Example # 8.22 Mccormac's book

Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.

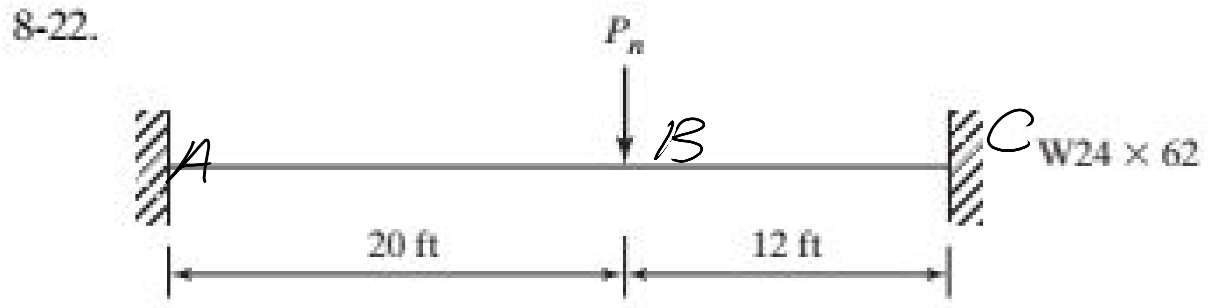


FIGURE P8-22

Nom-inal Wt.	Compact Section Criteria		Axis X-X			
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}
62	5.97	50.1	1550	131	9.23	153

study of the Sequence of Plastic Hinges and Corresponding Pn values

Example # 8.22 Mccormac's book

Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.

8-22.

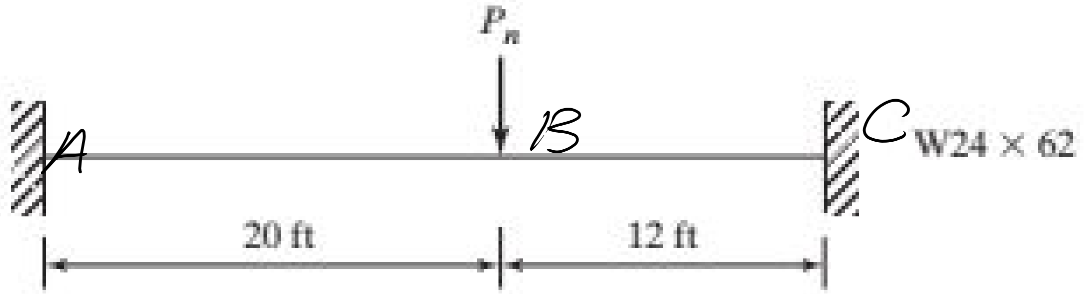


FIGURE P8-22

Nom-inal Wt.	Compact Section Criteria		Axis X-X			
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}
62	5.97	50.1	1550	131	9.23	153

Solution

A992 steel $\Rightarrow F_y = 50$ ksi

Find $\rightarrow Z_x = 153$ inch^3

Use Table 1-1 From AISC

CM #15 Pg 1-19 For W Sections

W24X62

Prepared by Eng.Maged Kamel.

How many possible plastic Joints ?

8-22.

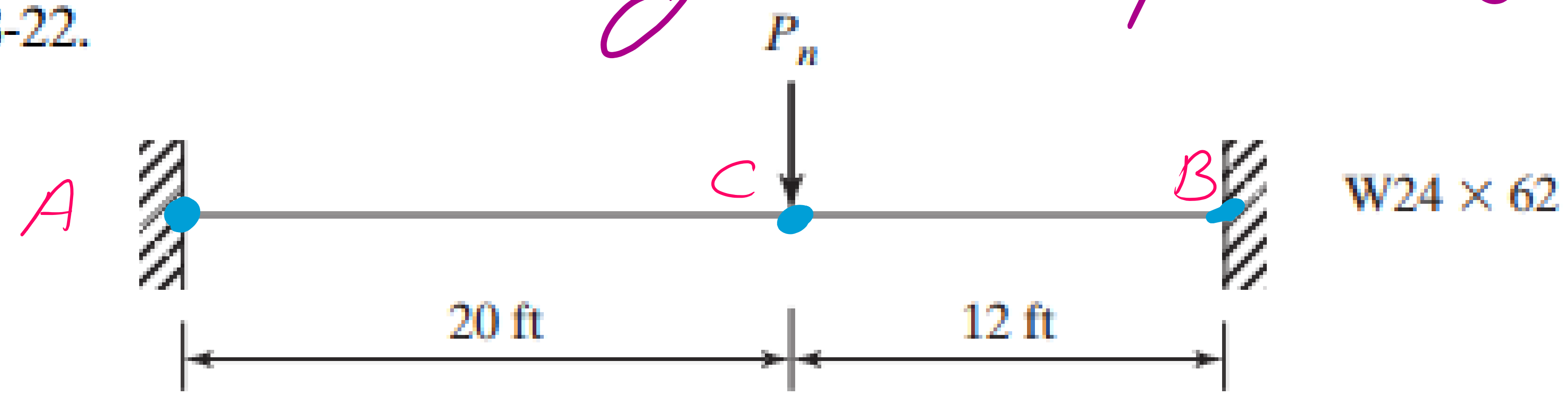
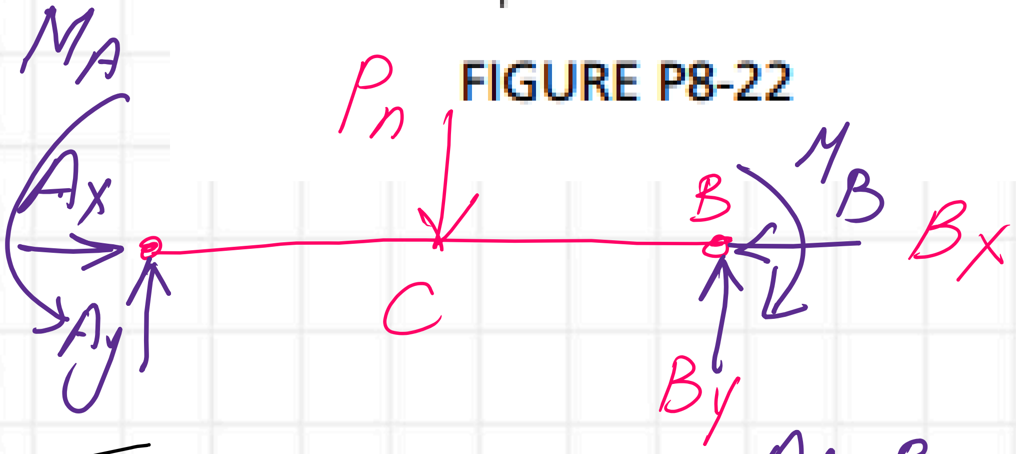


FIGURE P8-22



① Ignore HL Reactions

② Estimate Inderminancy I_D

$4 - 2 = 2$ add $1 = 3$

Four Unkowns A_y, B_y M_A, M_B

ΣY ΣM

Equilibrium Equ

3 possible plastic hinges

At A, B and C

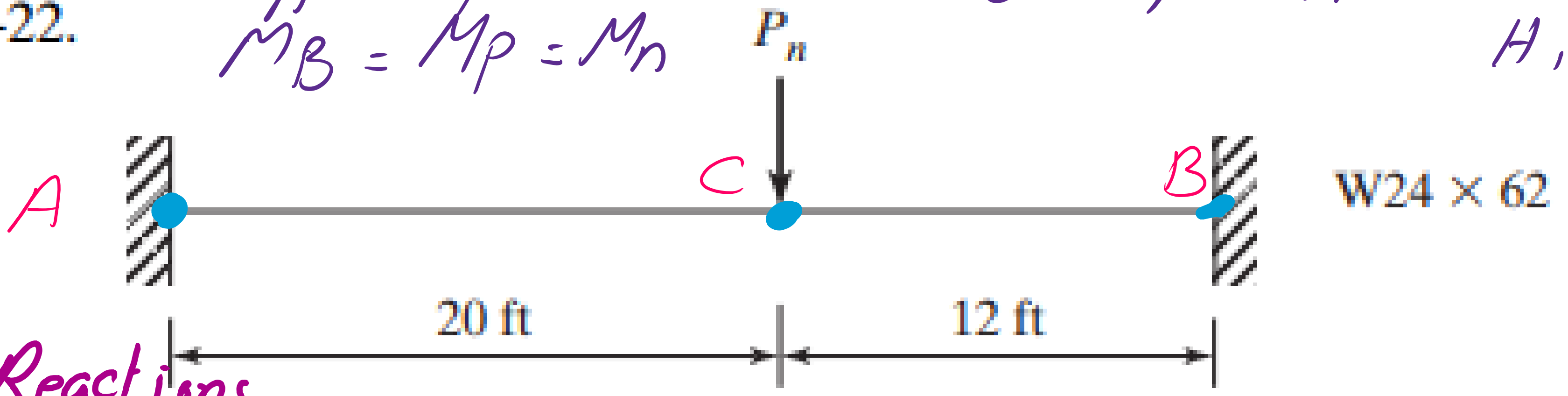
(A) Statical method

8-22.

$M_A = M_p = M_n$

$M_B = M_p = M_n$

$M_C = M_p = M_n$ Due to plastic Hinges



Find Reactions at A, B

FIGURE P8-22

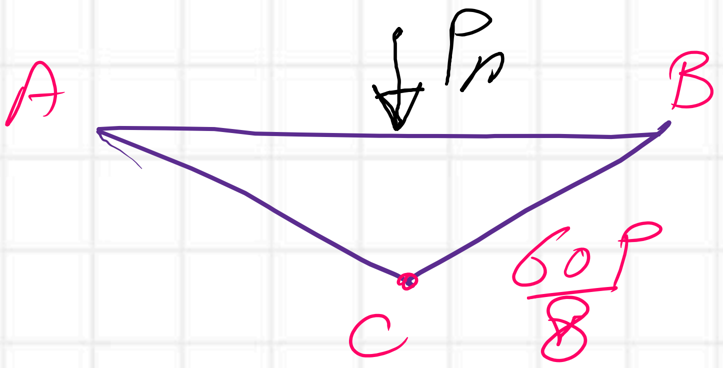
$A_y = \frac{P_n(12)}{32} = \frac{3}{8} P_n$

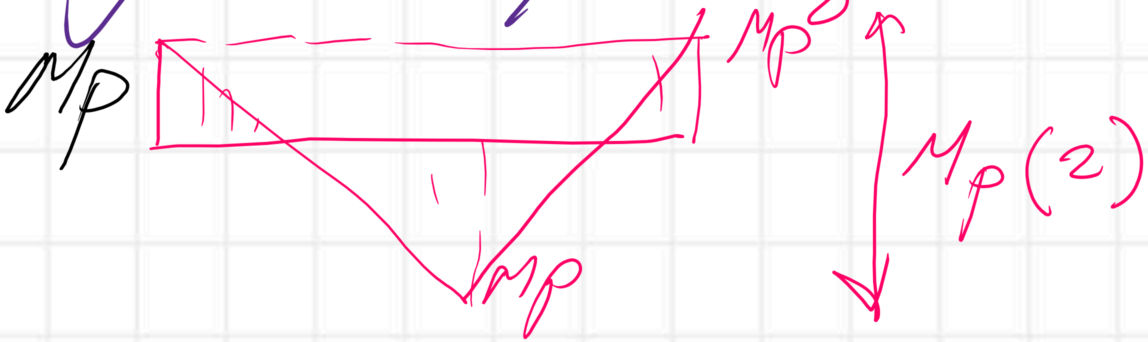
$B_y = \frac{P_n(20)}{32} = \frac{5}{8} P_n$

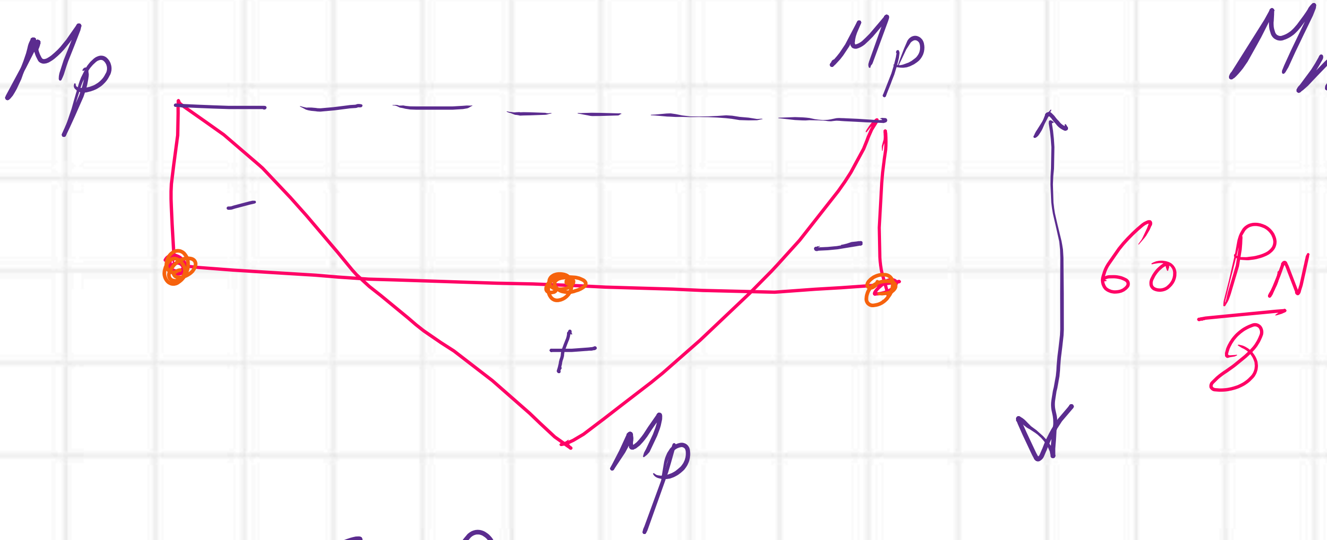
$M_C = A_y(20) = \frac{3}{8} P_n(20) = \frac{60}{8} P_n$

From left side

verify $M_C = B_y(12) = \frac{5}{8} P_n(12) = \frac{60}{8} P_n$







$M_n = M_p = F_y Z_x$

$= \frac{50(153)}{12}$

$= 637.50$ FT. kips

Equate $60 \frac{P_n}{8} = 2M_p = 2(637.50)$

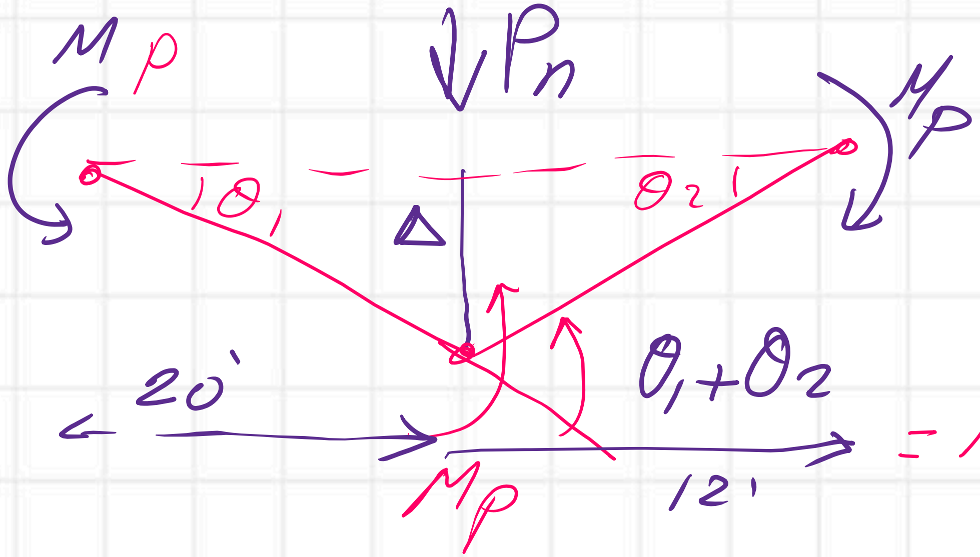
$P_n = \frac{8(2)(637.50)}{60} = 170$ kips

Nominal Load required For developing three Plastic Hinges

Prepared by Eng.Maged Kamel.

Check by Using upper bound

External work $= P_n \Delta$



Internal work $=$

$M_p \theta_1 + M_p \theta_2 + M_p(\theta_1 + \theta_2)$

$= M_p (2\theta_1 + 2\theta_2)$

$\theta_1 = \dfrac{\Delta}{20} \Rightarrow 2\theta_1 = \dfrac{2\Delta}{20}$

$\theta_2 = \dfrac{\Delta}{12} \Rightarrow 2\theta_2 = \dfrac{2\Delta}{12}$

$2\theta_1 + 2\theta_2 = \dfrac{40 + 24\Delta}{(12)(20)} = \dfrac{64\,\Delta}{240}$

$M_p = 637.50$ FT. kips

$P_n \Delta = M_p \dfrac{(64\,\Delta)}{240}$

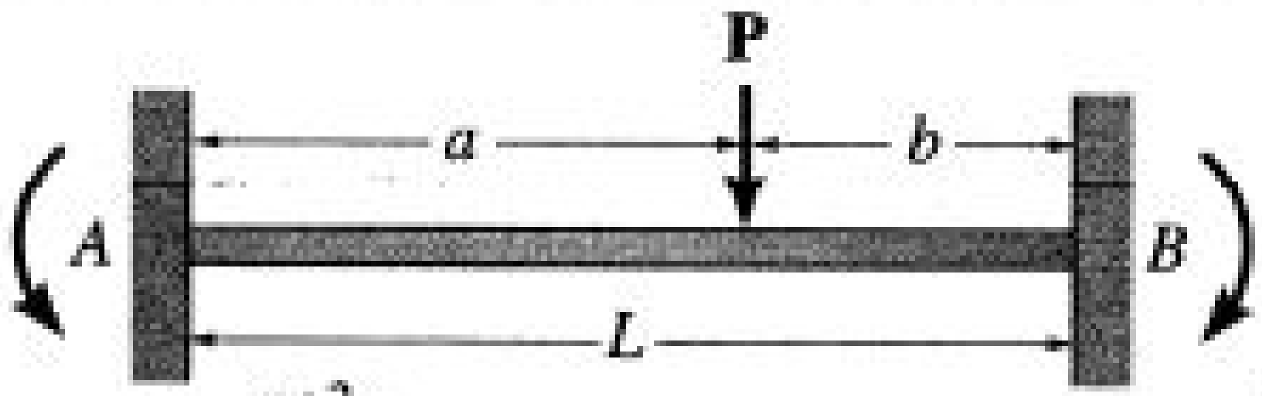
$P_n = \dfrac{4}{15} M_p = \dfrac{4}{15}(637.50)$

$= 170$ kips

matches with Lower bound

Prepared by Eng.Maged Kamel.

Fixed end moment values



$$(FEM)_{AB} = \frac{Pb^2a}{L^2} \qquad (FEM)_{BA} = \frac{Pa^2b}{L^2}$$

$M_B > M_A \quad a > b$

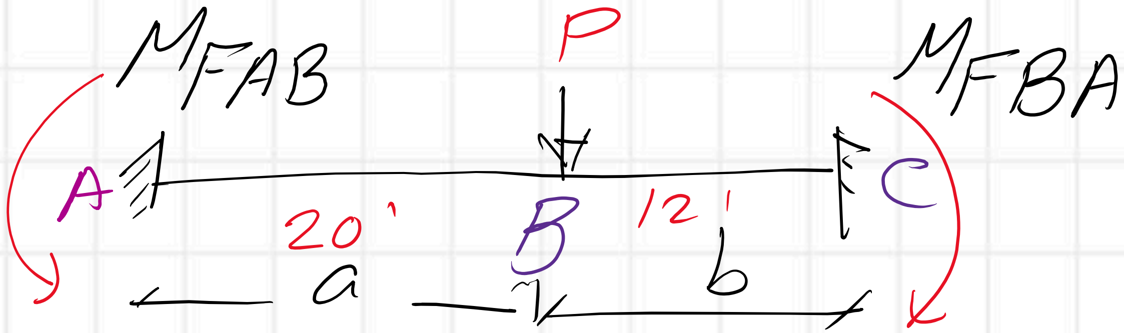
Case of Load P acting at a distance a from the Left support

$a = 20'$
$b = 12'$

$L = a + b = 32'$

First: Check Fixed end moments For $P < P_n$

$F < F_y$

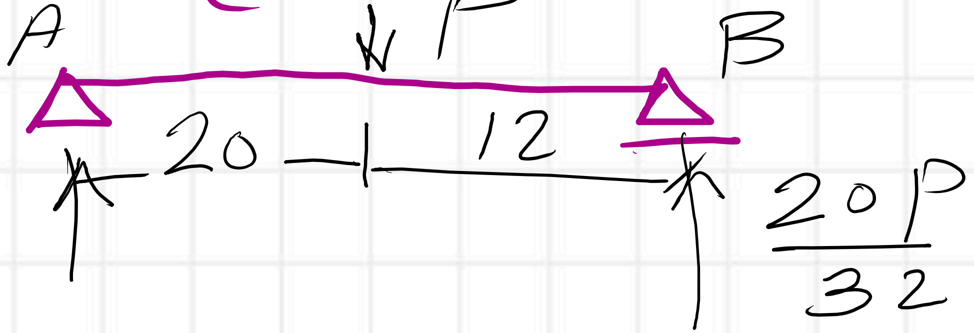


$$M_{FA\,left} = \frac{Pb^2a}{L^2} = \frac{P(12)^2(20)}{(32)^2} = \frac{45}{16}P$$

$$M_{FC\,R} = \frac{Pa^2b}{L^2} = \frac{P(20)^2(12)}{(32)^2} = \frac{75}{16}P$$

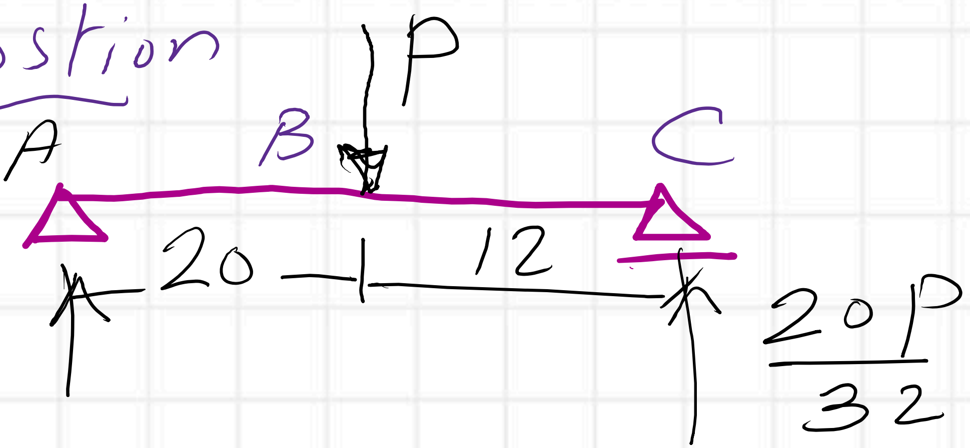
Find M_B

Find $M_{Positive}$



+ve

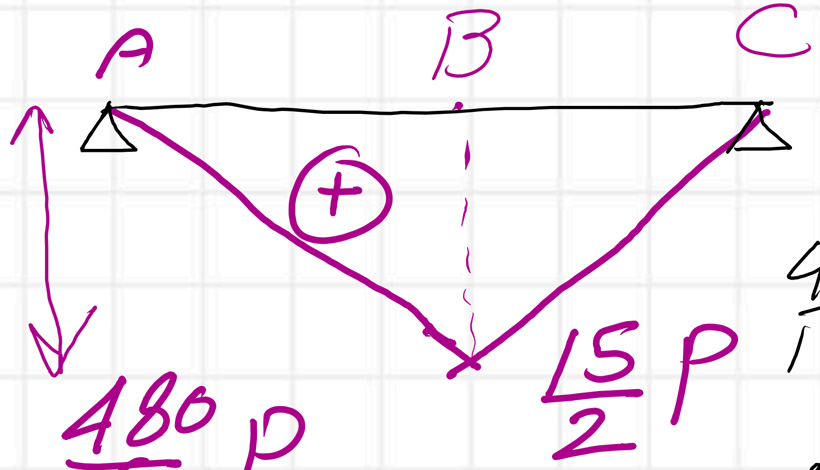
Using supur postion

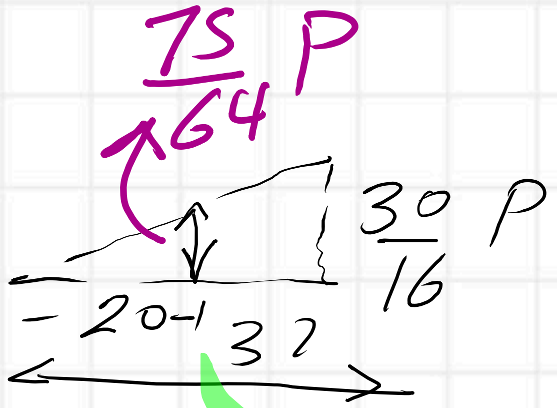


$$M_C = \frac{12P(20)}{32}$$

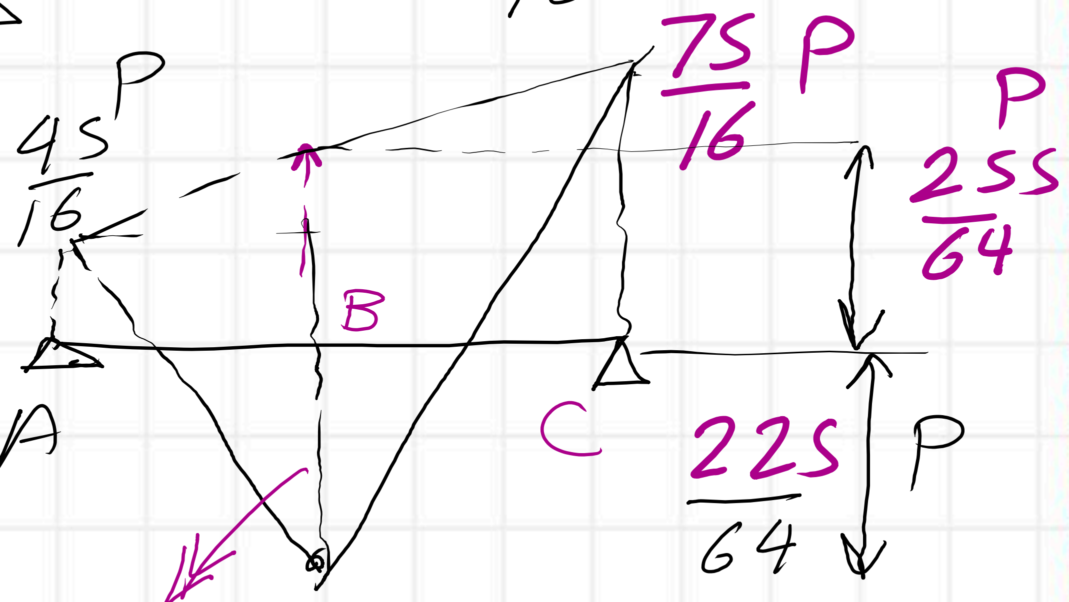
$$= \frac{240P}{32}$$

$$= \frac{15}{2}P \Rightarrow \frac{480}{64}P$$





add $\frac{45}{16}P$

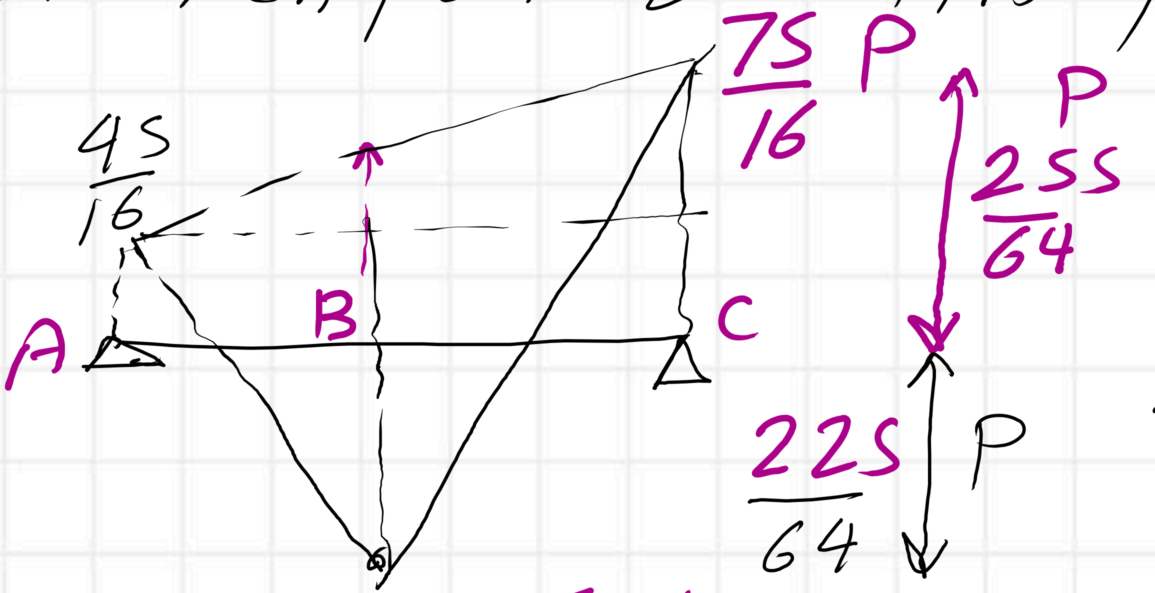


$\simeq 3.516 P$

Draw Find BM

Prepared by Eng.Maged Kamel.

In which point the First plastic Hinge will develop?



$M_C > M_B > M_A$

$M_C = \frac{75}{16} P \Rightarrow \frac{300 P}{64}$

$M_B = \frac{225 P}{64}$

$M_C = \frac{45}{16} P \Rightarrow \frac{180 P}{64}$

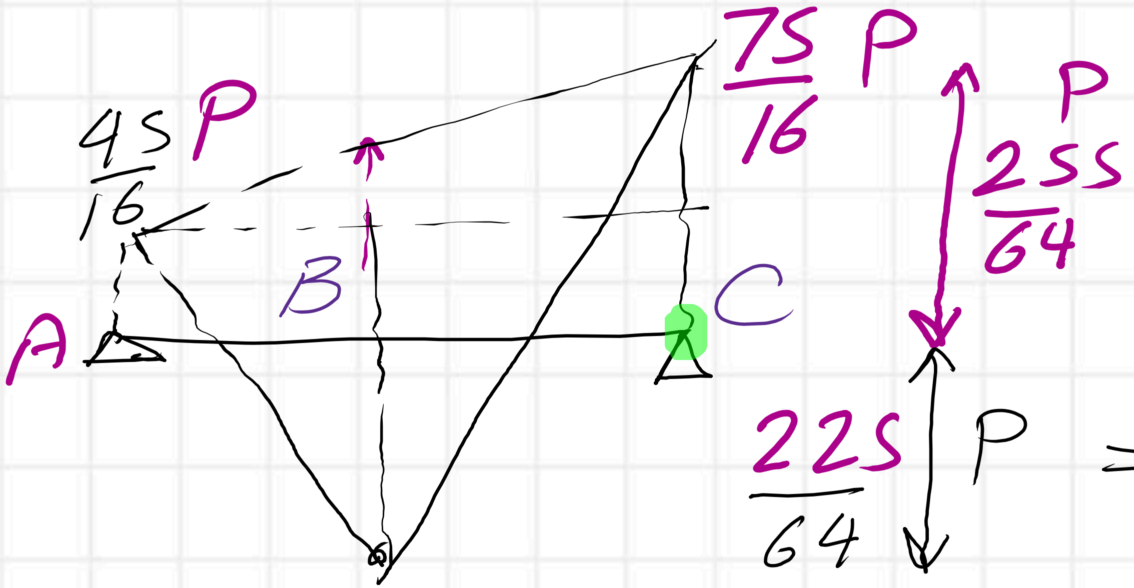
What is the First Collapse Load value?

Equate $\frac{75 P}{16} = M_p = 637.5$ FT.Kips

$P = \frac{16}{75}(637.5) = 136$ Kips → P. Hinge at B ⇒ First Load for P.H

For $P = 136$ Kips $\Rightarrow$ First plastic hinge is developed at $\boxed{C}$

Find M_B & M_C values

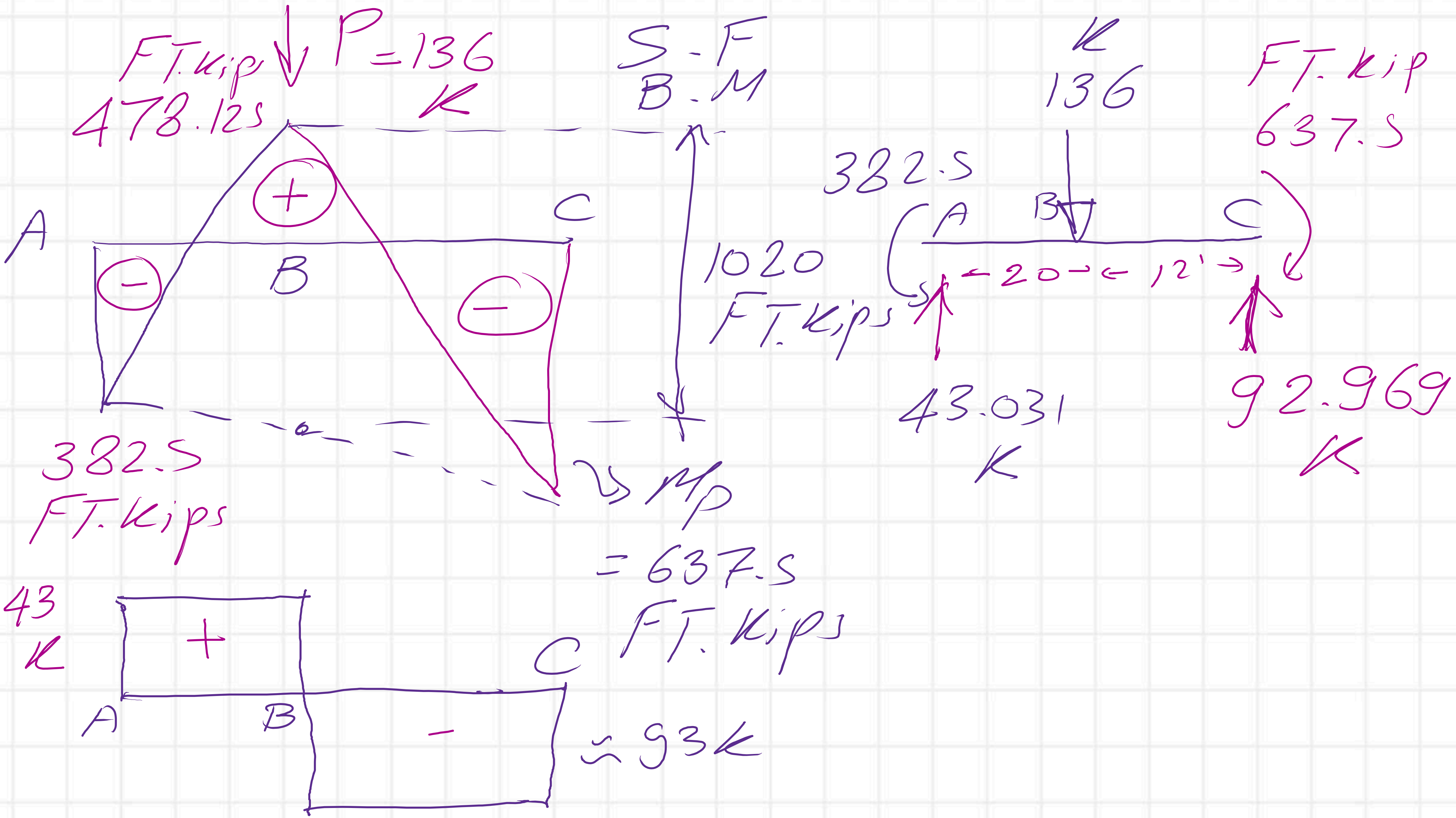


$M_p = 637.50$ FT.Kips

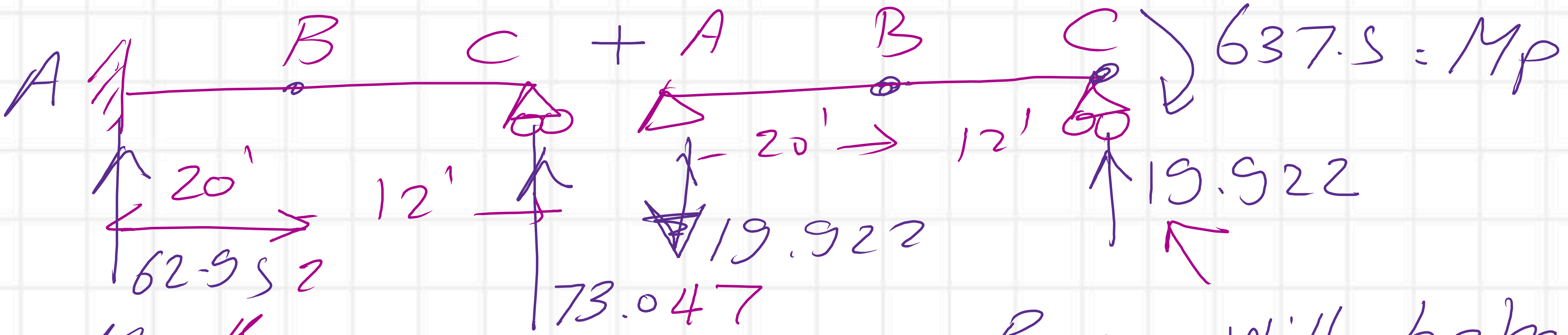
$\Rightarrow M_B = \frac{225}{64}(136) = 478.125$ FT. Kips < 637.50 FT.Kips

$M_A = \frac{45}{16}(136) = 382.50$ FT.Kips

M_A & $M_B < M_P$



FT.kip
P = 136 K
478.125
S.F
B.M
A
B
C
1020 FT.Kips
382.5 FT.Kips
Mp = 637.5 FT.Kips
43 K
A
B
C
≃ 93K
K 136
FT.kip 637.5
382.5
A
B
C
20
12'
43.031 K
92.969 K



$M_C \overset{K}{=}$

$M_B = 478.125$ FT.kips

$< M_p \rightarrow 637.5$ FT. kips

Beam will behave as a propped Cantilever

ΔM

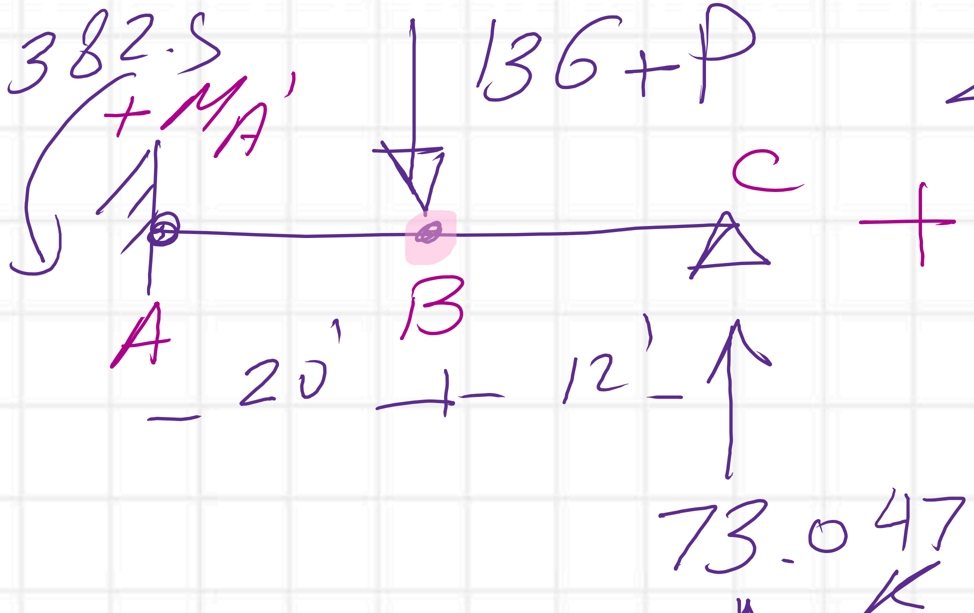
Increase Load

Difference $M_p - M_B$

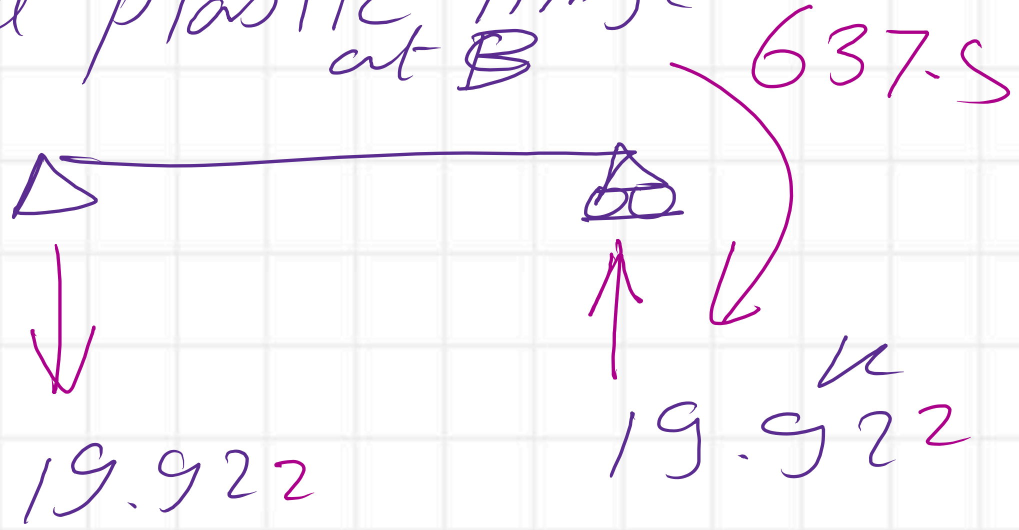
$637.5 - 478.125 = 159.375$ FT.kips

$\rightarrow /12 = 13.281K$ at B $\leftarrow$ We need to add

2nd Plastic Hinge at B



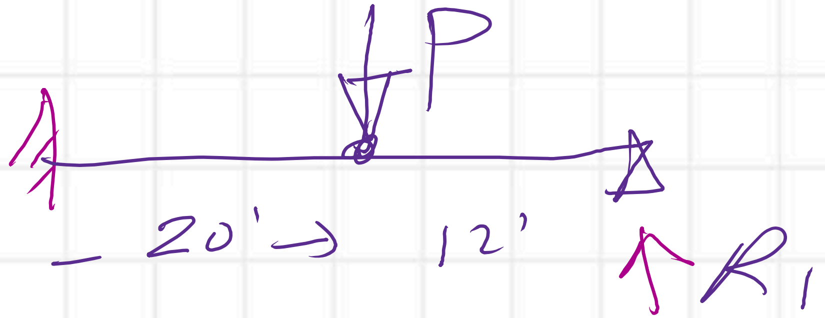
+



R_1 13.281 K

Find P

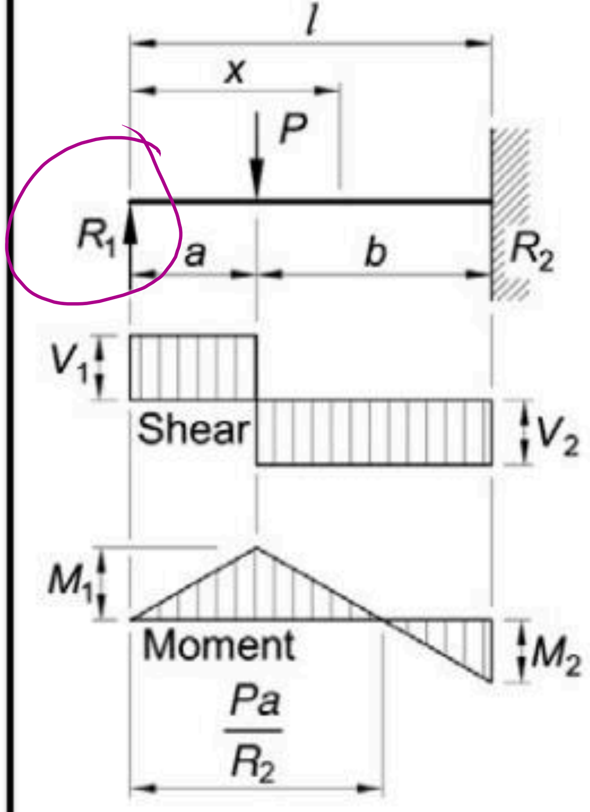
Reaction at C = 13.281 kips



Refer to Table

Increase Load $\Rightarrow R_1 = 13.281K = \dfrac{Pb^2}{2L^3}(a+2L)$

$P =$



$R_1 = V_1$ .... $= \dfrac{Pb^2}{2l^3}(a+2l)$

$R_2 = V_2$ .... $= \dfrac{Pa}{2l^3}(3l^2 - a^2)$

M_1 (at point of load) .... $= R_1 a$

M_2 (at fixed end) .... $= \dfrac{Pab}{2l^2}(a+l)$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $x > a$) .... $= R_1 x - P(x-a)$

Δ_{max} $\left[\text{when } a < 0.414l \text{ at } x = l\dfrac{(l^2+a^2)}{(3l^2-a^2)}\right]$ .. $= \dfrac{Pa}{3EI}\dfrac{(l^2-a^2)^3}{(3l^2-a^2)^2}$

Δ_{max} $\left(\text{when } a > 0.414l \text{ at } x = l\sqrt{\dfrac{a}{2l+a}}\right)$ ..... $= \dfrac{Pab^2}{6EI}\sqrt{\dfrac{a}{2l+a}}$

Δ_a (at point of load) .... $= \dfrac{Pa^2b^3}{12EIl^3}(3l+a)$

Δ_x (when $x < a$) .... $= \dfrac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$

$b = 20'$

$a = 12'$

$L = 32$

hinge length

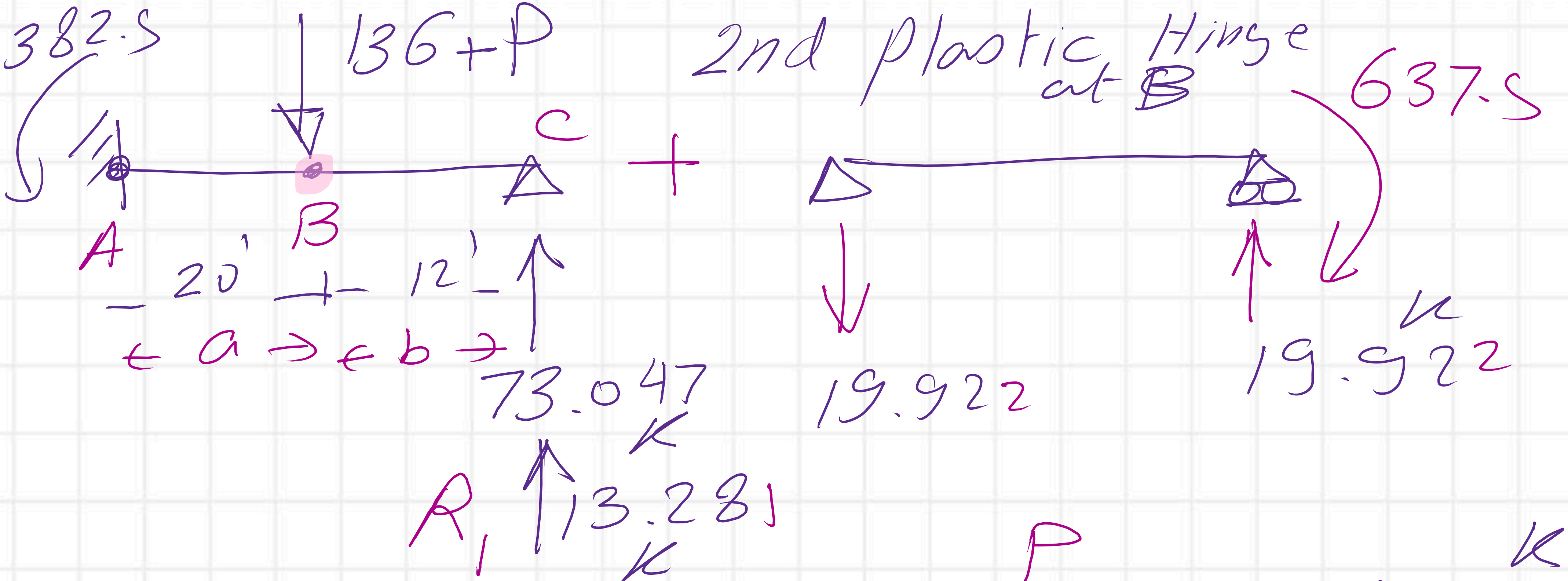
L^3

R_1

$$P = \frac{2(32)^3(13.281)}{(20)^2(12+64)}$$

$= 28.631$ Kips

2nd Plastic Hinge at B



$$P = \text{2nd Load} = 136 + \underbrace{28.629}_{P} = 164.629^{K}$$

Check if $P = 28.629$

$$\Delta M = \frac{Pab(b+L)}{2L^2} = \frac{28.629(20)(12)(44)}{2(32)^2} = 147.618 \text{ Ft.Kips}$$

Fixed $\qquad M_{Fixed} =$

Summary For

First Plastic hinge at C $\Rightarrow M_p = 637.5$ FT. kips $P = 136$ kips

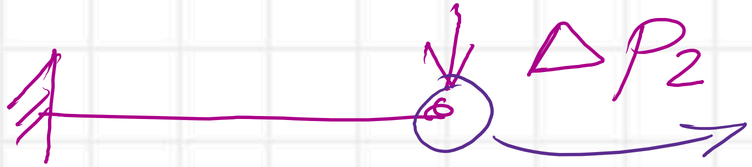
From Fixed To propped $\Rightarrow M_p = 637.5$ FT. kips $P = 164.629$ kips

2nd Plastic hinge at B

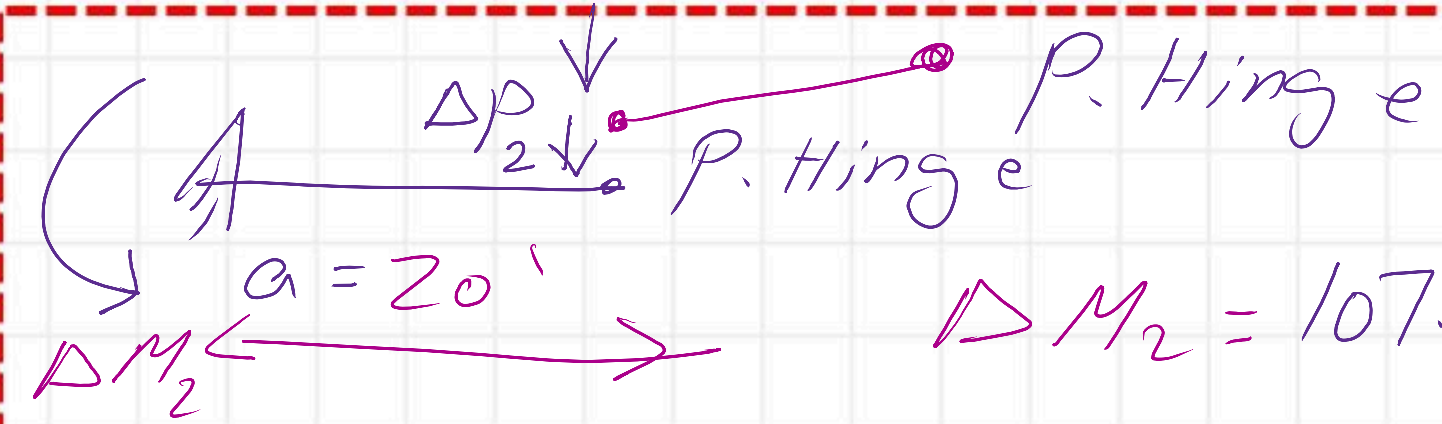
Moment at A $= 382.5 + 147.618 = 530.118$ FT. kips

We need more Load For P.H at A

$\Delta M = 637.5 - 530.118 = 107.381$ FT. kips



$\rightarrow$ Plastic hinge



$\Delta M_2 = 107.381$ FT. kips

$\Delta P_2 (20) = 107.381$

$\Delta P_2 = 5.369$ kips

MA

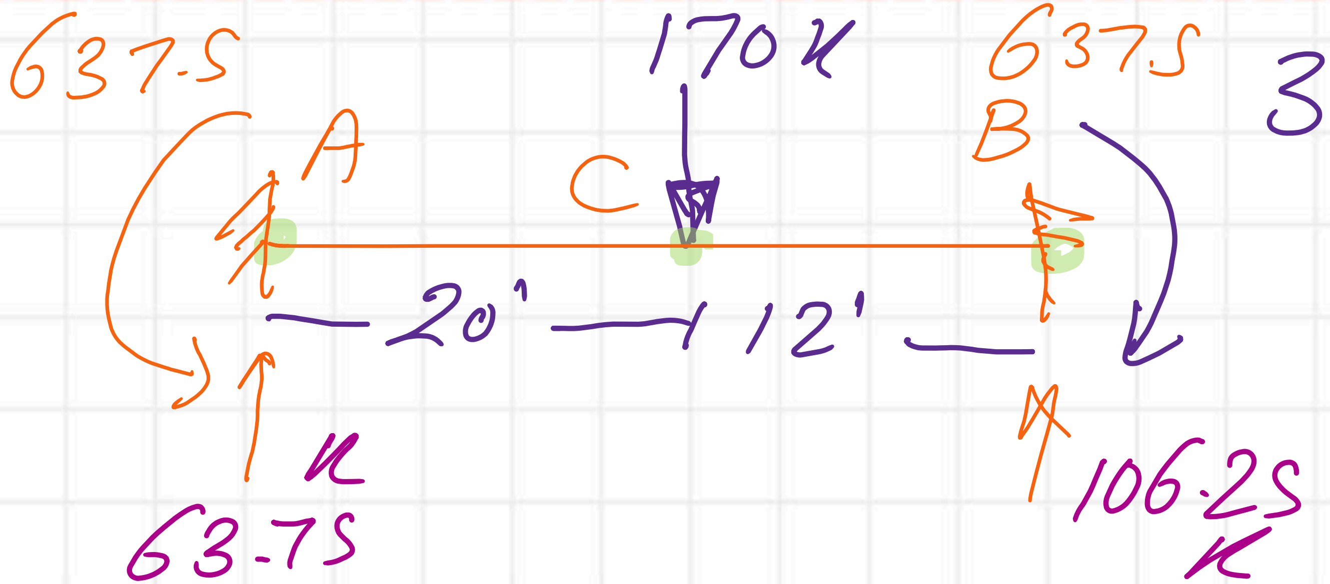
① 136 K First hinge at C → 382.5 FT. kips

② 164.629 K 2nd at B → 530.118 FT. kips

PL

③ 169.9981 kips ≃ 170 kips → 3rd Hinge A

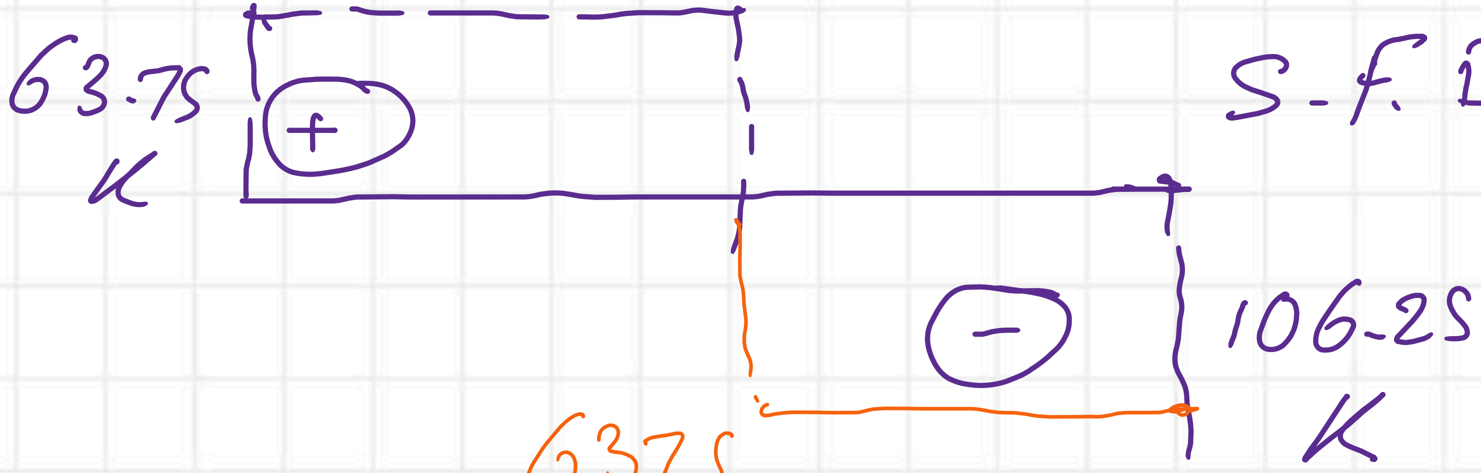
Same as Estimated before



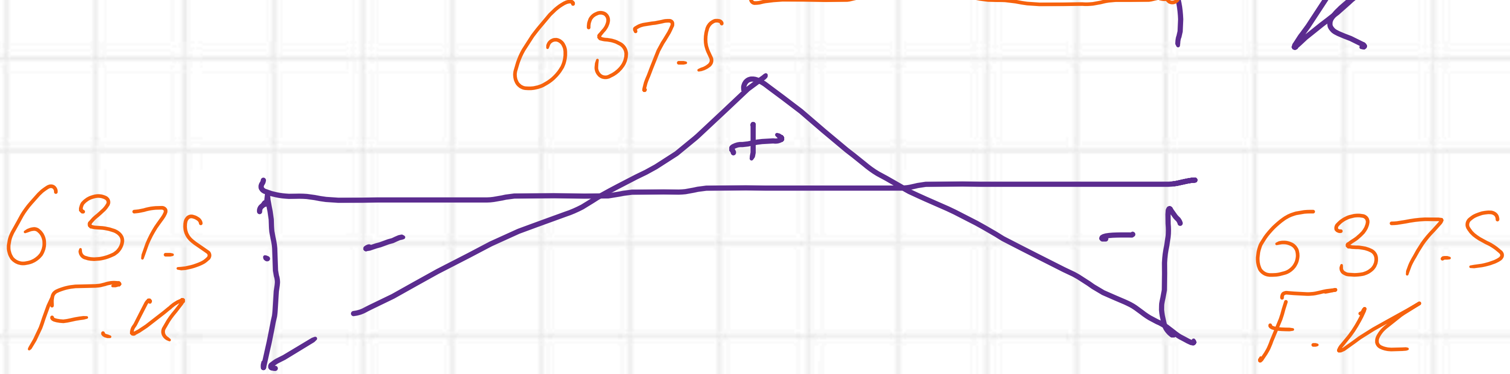
3ree Plastic hinges

at A, B, C

$Z_x = 153$
inch^3



S.F.D



B.M