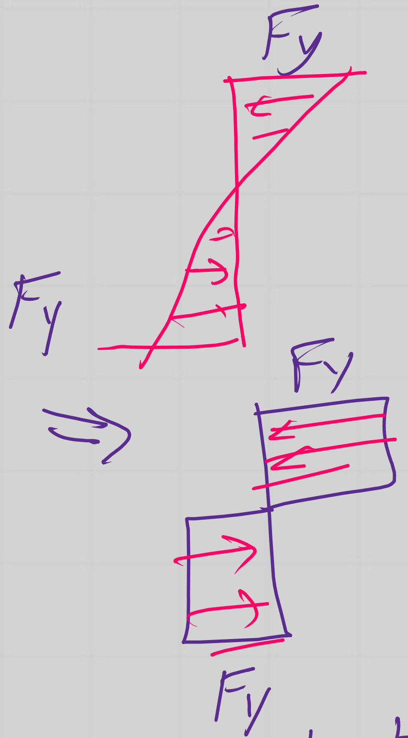
Estimate Colapse Load For shown beam

$P \to P_y \Rightarrow P_p$



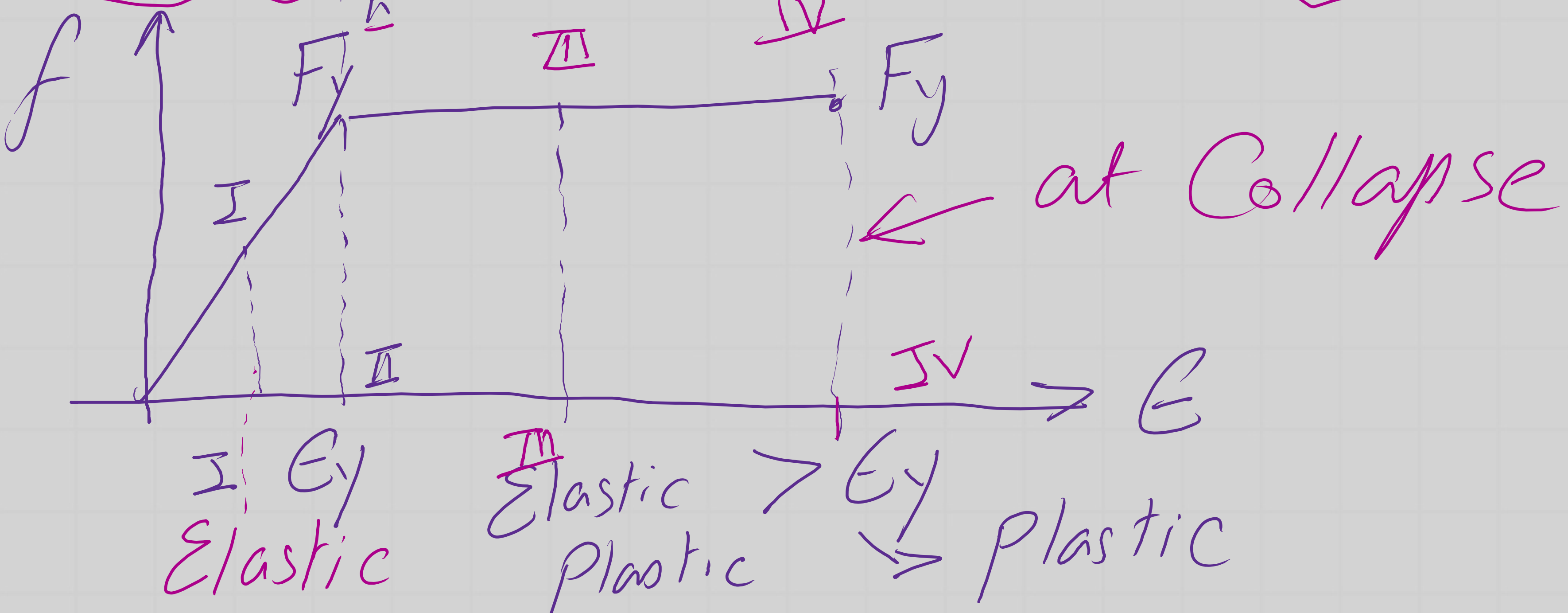
What is the Plastic Hinge Length?



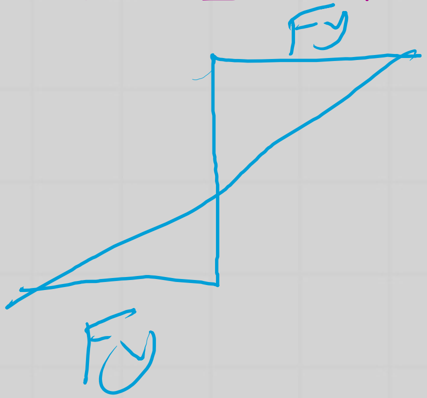


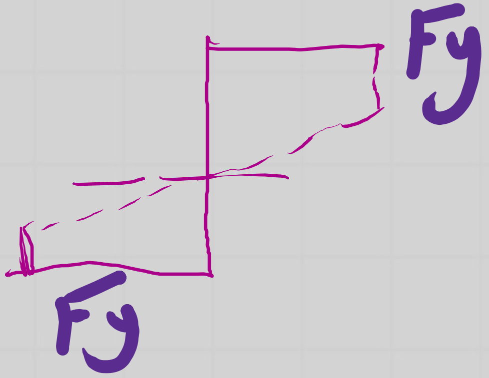
At plastic hinge

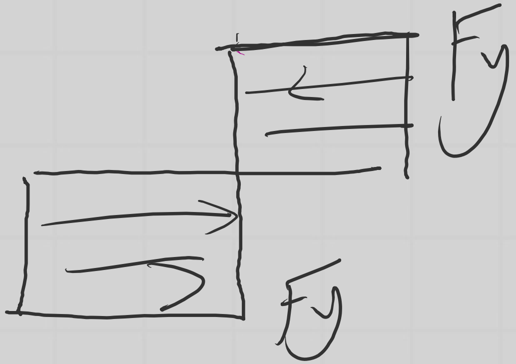
We refer to strain versus stress Graph



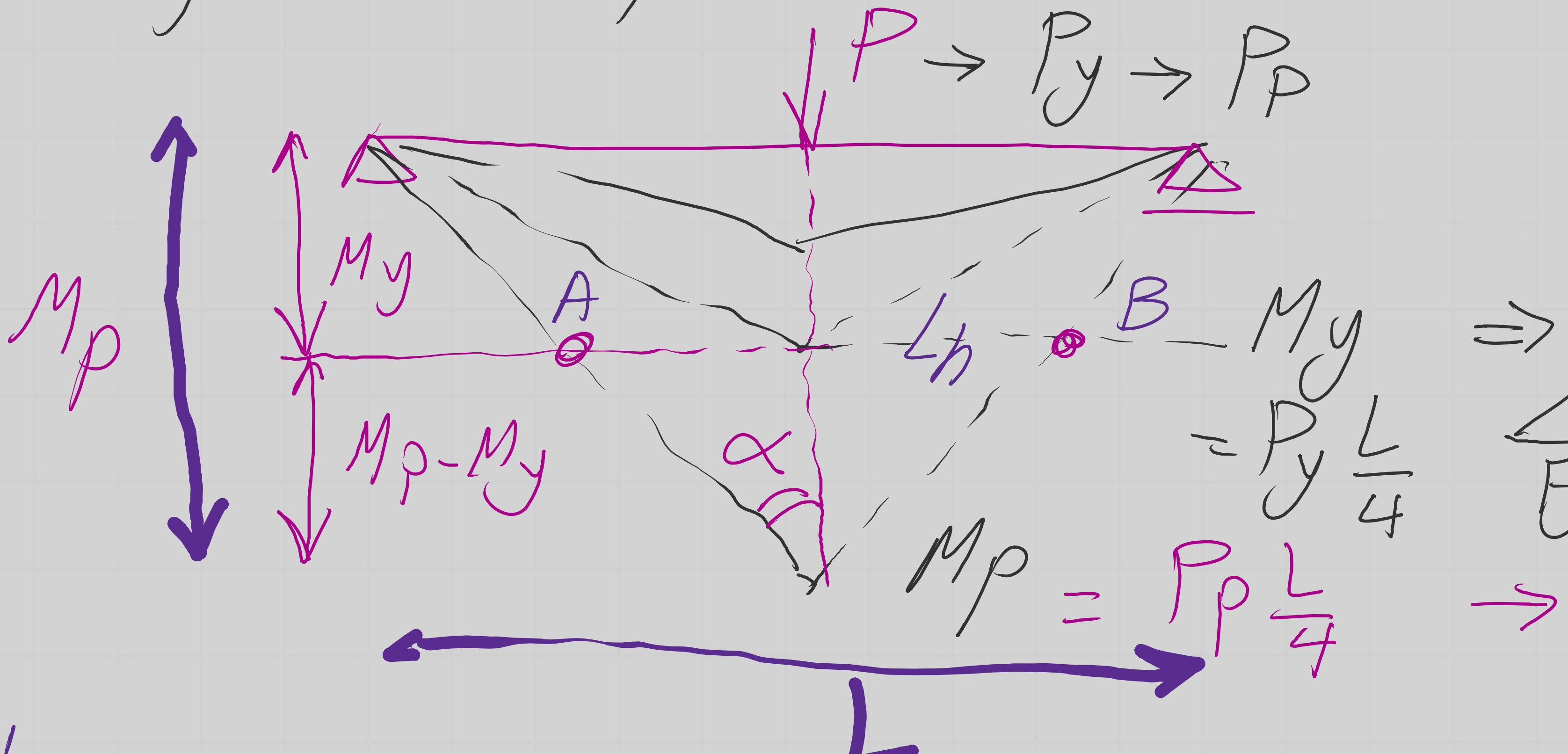
Case I-I Case II-II Case III-III Case IV-IV

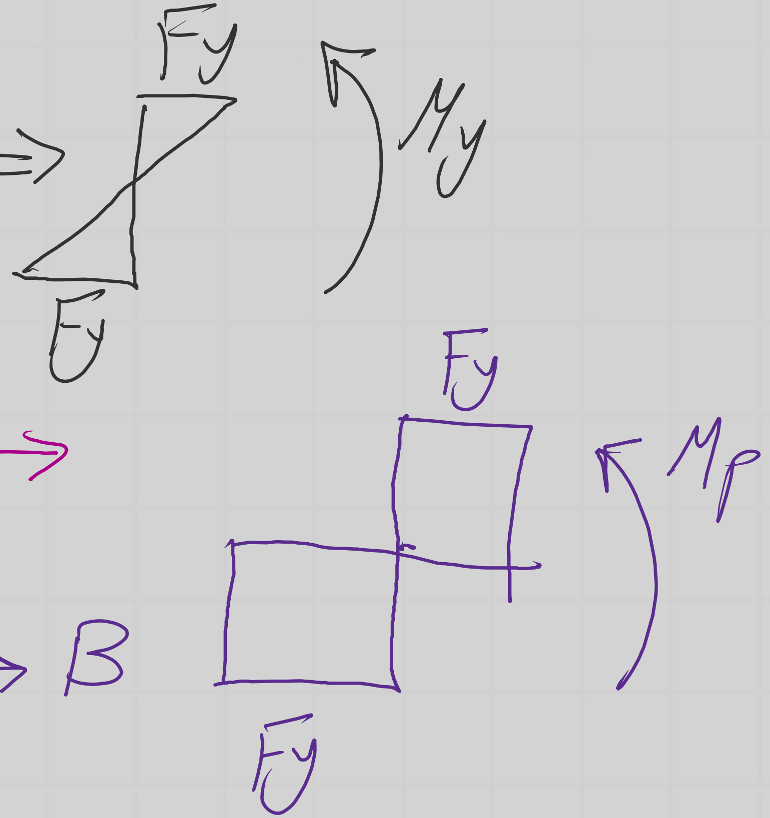




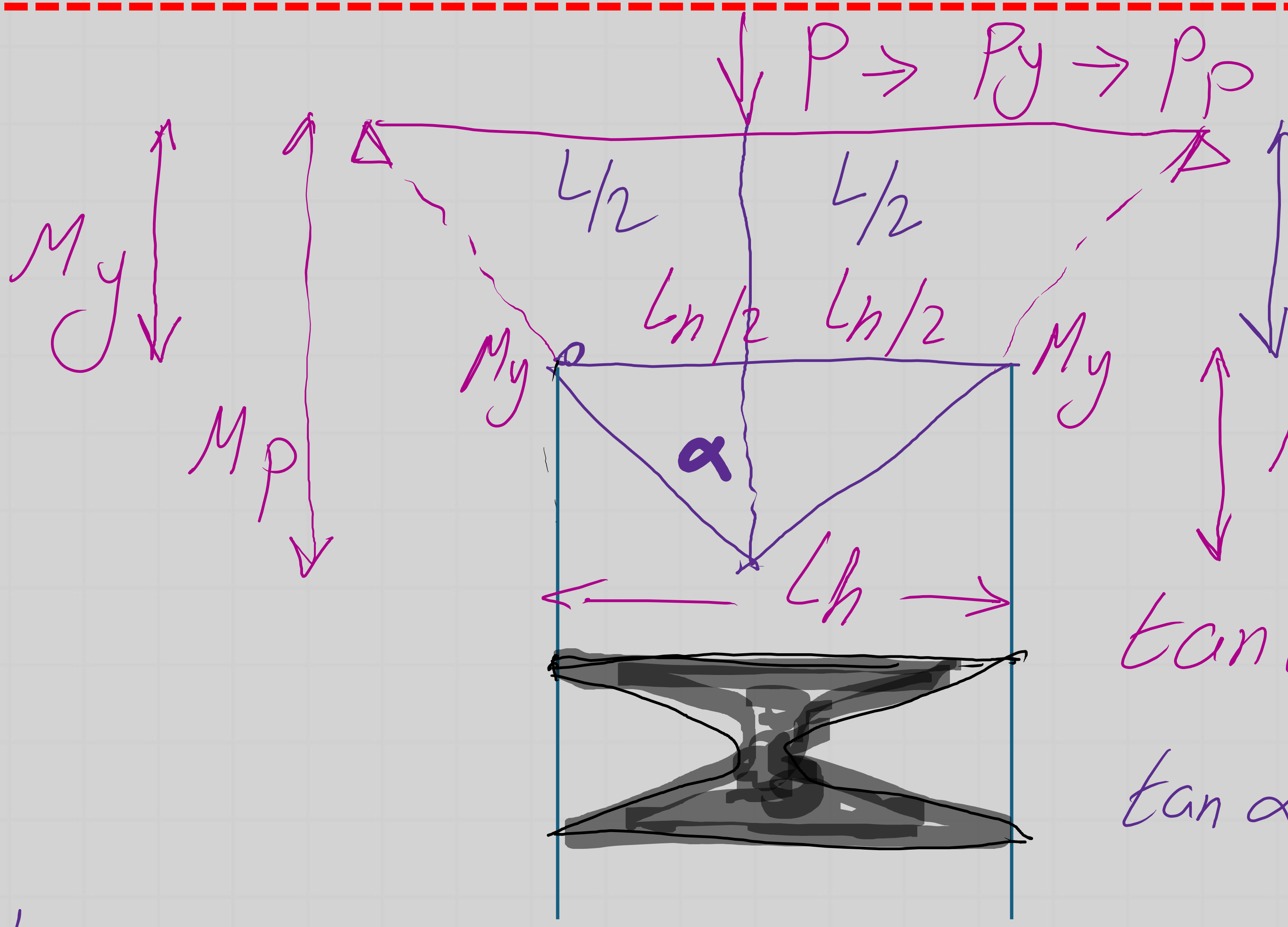


Let us Examine the Bending moment diagram of the simple beam Under CL of length L





L_h: Length of Hinge From Point $A \to B$



Find plastic hinge width

$$\tan\alpha \approx \alpha = \frac{L_h/2}{M_p - M_y}$$

$$\tan\alpha = \alpha = \frac{L/2}{M_p}$$

$$\frac{L_h}{2} = (M_p - M_y)\alpha$$

$$\frac{L_h}{2} = \frac{L}{2M_p}(M_p - M_y) = \frac{L}{2}\left(1 - \frac{M_y}{M_p}\right)$$

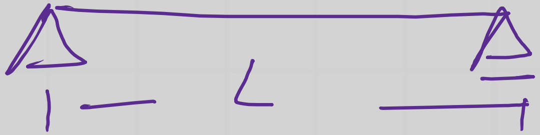
$M_p = P_p \frac{L}{4}$ A

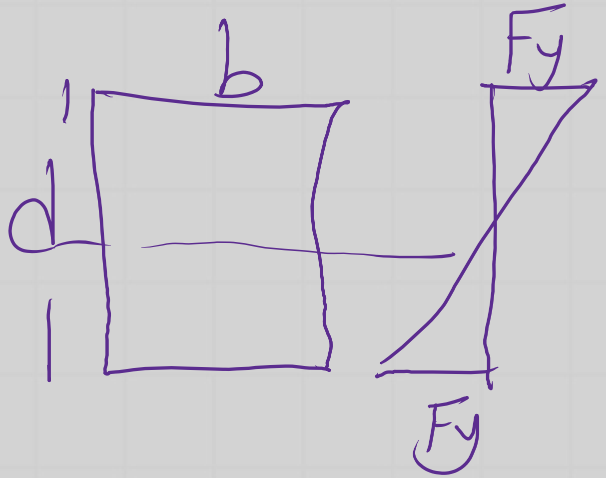
$M_p = F_y Z_x$
$M_y = F_y S_x$

From α Equation $\frac{L_h}{2} = \frac{L}{2}\left(1 - \frac{M_y}{M_p}\right)$

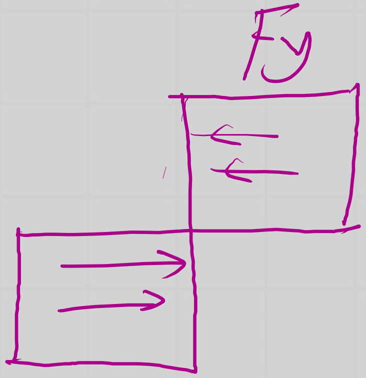
Length of plastic hinge $= L_h = L\left(1 - \frac{M_y}{M_p}\right)$

For a rectangle section





$M_y = F_y \frac{bd^2}{6}$, $M_p = F_y \frac{bd^2}{4}$



$L_h = L\left(1 - \frac{bd^2}{6}\left(\frac{4}{bd^2}\right)\right) = \frac{L}{3}$ Fy

What is hinge Length for a W-Section?

W18X76 $\quad F_y = 50$ ksi $\quad$ A992 steel

Area = 22.30 inch^2 $\quad S_x = 146$ inch^3 $\quad Z_x = 163$ inch^3 $\quad \frac{S_x}{Z_x} = 0.8957$

Length of plastic hinge $= L_h = L\left(1 - \frac{M_y}{M_p}\right)$

$M_y = F_y S_x$

$M_p = F_y Z_x$

$$L_h = L\left(1 - \frac{F_y S_x}{F_y Z_x}\right)$$

L: span length $\quad L(1 - 0.8957) = 0.1042\,L$

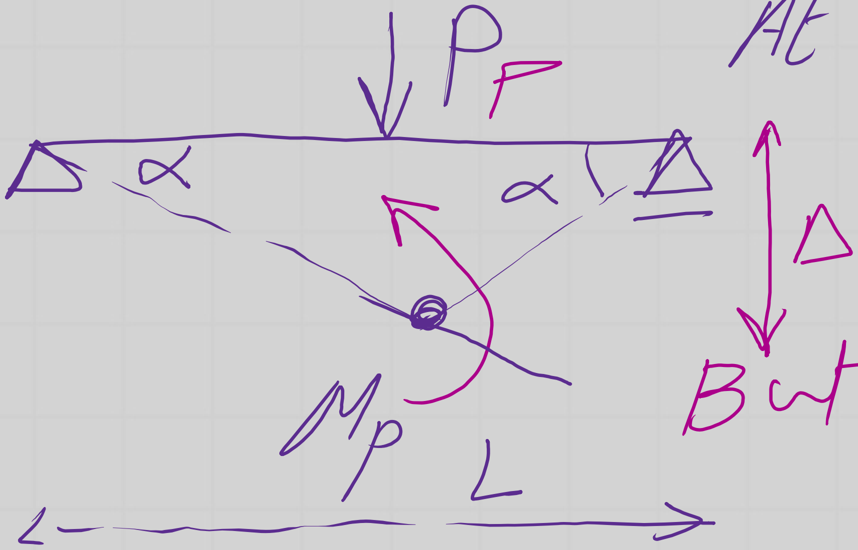
There are Two methods to Find the relation between P_p & M_p

① Virtual Work – upper bound

External Energy

At Collapse Load: $\Sigma W_e = \Sigma W_i$

internal energy



$$P_p \Delta = M_p (2\alpha)$$

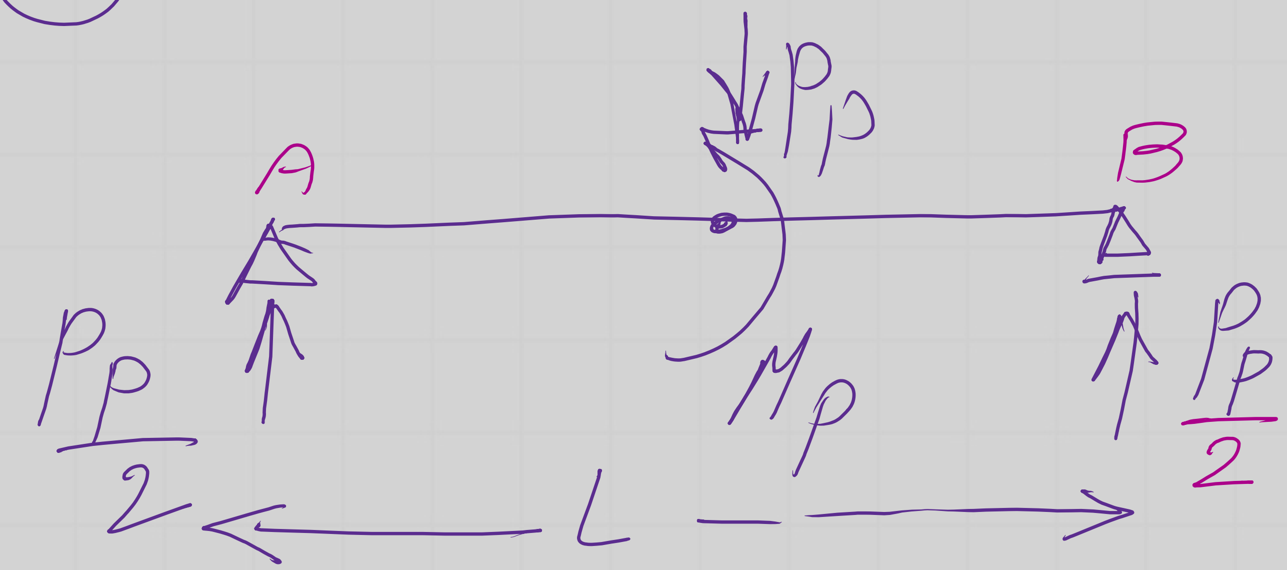
But $\alpha = \tan\alpha = \frac{\Delta}{L/2} = \frac{2\Delta}{L}$

$$P_p = \frac{M_p}{\Delta}(2)\left(\frac{2\Delta}{L}\right) = 4\frac{M_p}{L}$$

or $M_p = \frac{L}{4} P_p$

② Lower bound : 2nd method

at hinge we have $M = M_p$



$$\boxed{M_p - \frac{P_p}{2}\left(\frac{L}{2}\right) = 0}$$

$$M_p = \frac{P_p L}{4}$$

Or $P_p = \frac{4M_p}{L}$ which is the same result as obtained from Virtual Work method

$$M(x) = \frac{P_p}{2}\frac{L}{2} = M_p$$

Mccormac Fifth edition

8-21 to 8-39. *Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.* Modified $\underline{AB = BC}$

8-21. (~~Ans. 90.68 k~~)

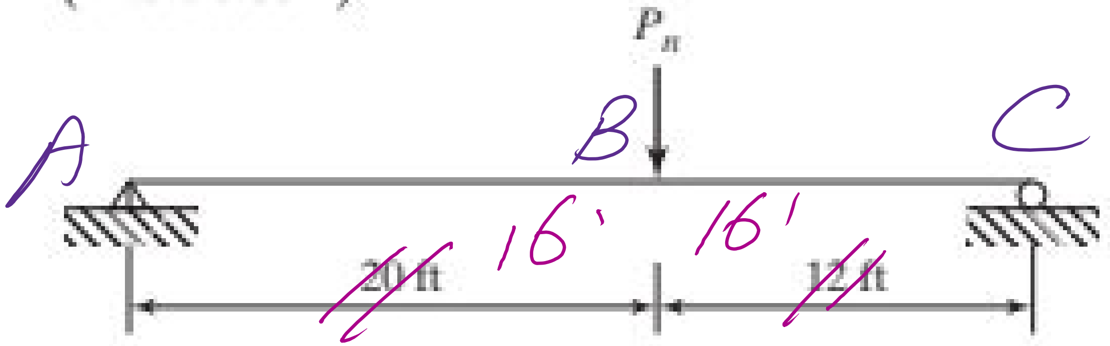
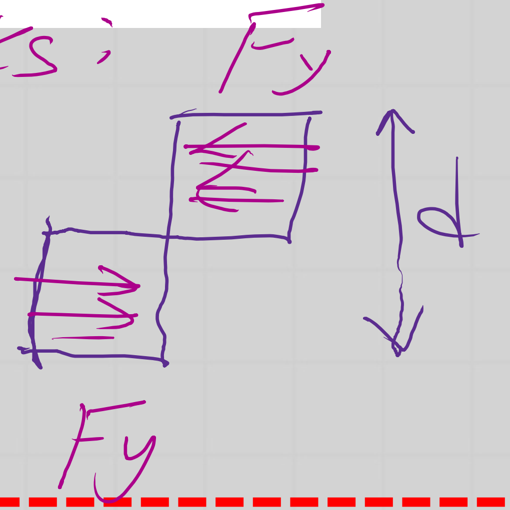


FIGURE P8-21

FT.kips = 12 inch.kips

Solution

A992 steel $\Rightarrow F_y = 50$ ksi



W18X76 $\Rightarrow Z_x = 163\ \text{inch}^3$

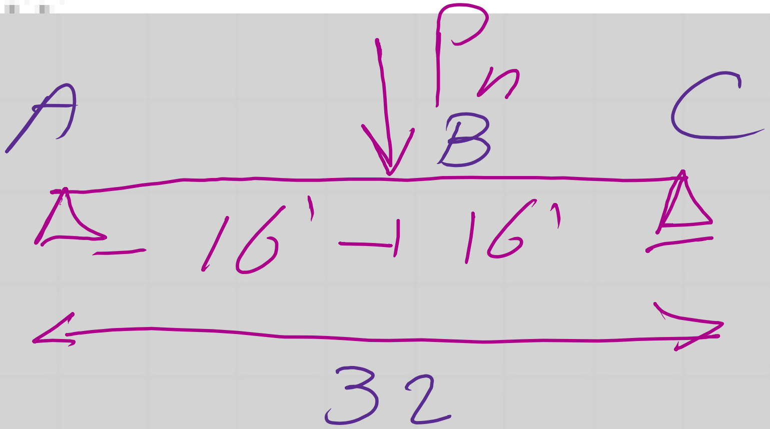
Area $= 22.30\ \text{inch}^2$

$M_p = F_y Z_x = 50(163) = 8150\ \text{inch.kips}$ / 12

8.21 Mccormac

Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.

① Use Lower bound.



$W_{18} \times 76$

$A = 22.30$ inch^2

$Z_x : 163$ inch^3

$F_y = 50$ ksi

$$A_y = \frac{P_n(16)}{32} = \frac{P_n}{2}$$

M_{max} at B $= P_n \cdot \left(\frac{L}{2}\right) = \frac{P_n}{2}(16) = 8\,P_n \rightarrow$ FT.Kip

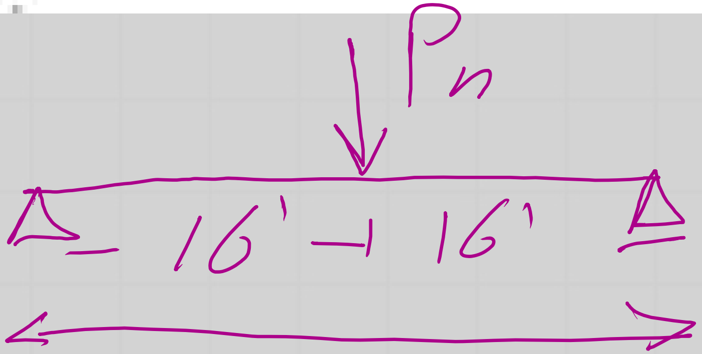
Equate M_{max} to $MP = (8150/12)$ FT.Kips

$\rightarrow P_n = 84.90$ Kips

8.21 Mccormac

Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.

Upper bound.



W18X76

$A = 22.30 \text{ inch}^2$

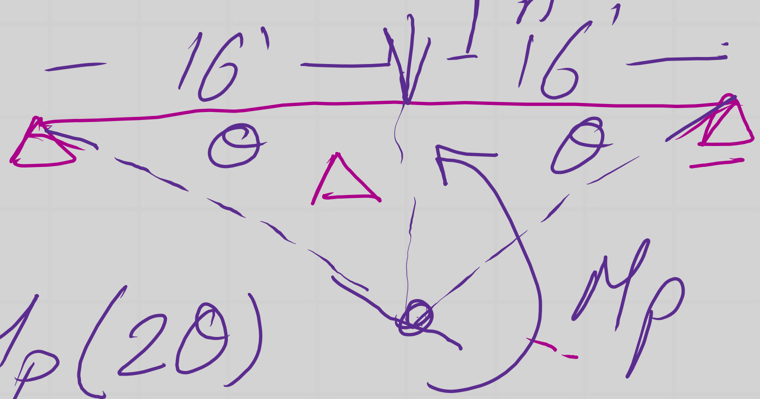
$Z_x : 163 \text{ inch}^3$

$F_y = 50 \text{ ksi}$

$M_p = F_y Z_x$

$M_p = M_n = \frac{50(163)}{12} = 679.16$ FT.Kips

Using virtual work



$P_n \cdot \Delta = M_p(2\theta)$

$\theta = \frac{\Delta}{16}$

$P_n \Delta = 679.16\left(\frac{2\Delta}{16}\right) \Rightarrow P_n = 48.896 \text{ kips} \simeq 48.9 \text{ k}$

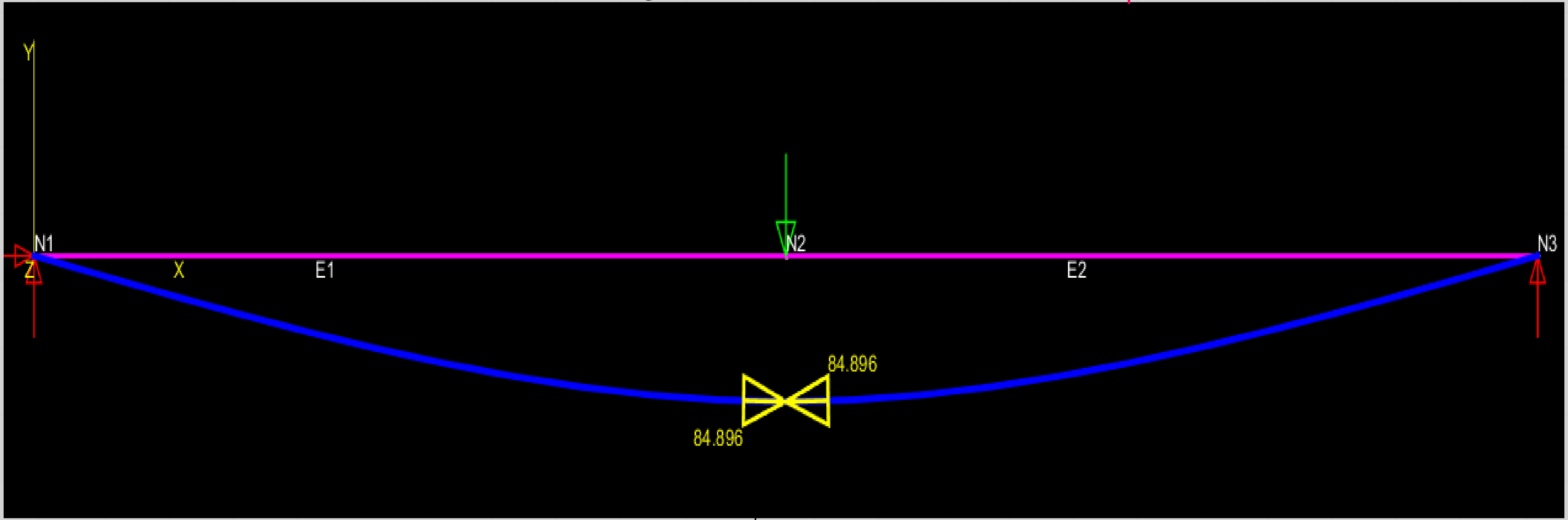
Using MASTAN2

$\downarrow P$ starts as Equal 1 kips

increment Size 10, Max no. of increment = 12

Max ratio : 120

W18X76



increment#9 (80→90) → Load = 84.896 kips

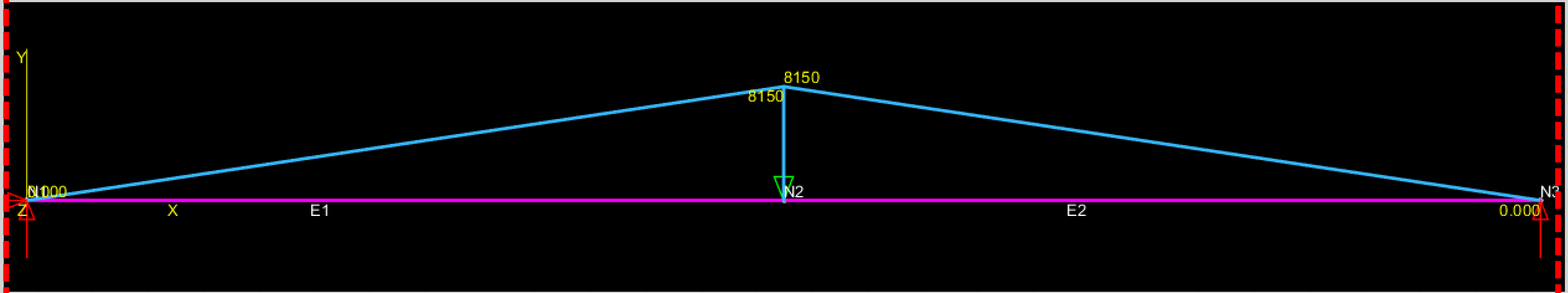
Using MASTAN 2

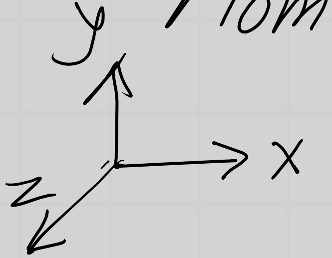
$\downarrow P$ starts as Equal 1 kips

increment Size 10, Max no. of increment = 12

Max ratio = 120

W18 X 76





Moment M_Z = 8150 kip.inch

$\left.\begin{array}{l} F_y = 50 \text{ ksi} \\ Z_x = 163 \text{ inch}^3 \end{array}\right) \longrightarrow M_p = 50(163)$

= 8150 kips.inch

Shear Value V_y $R_A = 42.45$ kips $+ve = \frac{P_n}{2}$