Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.

Solution

A992 steel
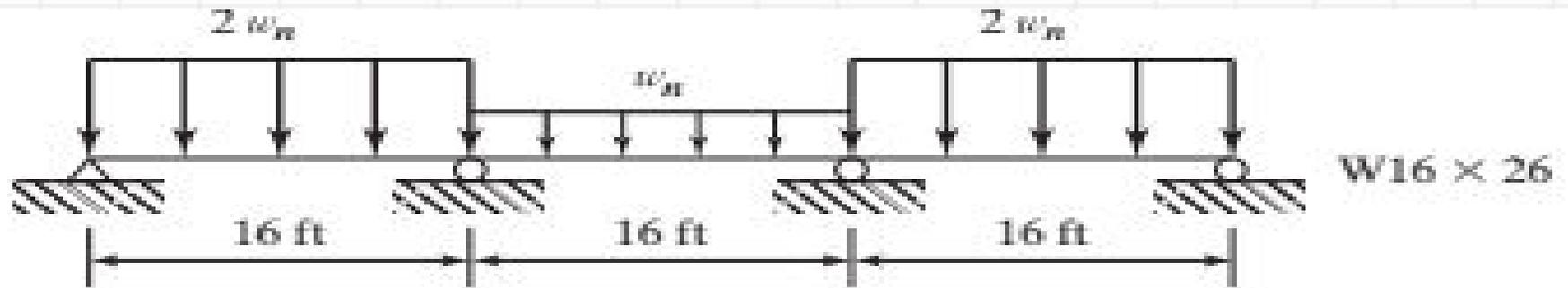

2 w_n

w_n

2 w_n

W16 × 26

16 ft   16 ft   16 ft

FIGURE P8-33

Section W16 X 26

Find M_n

$Z_x = 44.20$ inch^3

$F_y = 50$ kSi

$M_p = f_y Z_x = 50 k \frac{lb}{inch^2} \cdot (44.20) inch^3$

$M_n = M_p = 2210$ Kips. inch

or $= \frac{2210}{12} = 184.167$ kips. FT

A992 steel

Shape	Steel Type		
	ASTM Designation	F_y, ksi	F_u, ksi
Wide flanged beams	A992	50–65	65
Miscellaneous beams	A36	36	58–80
Standard beams	A36	36	58–80
Bearing piles	A572 Gr. 50	50	65
Standard channels	A36	36	58–80
Miscellaneous channels	A36	36	58–80
Angles	A36	36	58–80
Ts cut from W-shapes	A992	50–65	65
Ts cut from M-shapes	A36	36	58–80
Ts cut from S-shapes	A36	36	58–80
Hollow structural sections, rectangular	A500 Gr. B	46	58
Hollow structural sections, square	A500 Gr. B	46	58
Hollow structural sections, round	A500 Gr. B	42	58
Pipe	A53 Gr. B	35	60

Note: F_y = specified minimum yield stress; F_u = specified minimum tensile strength

W-Shapes   M-Shapes   S-Shapes   HP-Shapes   C-Shapes

L-Shapes   WT-Shapes   ST-Shapes   HSS-Shapes   Pipe

Prepared by Eng.Maged Kamel.

8.33

Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.
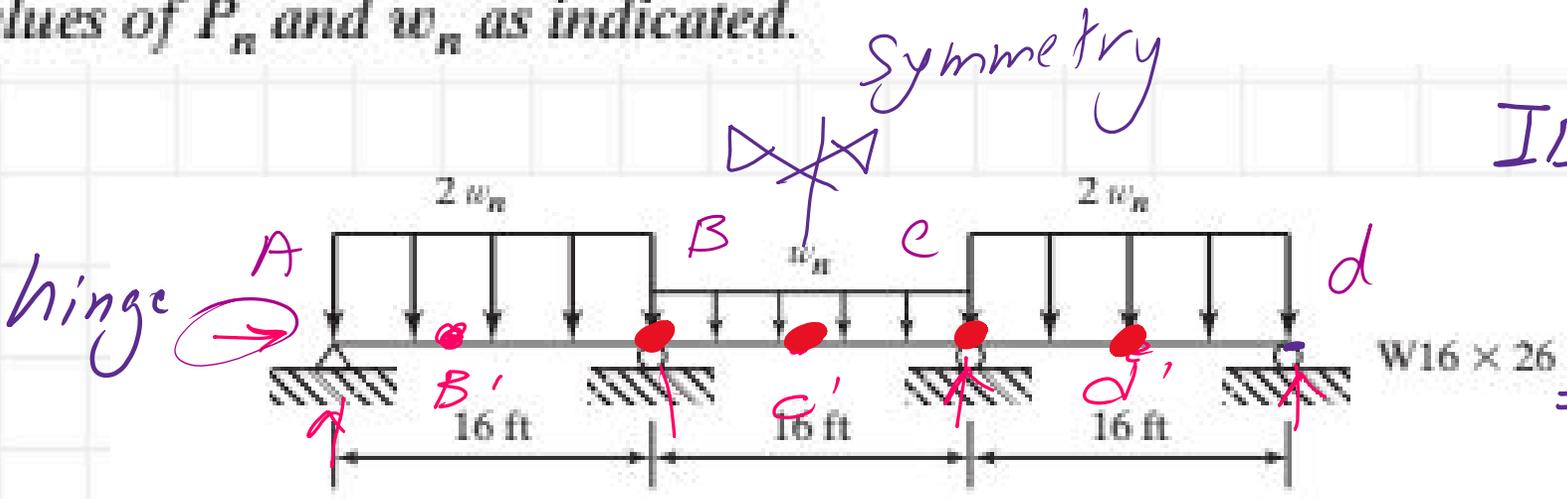
Symmetry

hinge

$2w_n$ A   B w_n e $2w_n$ d

B'   C'   d'   W16 × 26

16 ft   16 ft   16 ft

FIGURE P8-33

ID: Number of Indeterminacy

$= 4 - 2 = 2$

$\Sigma y = 0$
$\Sigma \mu = 0$

Except All Loads   same result

$ID = (2m + r) - 2n$

$= 2(3) + 4 - 2(4) = 2$

Possible hinge Locations: $B', B, C', C, d' = 5$

Number of Mechanism $= 5 - 2 = 3$

$\begin{array}{ccc} A & B' & B \\ B & B' & C \\ C & d' & d \end{array}$ Same
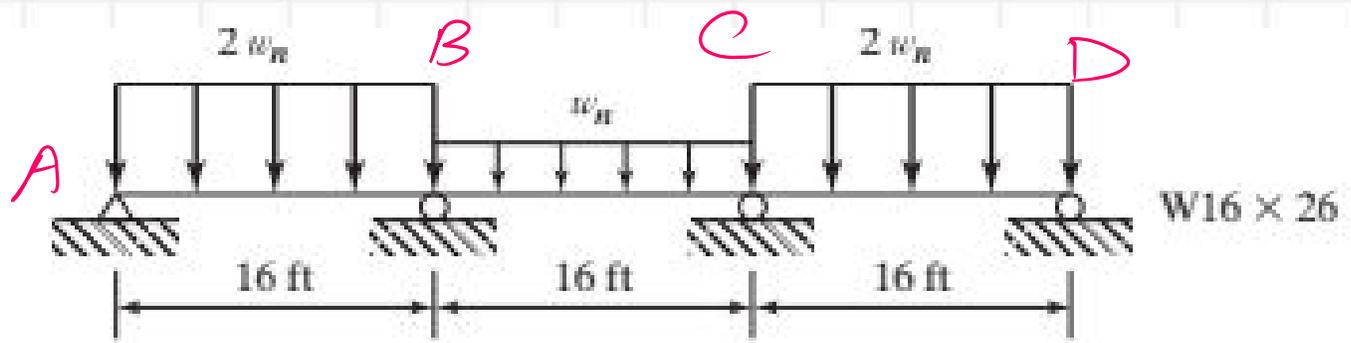
Prepared by Eng.Maged Kamel.

FIGURE P8-33

2 w_n B   C   2 w_n D

w_n

A

W16 × 26

16 ft   16 ft   16 ft

Real hinge

Possible Mechanisms

A   B

1st Mechanism
3ree plastic hinger

B'

same
results

B   C

2nd Mechanism

C'

Real hinge

C   D

3rd mechanism

D'

FIGURE P8-33

Labels on figure: $2w_n$, B, C, $2w_n$, d, A, w_n, W16 × 26, 16 ft, 16 ft, 16 ft

First panel

$$R_A = 2w_n\left(\frac{\ell}{2}\right) - \frac{M_P}{L}$$
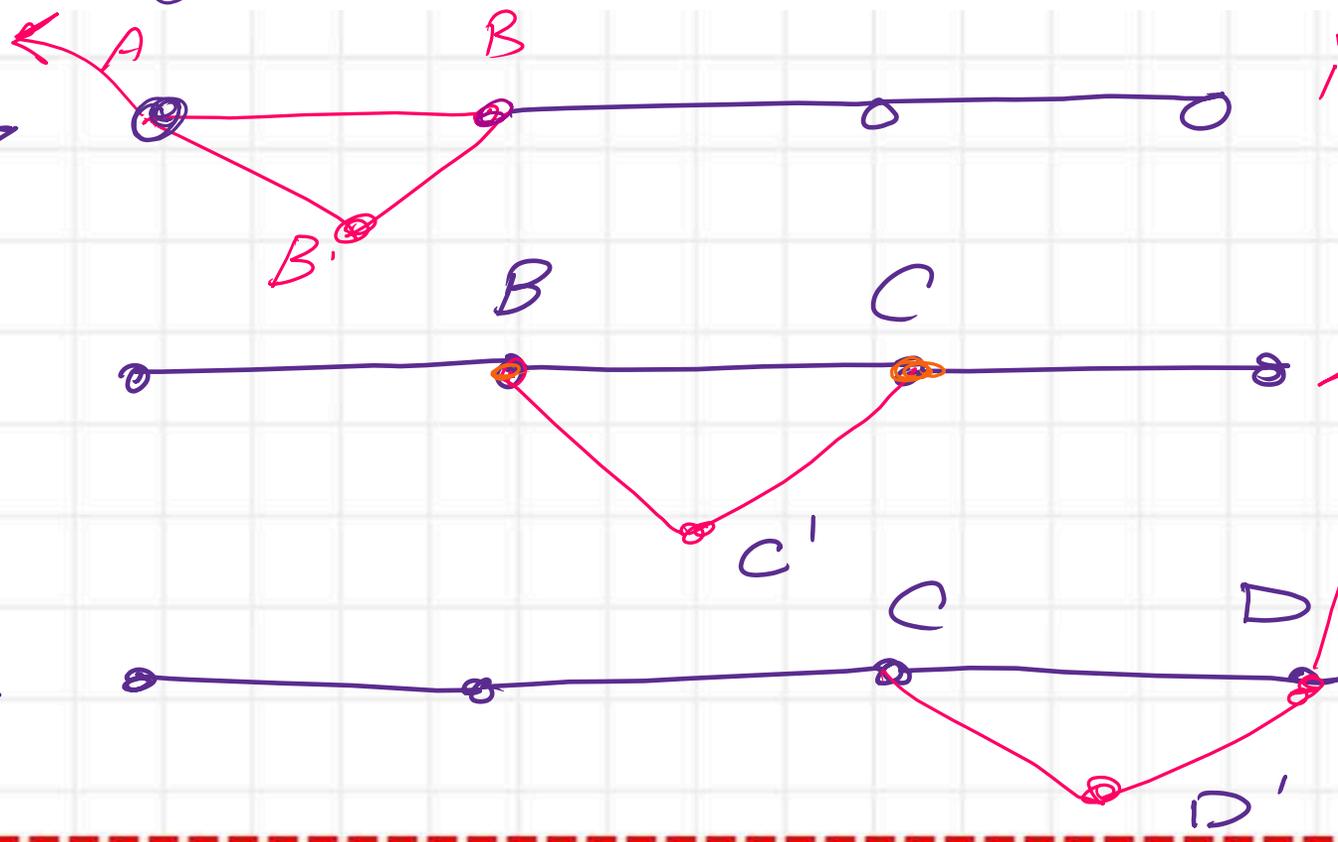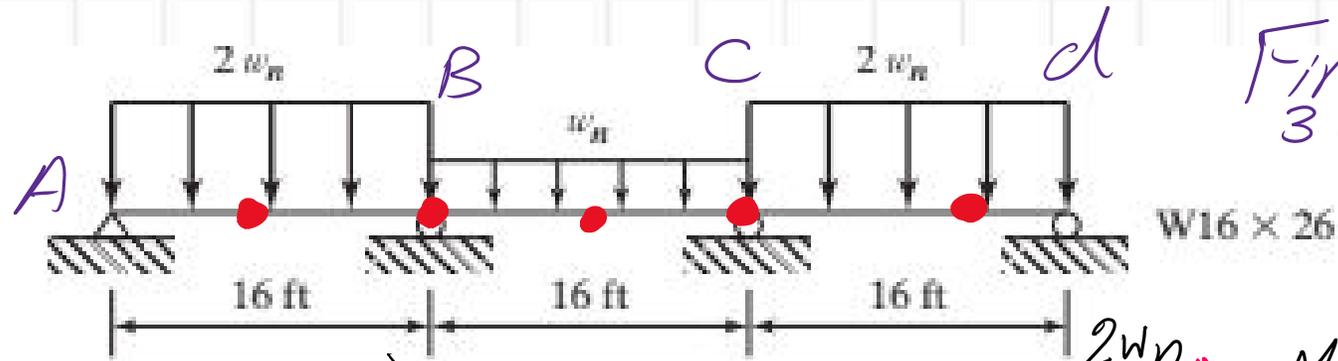
First panel $L \Rightarrow 2$ P. Hinges
3rd

Derive the expression
For x distance
For shear = 0
$$dM/dx = 0$$

$2w_n$, M_P

$\uparrow 2w_n\,\ell/2$

$\downarrow \dfrac{M_P}{L}$
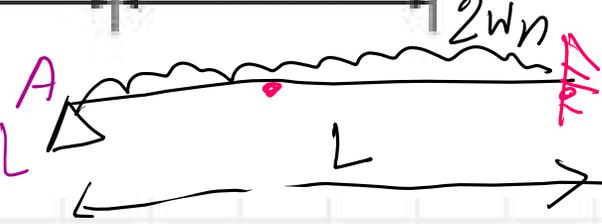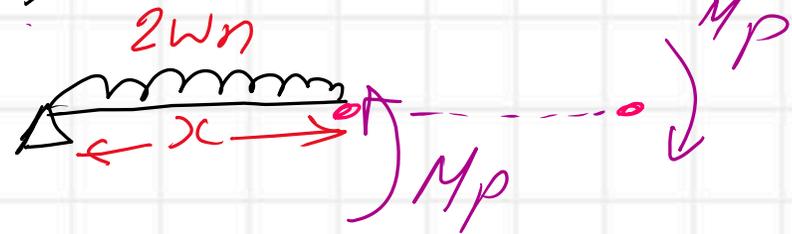
$2w_n$, $\leftarrow x \rightarrow$, M_P, M_P

For $Q = 0 \Rightarrow$ $w_n\ell - \dfrac{M_P}{L} = 2w_n(x)$

$$\underline{M_P} = w_n\ell - 2w_n x \Rightarrow \text{Expression for } M_P$$
$$L\quad M_P = w_n L^2 - 2w_n x L$$

For $M_x = M_P$ L.H.S $\Rightarrow$ $w_n Lx - \dfrac{M_P}{L}(x) - w_n x^2$

R.H.S $\Rightarrow$ M_P

FIGURE P8-33

$2w_n$ B   C $2w_n$ D
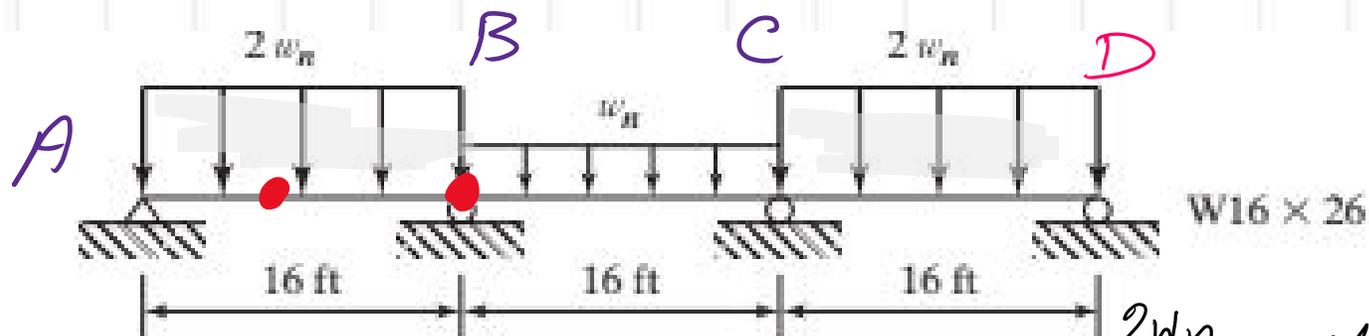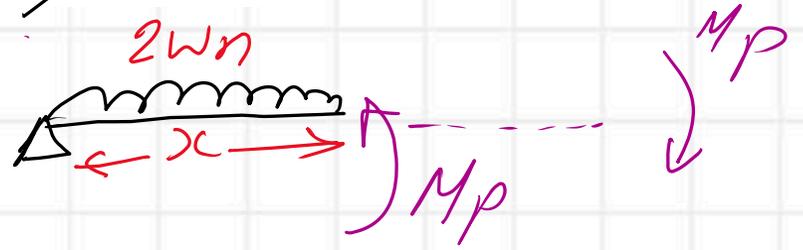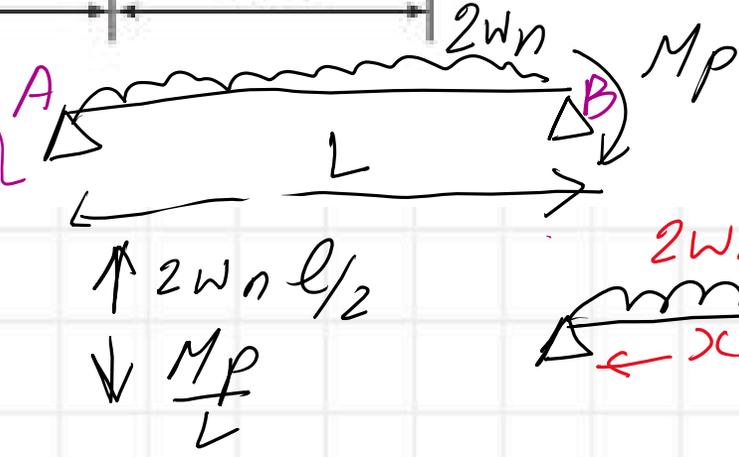
A w_n W16 × 26

16 ft   16 ft   16 ft

Continue First and third

Derive the expression
For x distance
For shear = 0

First panel

$$w_n L x - M_p\left(\frac{x}{L}\right) - w_n x^2 = M_p$$

$2w_n \ell/2 \uparrow$

$\frac{M_p}{L} \downarrow$

$$M_p + M_p\left(\frac{x}{L}\right) = -w_n x^2 + w_n L x \rightarrow \text{but } M_p = w_n L^2 - 2w_n x L$$

$$w_n L^2 - 2w_n x L + w_n L x - 2w_n x^2 = -w_n x^2 + w_n L x$$

$$w_n x^2 + 2 w_n x L - w_n L^2 = 0$$

$$x^2 + 2 x L + L^2 = 0$$

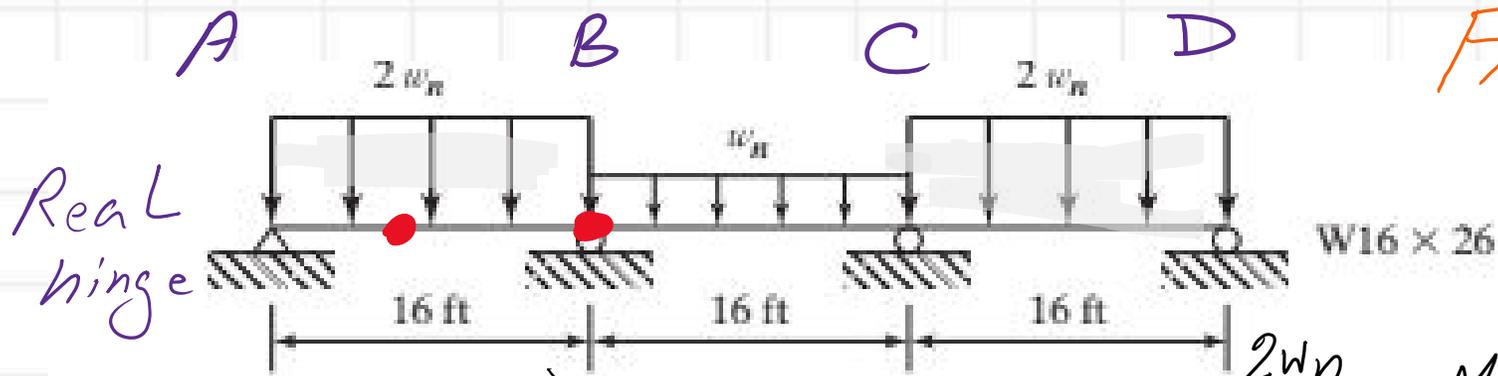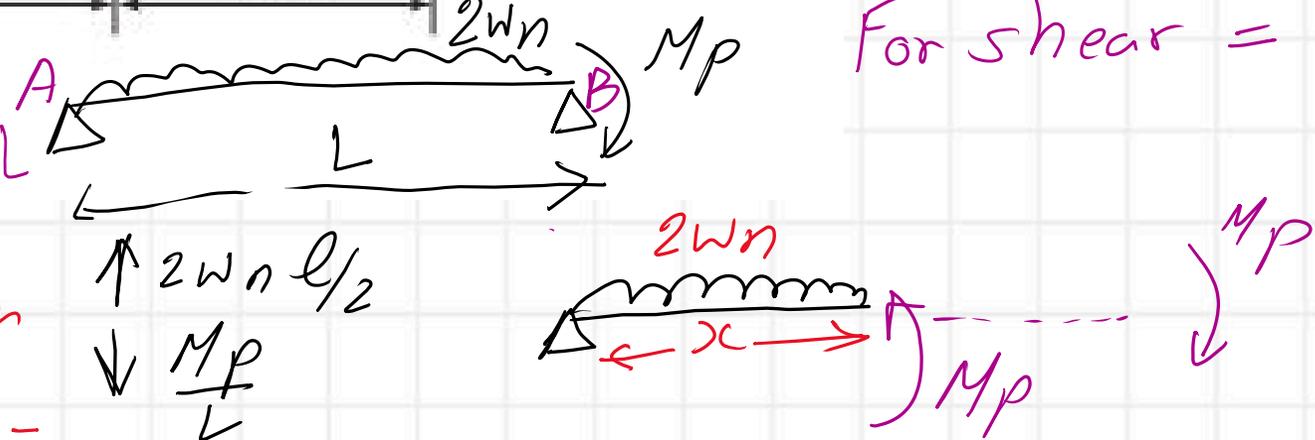Divide
w_n

$$x = \frac{-2 \pm \sqrt{4+4}}{2} = 0.414 \ L$$

A   B   C   D

$2w_n$ w_n $2w_n$

Real
hinge

16 ft   16 ft   16 ft

W16 × 26

FIGURE P8-33

First panel

Derive the expression
For x distance
For shear = 0

A ⟿⟿⟿⟿ $2w_n$ B M_P

L

Back to the Expression
For shear, put x =

↑ $2w_n \ell/2$
↓ $\frac{M_P}{L}$

0.414   L

$2w_n$
← x → M_P
M_P

$W_n L - 2W_n X - \frac{M_P}{L} = 0$

$W_n \ell - \frac{184.167}{L} - 2W_n (0.414 \ L) = 0$

$\frac{43}{250}$ $\frac{W_n L^2}{L^2} = 184.167 \Rightarrow W_n = \frac{184.167}{(16)^2} \frac{(250)}{43} = 4.193$

$M_P = 184.167$ Ft.kips
$\ell = 16'$

Kips / Ft

Prepared by Eng.Maged Kamel.

$2w_n$ $2w_n$ $2w_n$ $3rd$ **upper bound**

$2w_n$ w_n $2w_n$ W16 × 26

16 ft   16 ft   16 ft

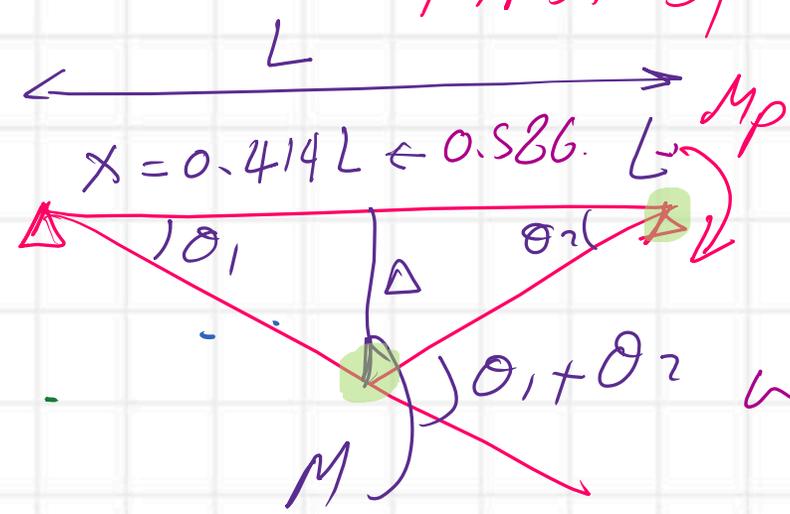**FIGURE P8-33**

$M_p = 184.167$

F.T. kips

$\dfrac{2w_n}{L/2}$ + $\dfrac{2w_n}{L/2}$

w_nL w_nL

$\dfrac{\Delta}{2}$ $\dfrac{\Delta}{2}$

First span   or   third span upper bound

$2w_n$ → M_p

Δ $X = 0.414 L$

L

$X = 0.414 L$ ← $0.586 L$ → M_p

θ_1 Δ θ_2

$\theta_1 + \theta_2$

M

P
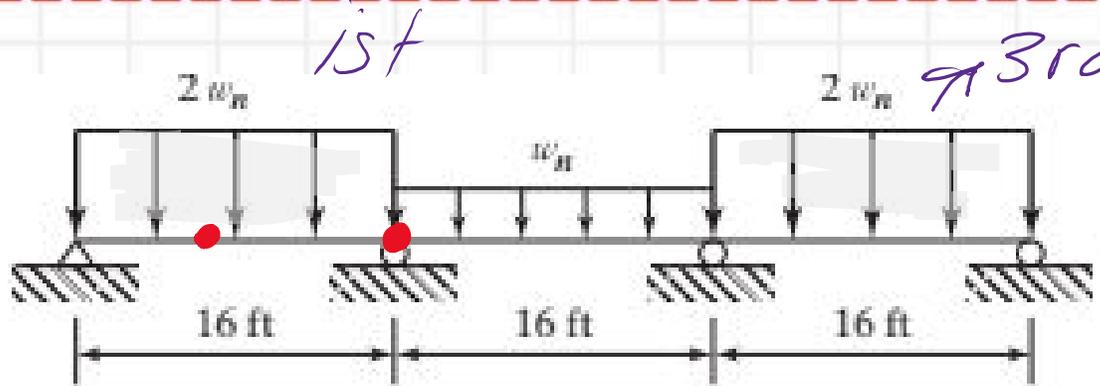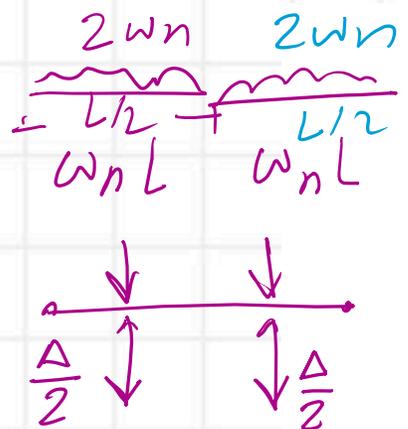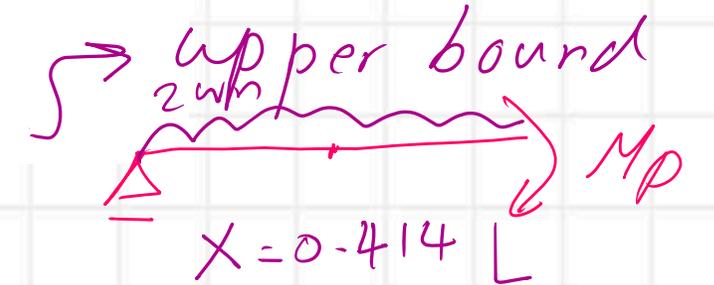
$\theta_1 = \dfrac{\Delta}{0.414 L}$

$\theta_2 = \dfrac{\Delta}{0.586 L}$

External work $= w_n L \dfrac{\Delta}{2}$

internal work $= M_p(\theta_1 + \theta_2 + \theta_2)$

$w_n L \Delta = M_p \left(\dfrac{5.8284}{L}\right)\Delta$

$\theta_1 + \theta_2 = \dfrac{4.1219}{L}\Delta$

$\theta_2 = \dfrac{1.7065}{L}\Delta$

$l = 16 m$

$M_p = 184.167$

$w_n = \dfrac{(184.167)(5.8284)}{(16)^2}$

$= 4.193 \ K/F$

Prepared by Eng.Maged Kamel.
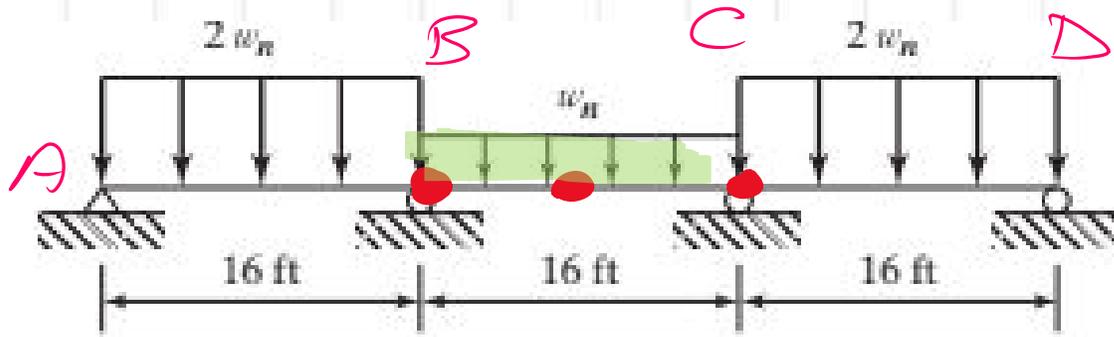
FIGURE P8-33

Top of figure labels: $2w_n$, B, C, $2w_n$, D, w_n, A, W16 × 26, 16 ft, 16 ft, 16 ft

STatic method

FT. kips

$$M_P = 184.167$$

2nd span 3ree plastic hinges

M_P

$L/2$ $L/2$ M_P

1θ Δ θ

$\theta = \dfrac{\Delta}{L/2}$ M_P 2θ

$4\theta = \dfrac{8\Delta}{L}$
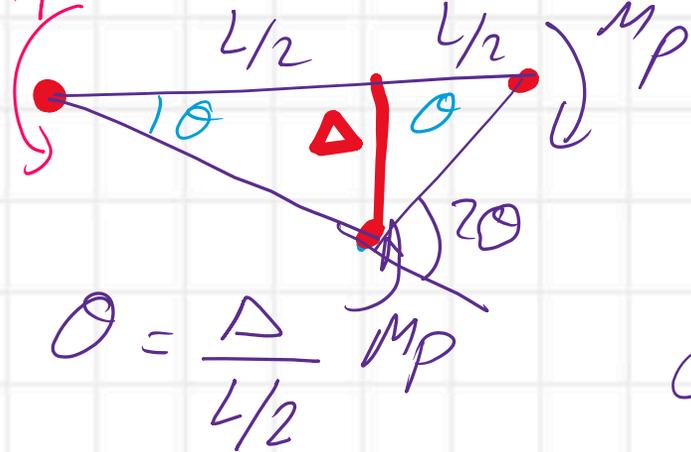
$W_n = min(4.193, 11.510)$

External work = $\dfrac{w_n L}{2} \dfrac{\Delta}{2}(2) = WL\dfrac{\Delta}{2}$

internal work

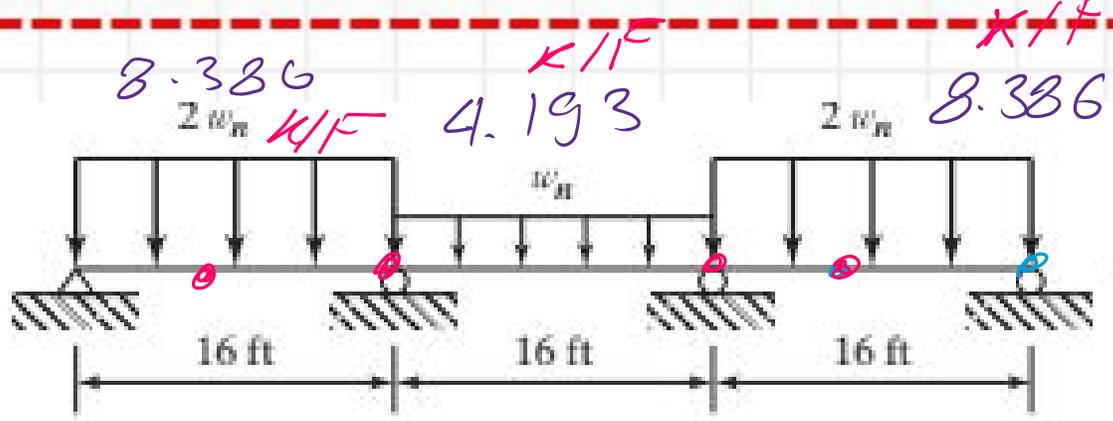$= M_P(\theta + \theta + 2\theta) = 4M_P(\theta) = M_P\dfrac{8\Delta}{L}$

$$w_n L \dfrac{\Delta}{2} = 8M_P\dfrac{\Delta}{L}$$

$$W_n = \dfrac{16\,M_P}{L^2} = \dfrac{16(184.167)}{(16)^2} = 11.510 \text{ kip/ft}$$
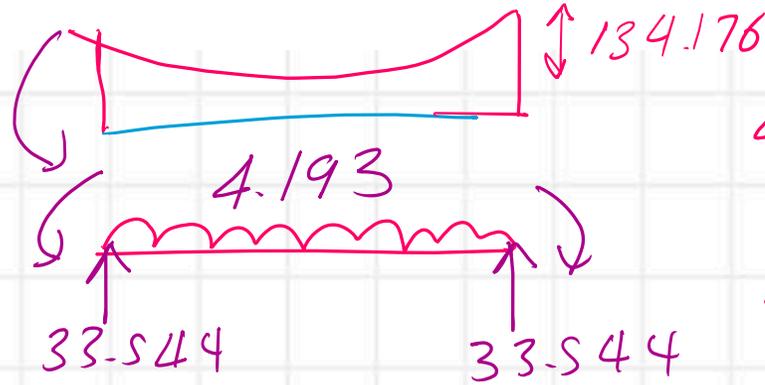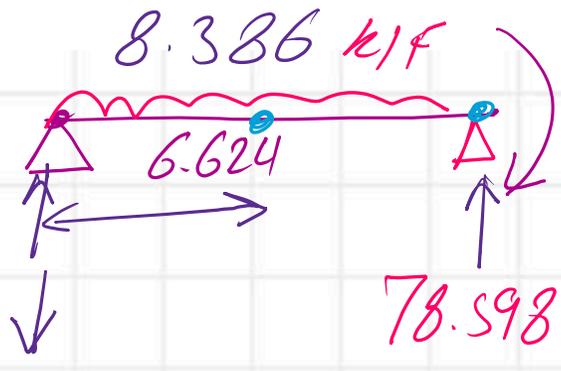
$$= 4.193 \text{ kip/FT}$$

FIGURE P8-33

Beam diagram labels: $2w_n$ kif, 8.386; w_n, 4.193 k/f; $2w_n$, 8.386 k/f; W16 × 26; 16 ft, 16 ft, 16 ft

$M_p = 184.167$ FT. kips

184.167   184.167   184.167   Check $2M_p = \dfrac{w_n L^2}{8}$

8.386 k/F

$\updownarrow$ 134.176

6.624

4.193

$\dfrac{4.193 (16)^2}{8}$

67.088

11.510

78.598

33.544   33.544

$= 134.176$

55.578

$55.578(6.624) - \dfrac{8.386(6.624)^2}{2}$

$= M_p = 184.17$ FT. kips

8.386   4.193   8.386

55.578   134.176   67.08   134.176   55.578

112.142   112.142
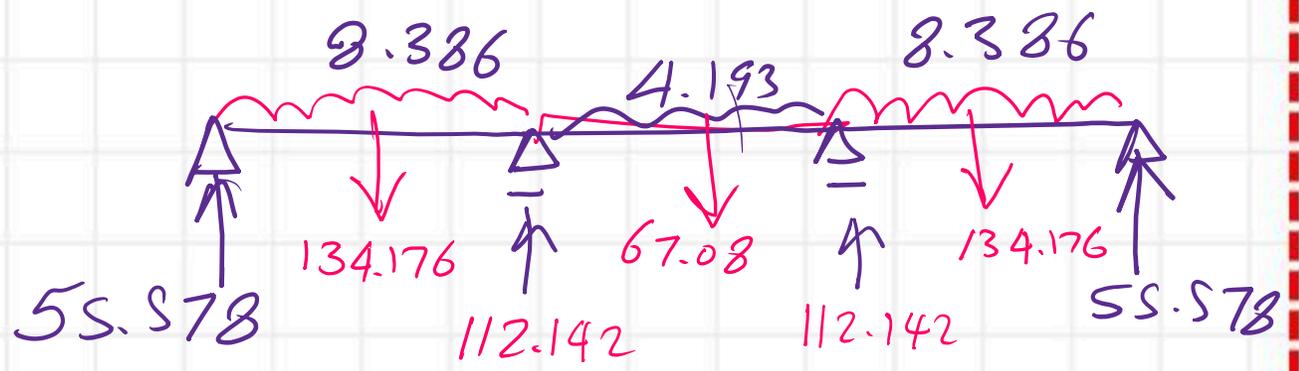
$$W_w = 0.20 \text{ K/inch} \qquad \lambda = 1.747 \Rightarrow 0.3494 \frac{k}{\text{inch}} \rightarrow 4.1928 \text{ k/ft} \text{ Same}$$

**** Deflected Shape: 1st-Order Inelastic, Incr # 18, Applied Load Ratio = 1.7472 ***

1.747

1.747

1.747

1.747

1.747

1.661 1.661

1.661

1.661

E1 X E2 E3 E4 E5 E6

Y

Z
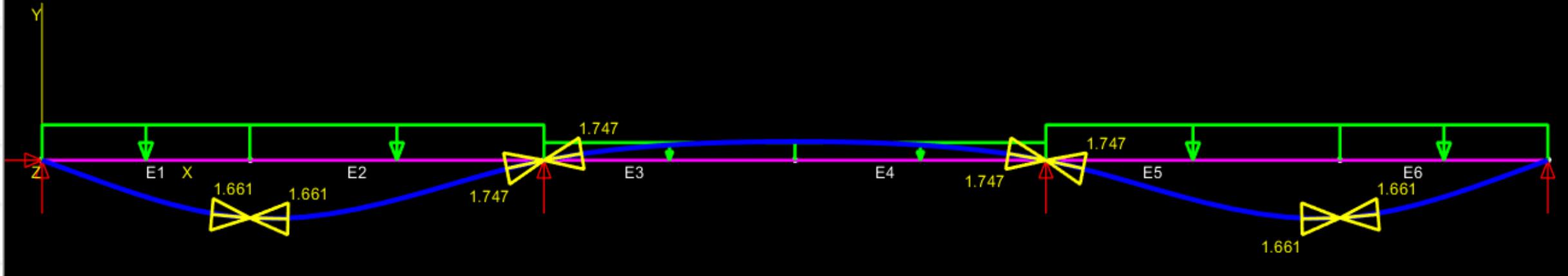
$$F_y = 50 \text{ ksi}$$
$$E = 29000 \text{ ksi}$$

Nominal Load
Four plastic hinges

$$\lambda_1 = 1.661 (0.20) = 0.3322 \text{ k/inch}$$
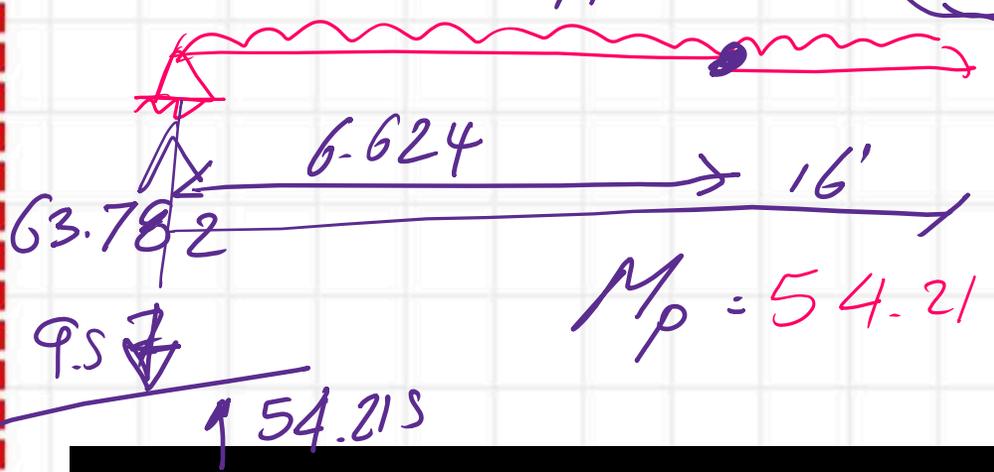$$\lambda_2 = 1.747 (0.2) = 0.3494$$

7.9728 k/F

First hinge at $w_n = 0.3322$ k/inch

First panel $= 0.6644$ k/inch

Load $\Rightarrow = 7.9728 \dfrac{k}{F}$

∠153.08

6.624

16'

63.78/2

9.5

↑54.215

$$M_p = 54.215 (6.624) - 7.9728 \dfrac{(6.624)^2}{2} = 184.20$$

M_p FT-kips
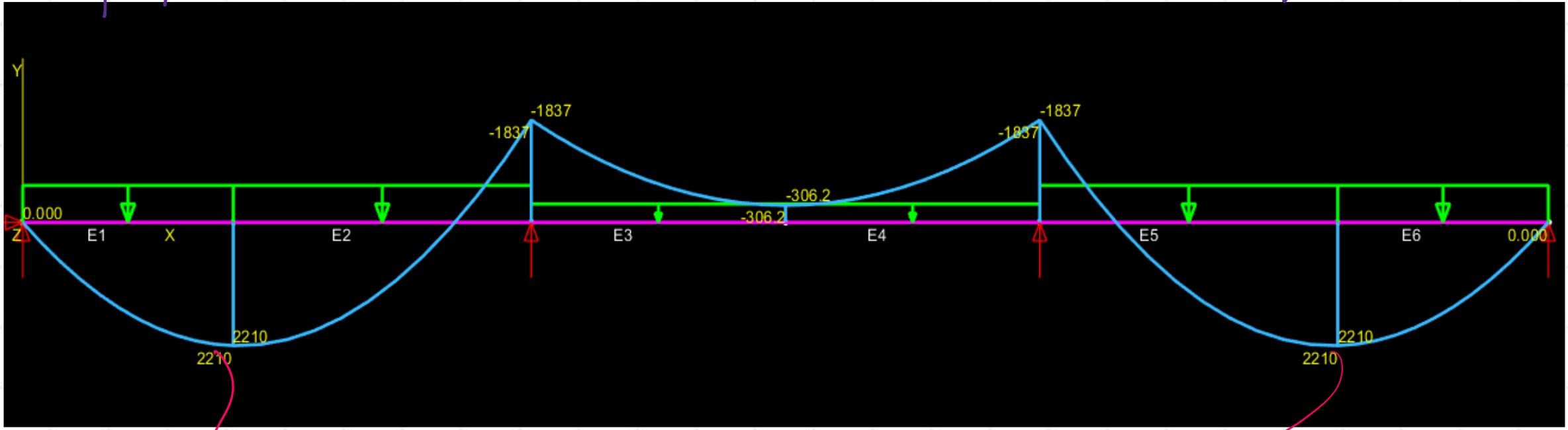
Two plastic hinges

$M_Z : M_n = 2210$ inch. kip $= M_n = F_Z Z_x$

MASTAN-2

Plastic
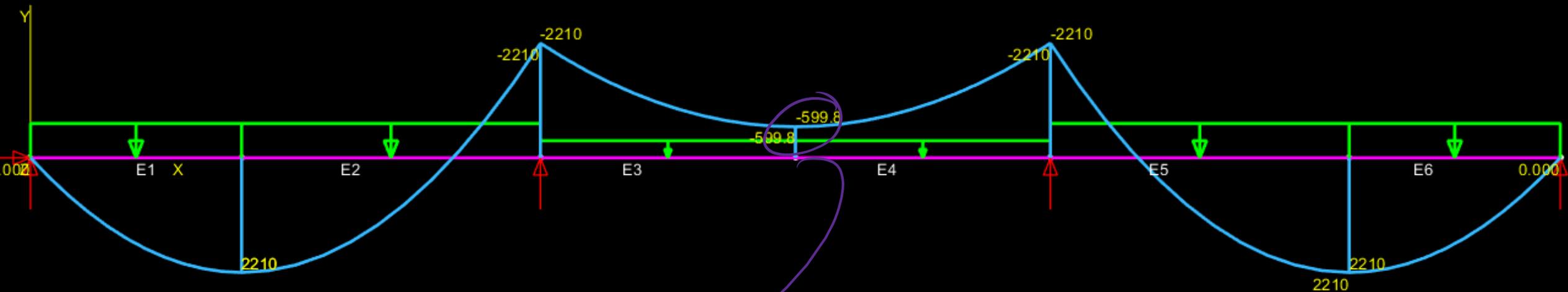
There is no hinge in the mid of span -2

0.6988 $k/inch$ 0.3494 $k/inch$ 0.6988 $k/inch$

Shear diagram

8.386 F/ı̊ k/f
4.193 klf
8.386 klf

16   16   16