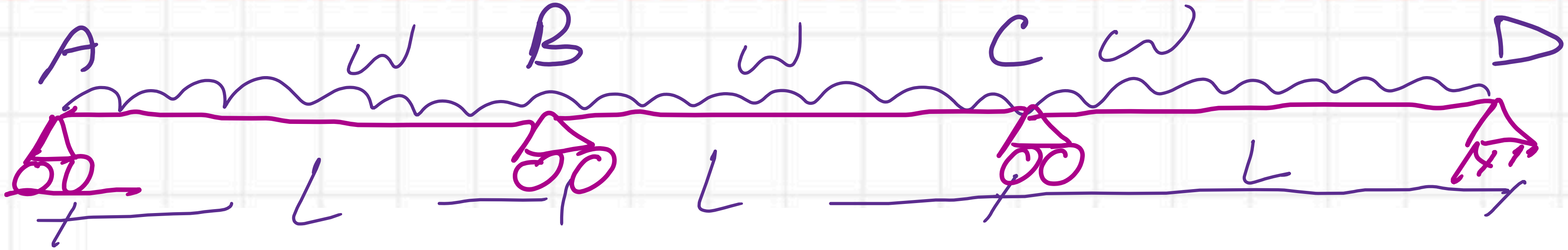


Statical method

Continuous Beam

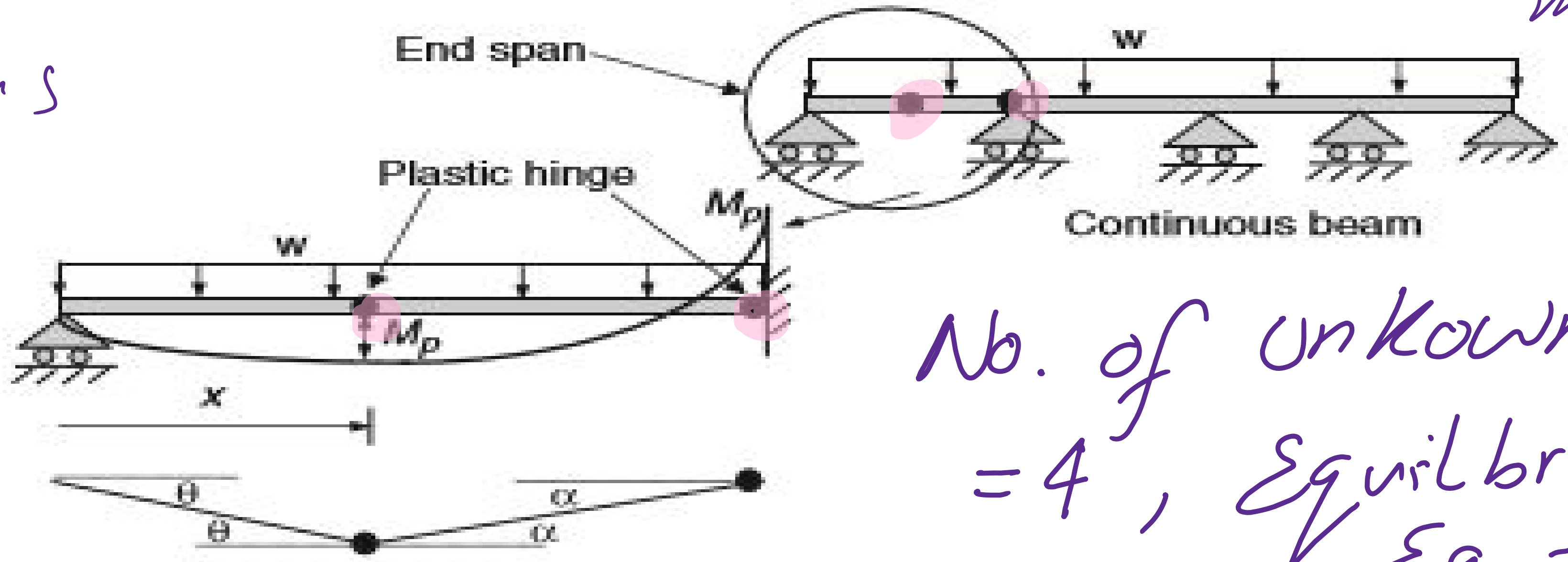


FIGURE 5.14. Collapse mechanism at end span of a continuous beam.

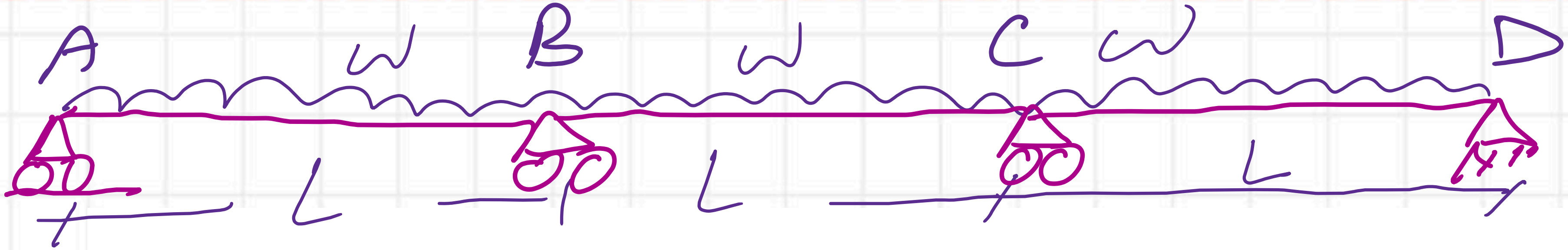
No. of unkowns =4, Equilbrium Eq = 3

add 1

To get Collapse we need two Plastic hinge

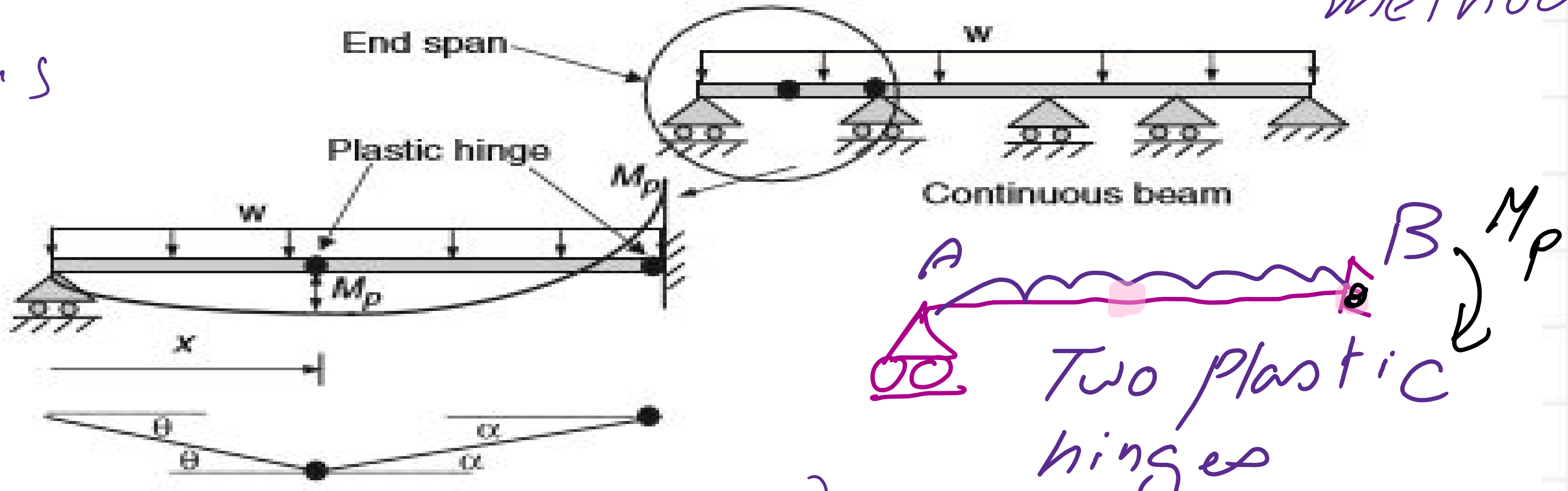
$4 - 3 + 1 = 2$

Prepared by Eng.Maged Kamel.



Statical method

Continuous Beam

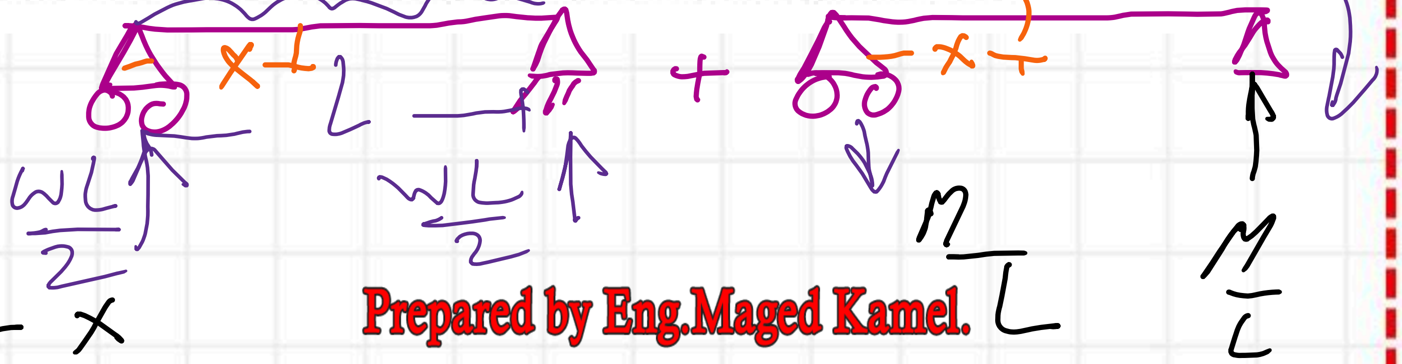


Two plastic hinges

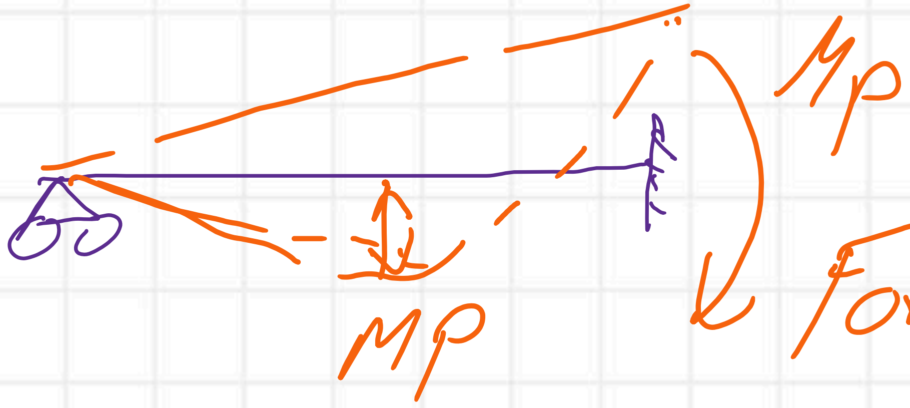
First span

FIGURE 5.14. Collapse mechanism at end span of a continuous beam.

Considered at Fixed at B with roller at A



$$M_x = \frac{wL}{2}(x) - \frac{wx^2}{2} - \frac{M}{L}x$$



$$M(x) = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{M}{L}(x)$$

For $\frac{dM(x)}{dx} = 0$ to get Mp

when $w = w_p$ $\Rightarrow$ $M(x) = M_p$

$$\frac{dM}{dx} = \frac{w_pL}{2} - \frac{2}{2}w_p x - \frac{M_p}{L} = 0 \quad \text{I}$$

$$\frac{M_p}{L} = w_p\left(\frac{L}{2} - x\right) \quad \text{II}$$

Back to I $\quad M(x) = M_p \quad w \Rightarrow w_p$

$$M_p = \frac{w_pL}{2}(x) - \frac{w_p x^2}{2} - w_p\left(\frac{L}{2} - x\right)x$$

$$M_p = w_p\left(\frac{Lx}{2} - \frac{x^2}{2} - \frac{\ell}{2}x + x^2\right)$$

$$M_p = w_p\left(\frac{Lx}{2} - \frac{x^2}{2} - \frac{\ell^2}{2}x + x^2\right)$$

but $\frac{M_p}{L} = w_p\left(\frac{L}{2} - x\right) \Rightarrow M_p = w_p\frac{L^2}{2} - w_p L x$

$$w_p\left(\frac{L^2}{2} - Lx\right) = w_p\left(\frac{x^2}{2}\right)$$

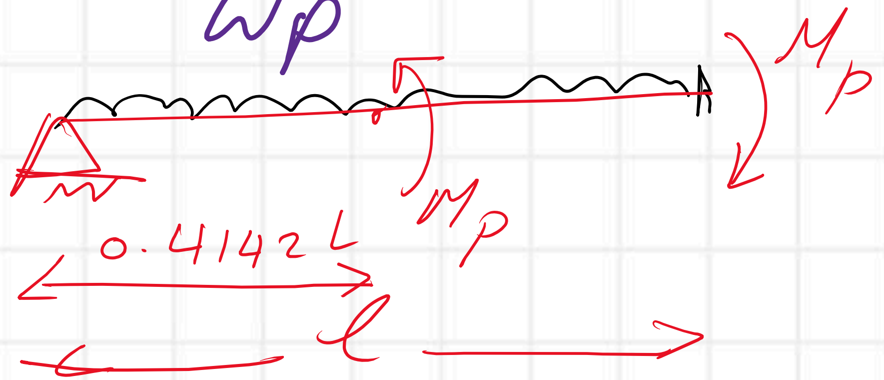
$$\frac{x^2}{2} + Lx - \frac{L^2}{2} = 0 \Rightarrow x^2 + 2Lx - L^2 = 0$$

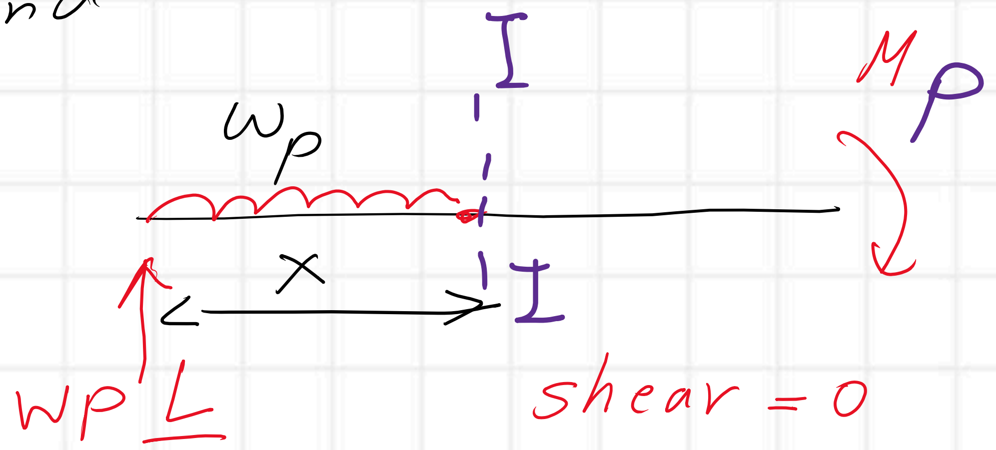
$$x = \frac{-2L \pm \sqrt{(4+4)L^2}}{2}$$

$$x = -L \pm \frac{2\sqrt{2}}{2}L$$

$$x = 0.4142L$$

First span Lower bound





shear = 0

$$\frac{M_p}{L} = w_p\left(\frac{L}{2} - x\right)$$

$$x = 0.4142\,L$$

$$\frac{M_p}{L} = 0.0858\,L\,(w_p)$$

$$M_p = W_p \frac{L}{2}(0.4142\,L) - \frac{M_p}{L}(x) - \frac{w_p}{2}(0.4142\,\ell)^2$$

$$M_p = W_p \ell^2 \left[0.2071 - 0.0355 - 0.08578\right]$$

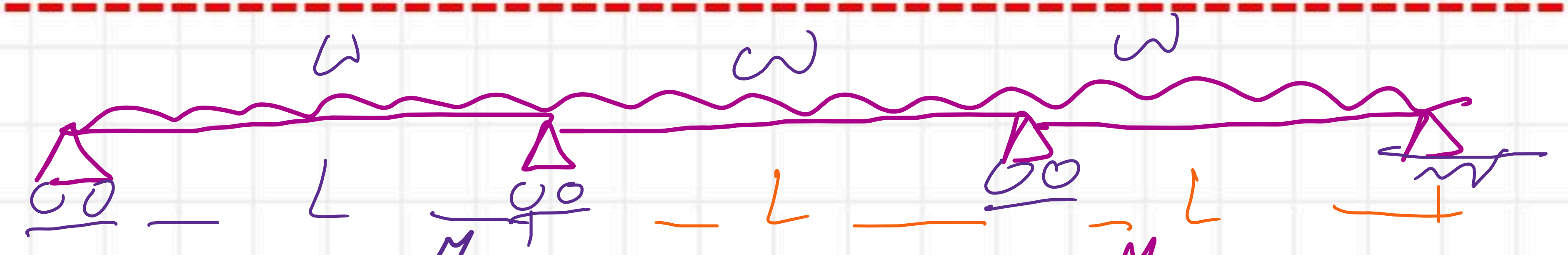
$$M_p = 0.08582\, W_p L^2$$

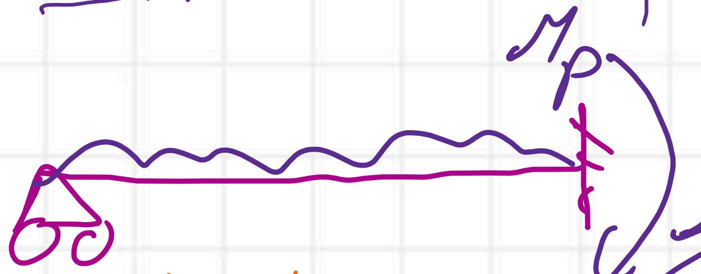
$$W_p = M_p(11.6523)/L^2$$

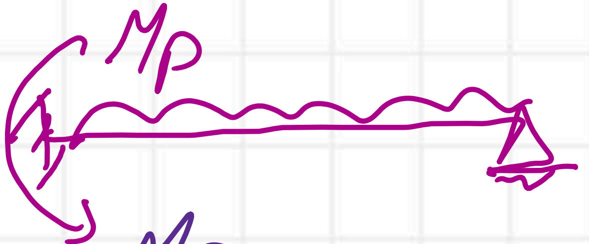
$$W_p \frac{L}{2} - \frac{M_p}{L} = W_p(0.5L - 0.08[illegible])$$

$$= W_p(0.4142)L$$

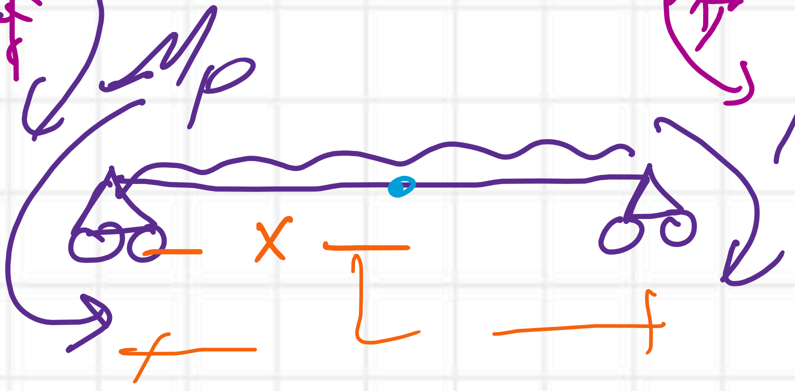
Shear = 0 I-I

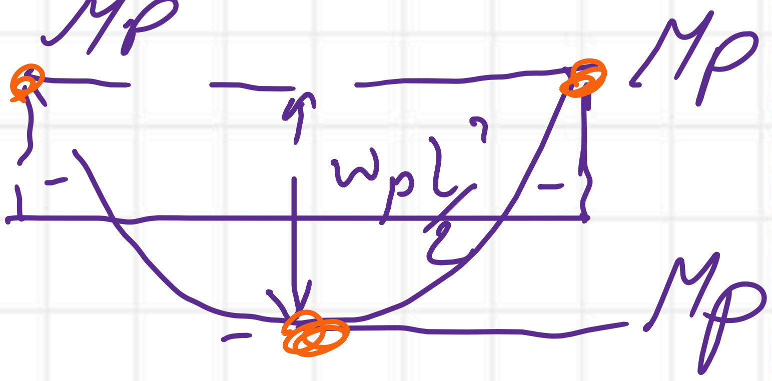






Intermediate Span



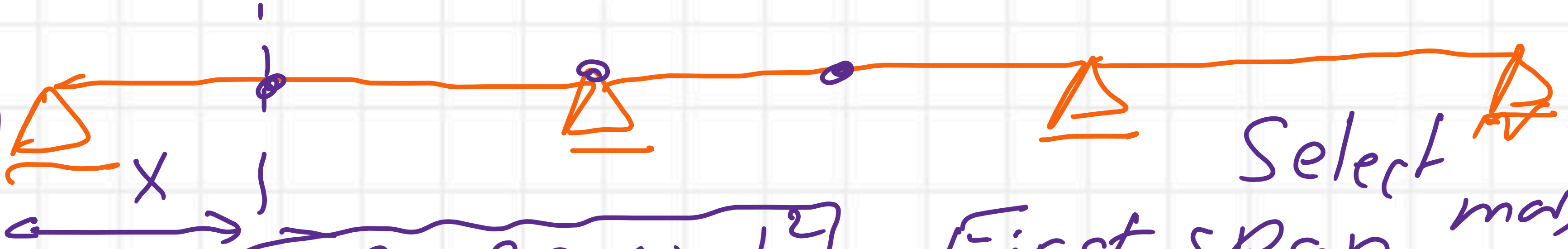


$$2M_p = w_p \frac{L^2}{8}$$

$$M_p = w_p \frac{L^2}{16}$$

$$= w_p L^2 (0.0625)$$

$x = 0.5L$



$x = 0.4142L$

$x = 0.5L$

$M_p = 0.08582\, w_p L^2$

$M_p = 0.0625\, w_p L^2$

Select max

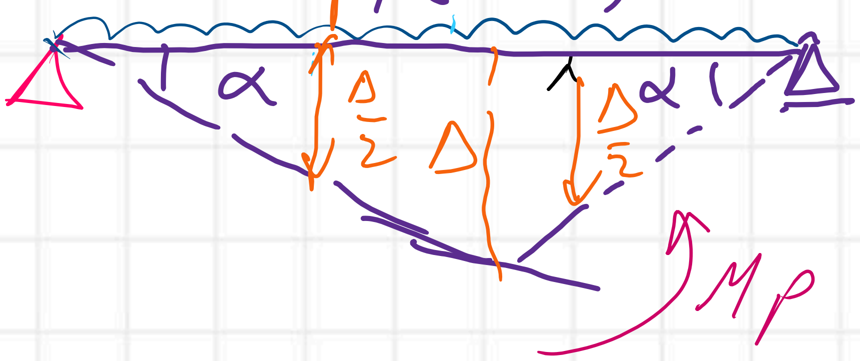
First span

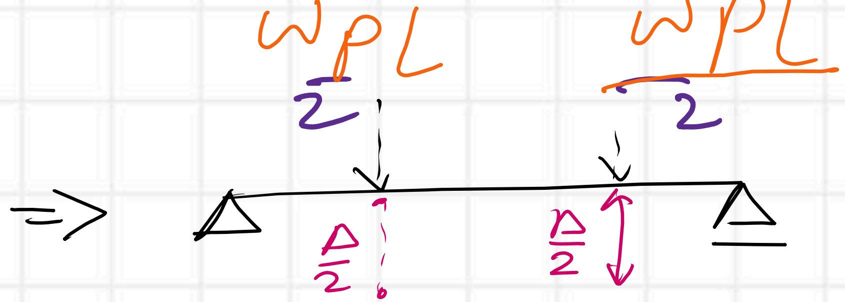
2nd span

Upper Limit Expression of external work

UL: Uniform Load

$w_p/(UL)$





Consider two cl

$$\text{External work} = 2\left(\frac{w_pL}{2}\right)\frac{\Delta}{2} = \frac{w_pL\Delta}{2} \Rightarrow \frac{1}{2}(UL)\,span(\Delta)$$

$$\text{internal work} = M_p(2\alpha)$$

$$\alpha = \frac{2\Delta}{L}$$

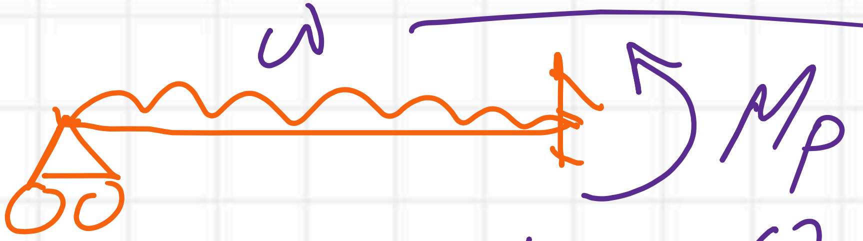
$$M_p(2\alpha) = M_p(2)\left(\frac{\Delta}{\frac{L}{2}}\right) = \frac{4M_p\Delta}{L}$$

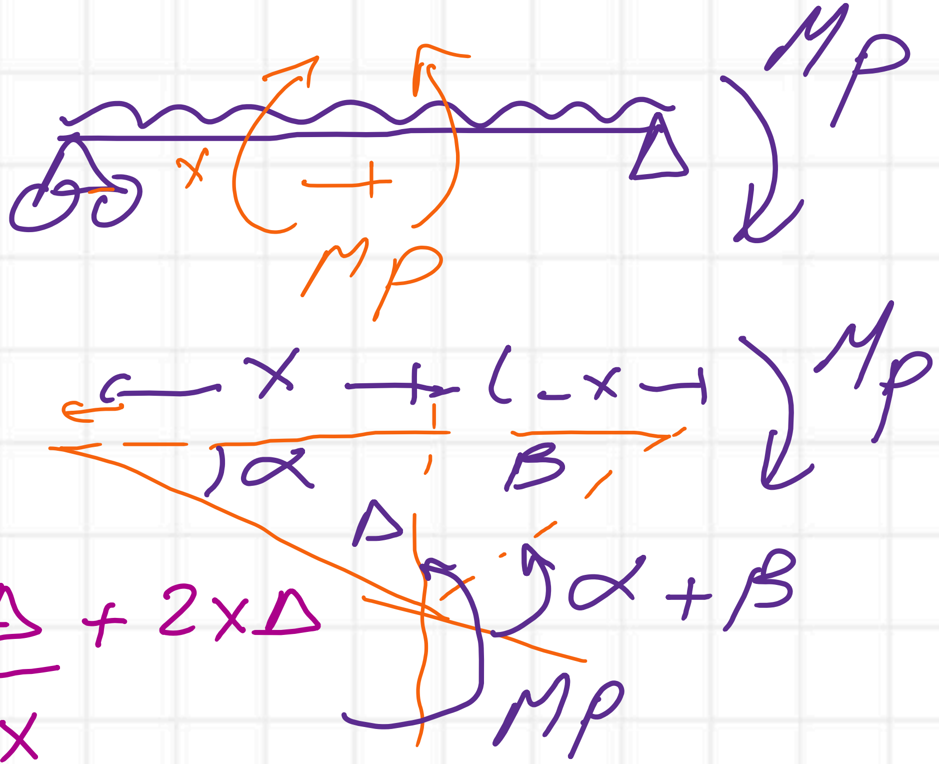
$$\frac{w_pL\Delta}{2} = 4M_p\frac{\Delta}{L}$$

$$M_p = w_pL^2/8$$

Prepared by Eng.Maged Kamel.

Using an upper bound For the Same Continuous beam





$$W_e = W_i \qquad W_e = \frac{w_p L \Delta}{2}$$

$$W_i = M_p(\beta) + M_p(\alpha + \beta)$$

$$\alpha = \frac{\Delta}{x} \;\&\; \beta = \frac{\Delta}{L-x} \qquad \alpha + 2\beta = \frac{(L-x)\Delta + 2x\Delta}{(L-x)x}$$

$$\frac{w_p L \Delta}{2} = M_p\left(\frac{\Delta(L+x)}{(L-x)x}\right) \qquad M_p = \frac{w_p L (L-x)x}{2(L+x)}$$

$$M_p = \frac{w_p L(L-x)x}{2(L+x)} = \frac{w_p(L^2x - Lx^2)}{2(L+x)}$$

For $\frac{dM_p}{d(x)} = 0$

$$0 = \frac{2(L+x)(L^2-2xL) - (L^2x - Lx^2)(2)}{4(L+x)^2}$$

$$2(L+x)(L^2-2xL) - 2L^2x + 2Lx^2 = 0$$

$$2L^3 + 2xL^2 - 4xL^2 - 4x^2L - 2L^2x + 2x^2L = 0$$

$$2L^3 - 4xL^2 - 2x^2L = 0 \quad \left(\cdot\frac{1}{2L}\right)$$

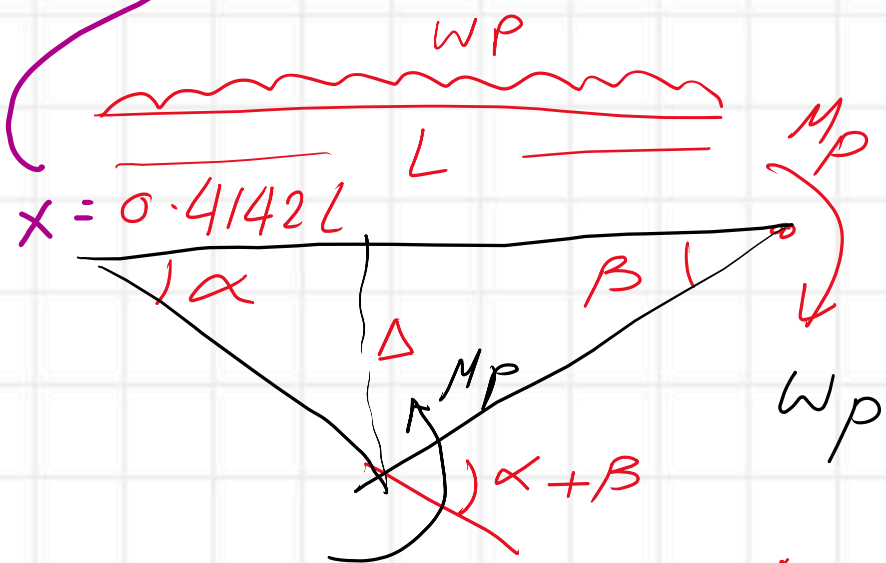
$$x^2 + 2xL - L^2 = 0$$

$$x = \left(\frac{-2 \pm \sqrt{(4+4)}}{4}\right)L = 0.4142L$$

Same as before

$x = 0.4142\,L$

Find w_p & M_p relation



$$w_p L \frac{\Delta}{2} = M_p(\alpha+\beta) + M_p(\beta)$$

$$w_p \frac{L}{2}\Delta = M_p\left(\frac{\Delta}{L}(4.1214 + 1.70707)\right)$$

$$w_p = \frac{M_p}{L^2}(11.6569)$$

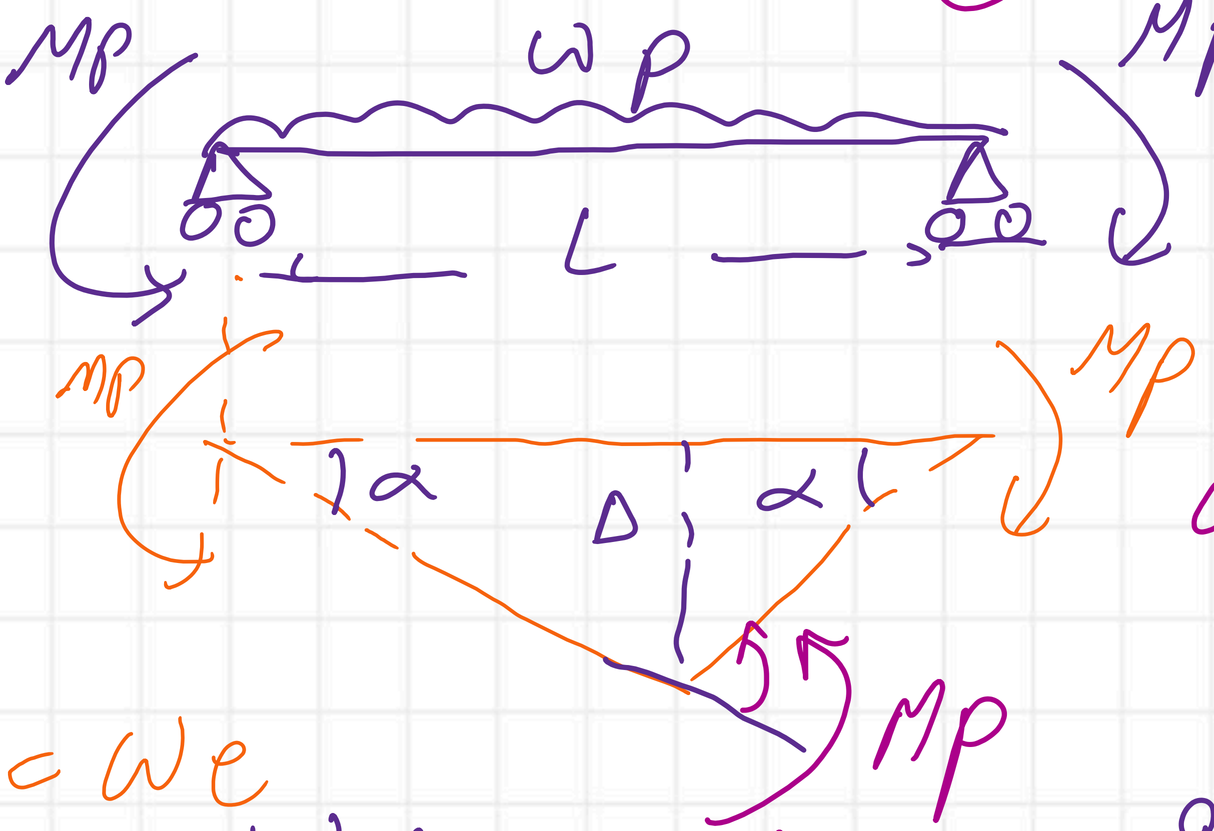
$$\Delta = 0.4142\,L(\alpha) \rightarrow \alpha = \frac{\Delta}{0.4142\,L}$$

$$\Delta = 0.5858\,L(\beta) \qquad \beta = \frac{\Delta}{0.5858\,L} = \frac{1.70707\,\Delta}{L}$$

$$\alpha+\beta = \frac{l\,\Delta}{(0.4142)(0.5858)L^2}$$

$$= 4.1214\,\Delta/L$$

Using upper bound theory for intermediate span



$$\alpha = \frac{2\Delta}{L}$$

$$W_e = w_p L \frac{\Delta}{2}$$

$$W_i = M_p(2\alpha) + M_p(2\alpha)$$

$$= 4M_p\alpha$$

$$= 4M_p\left(\frac{2\Delta}{L}\right)$$

$W_i = W_e$

$$w_p L \frac{\Delta}{2} = 4M_p\alpha = 8M_p\frac{\Delta}{L}$$

$$M_p = \frac{w_p L^2}{16}$$ Same result

$$w_p = 16M_p/L^2$$