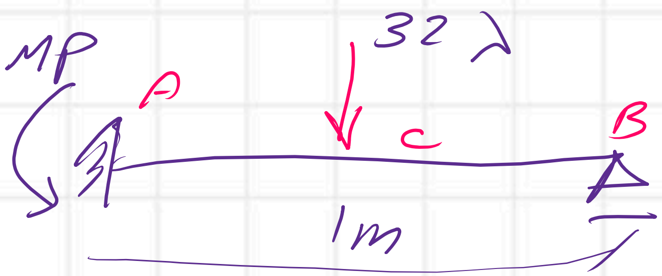
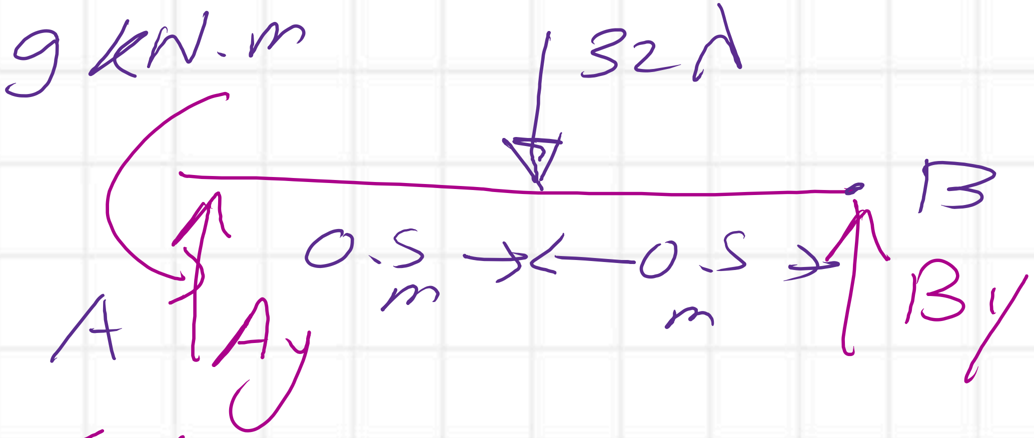


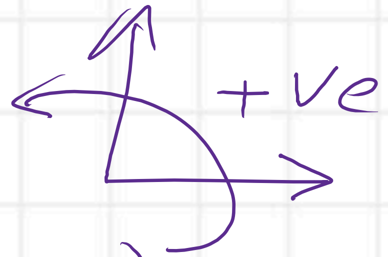
For Two Plastic hinges at A & C

$M_p = 9$ kN.m

We Can Use Statics and Find λ



$\Sigma M_A = 0$



$By(1) - 32\lambda(0.50) + 9 = 0$

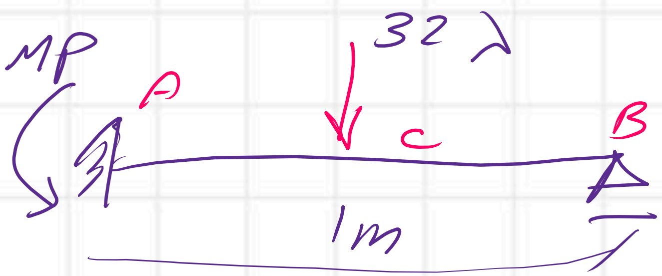
$By = 16\lambda - 9$

$\Sigma M_B = 0 \Rightarrow -Ay(1) + 32\lambda(0.5) - 9 = 0$

$Ay = 16\lambda + 9$

$\Sigma Y = 0$

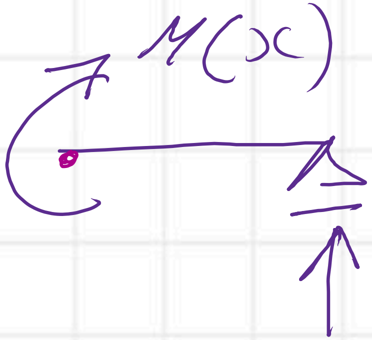
$Ay + By = 32\lambda$

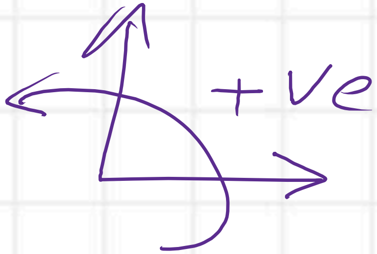


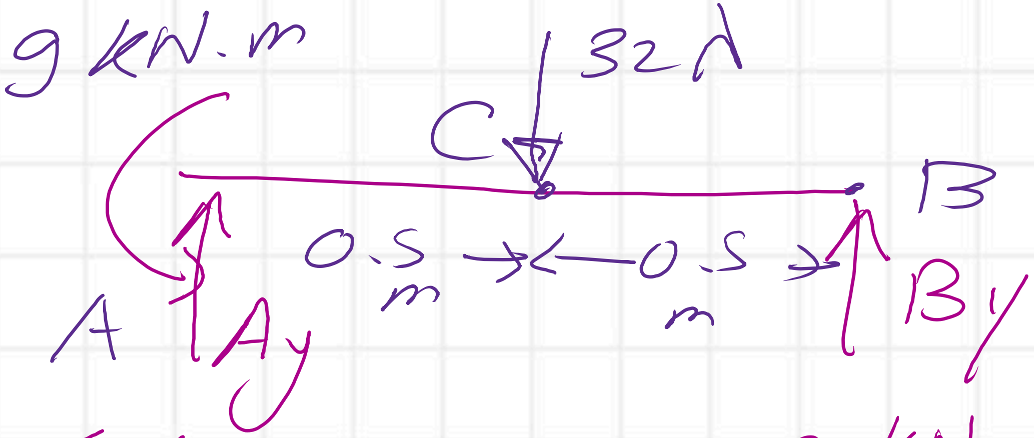
For Two Plastic hinges at A & C

$M_p = 9 \text{ kN.m}$

We Can Use Statics and Find λ







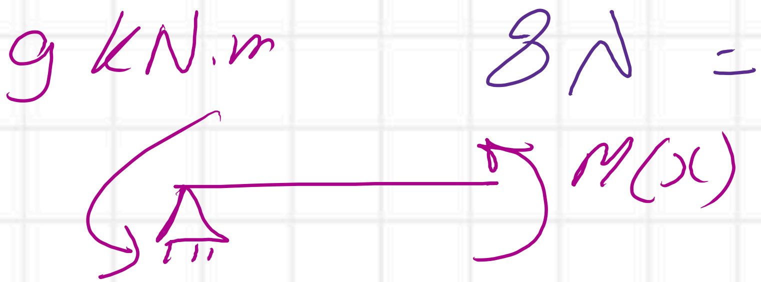
$M(x) = (16\lambda - 9)(0.5) = M_p$

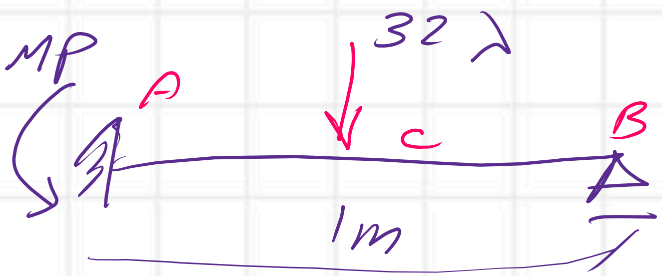
$8\lambda - 4.50 = 9$

$8\lambda = 13.5 \Rightarrow \lambda = 1.6875$

$\Sigma Y = 0$

$A_y + B_y = 32\lambda$

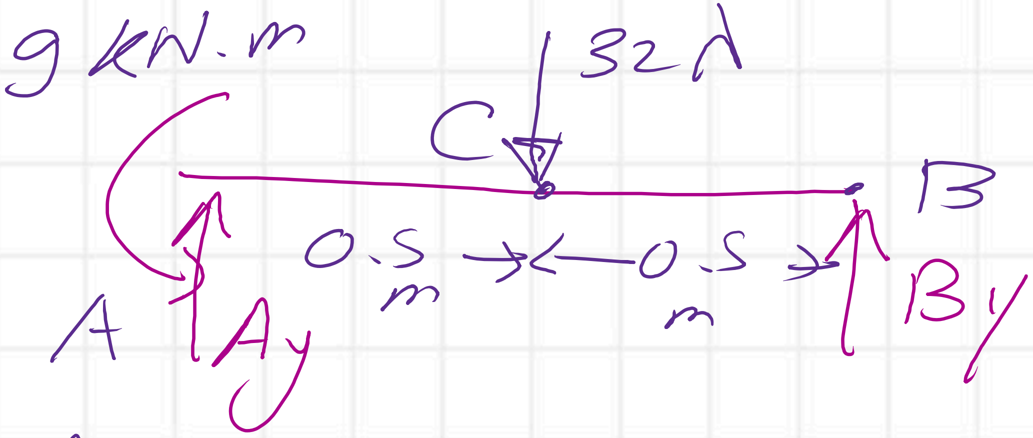


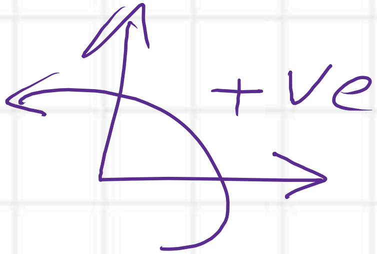


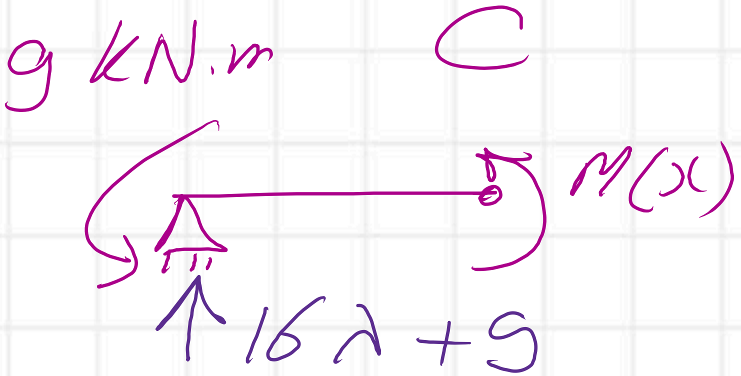
For Two plastic hinges at A&C

$M_p = 9\ kN.m$

We Can Use Statics and Find λ







$\Sigma M_C = 0$

$Ay = 16\lambda + 9$

$+9 - (16\lambda + 9)0.5 + M(x) = 0$

$M(x) = M_p = 9\ kN.m$

$M_p = -9 + 8\lambda + 4.5 = 9$

$8\lambda = 13.50 \rightarrow \lambda = 1.3875$

4.4 The Uniqueness Theorem

Linking the upper- and lower-bound theorems, we have:

If a bending moment distribution can be found which satisfies the three conditions of equilibrium, mechanism, and yield, then the corresponding load factor is the true load factor at collapse.

So to have identified the correct load factor (and hence collapse mechanism) for a structure we need to meet all three of the criteria:

1. Equilibrium;
2. Mechanism;
3. Yield.

(1) *Equilibrium condition*: The bending moments must represent a state of equilibrium between the internal forces in the structure and the applied loads.

(2) *Mechanism condition:* At collapse, the bending moment must be equal to the full plastic moment of resistance of the cross-section at a sufficient number of sections of the structure for the associated plastic hinges to constitute a mechanism involving the whole structure or some part of it.

(3) *Yield condition:* At every cross-section of the structure, the bending moment must be less than, or equal to, the full plastic moment of resistance.

$|M| \leq M_p$

4.2 The Upperbound (Unsafe) Theorem

Equilibrium + Mechanism

but Possible Some Places

This can be stated as:

> *If a bending moment diagram is found which satisfies the conditions of equilibrium and mechanism (but not necessarily yield), then the corresponding load factor is either greater than or equal to the true load factor at collapse.*

with $M > M_P$

This is called the unsafe theorem because for an arbitrarily assumed mechanism the load factor is either exactly right (when the yield criterion is met) or is wrong and is too large, *leading a designer to think that the frame can carry more load than is actually possible.*

Think of it like this: **unless it's exactly right, it's dangerous.**

1. If the collapse loads are determined for all possible mechanisms, then the actual collapse load will be the lowest of these (Upperbound Theorem);

4.3 The Lowerbound (Safe) Theorem (static)

This can be stated as: Equilibrium yield ← satisfies $M \leq M_p$

If a bending moment diagram is found which satisfies the conditions of equilibrium and yield (but not necessarily that of mechanism), then the corresponding load factor is either less than or equal to the true load factor at collapse.

This is a safe theorem because the load factor will be less than (or at best equal to) the collapse load factor once equilibrium and yield criteria are met leading the designer to think that the structure can carry less than or equal to its actual capacity.

Think of it like this: **it's either wrong and safe or right and safe.** Safe theorem

2. The collapse load of a structure cannot be decreased by increasing the strength of any part of it (Lowerbound Theorem);

The permutations of the three criteria and the three theorems are summarized in the following table:

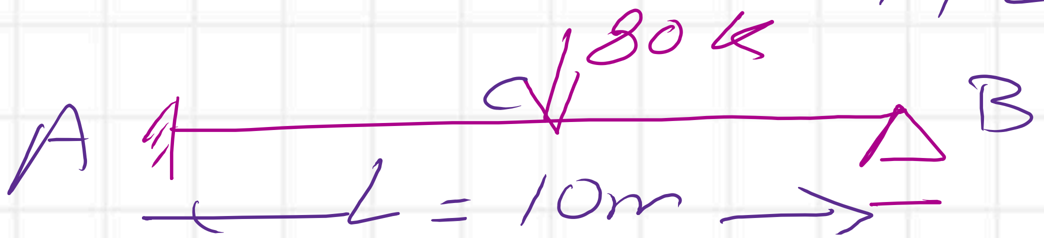
<table>
<tr><th>Criterion</th><th>Upperbound (Unsafe) Theorem</th><th>Lowerbound (Safe) Theorem</th><th>Unique Theorem</th></tr>
<tr><td>Mechanism</td><td rowspan="2">$\left.\right\} \lambda \geq \lambda_c$</td><td></td><td rowspan="3">$\left.\right\} \lambda = \lambda_c$</td></tr>
<tr><td>Equilibrium</td><td rowspan="2">$\left.\right\} \lambda \leq \lambda_c$</td></tr>
<tr><td>Yield</td><td></td></tr>
</table>

Can give higher Loads than actual

① Point B

if Some Guess that P_{PL}

$$M_C = \frac{3PL}{16} = \frac{3(80)(10)}{16} = -150 \text{ kN.m}$$

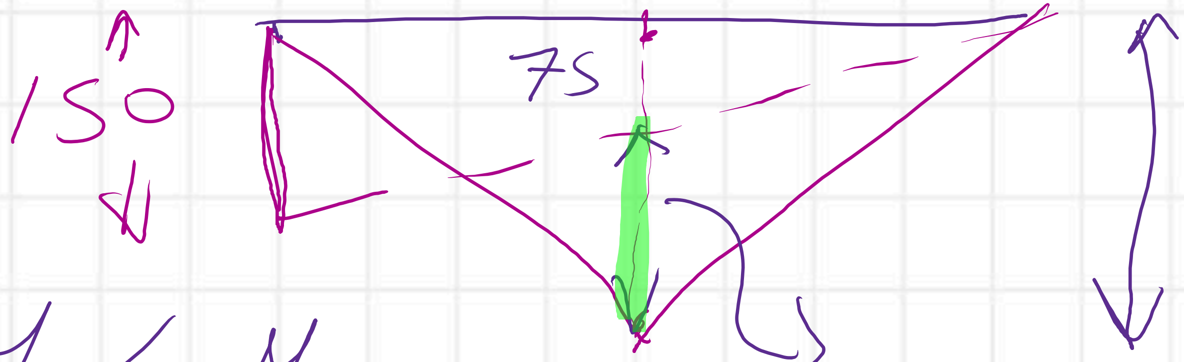


$$\frac{PL}{4} = 200 \text{ kN.m}$$

$$M_A = -150 \text{ kN.m}$$

$$M_C = \frac{5PL}{32} = \frac{5(80)(10)}{32} = +125 \text{ kN.m}$$

$M_p = 266.67$ kN.m



$PL/4 = 200$ kN.m

$M_C < M_p$

$M_A < M_P$

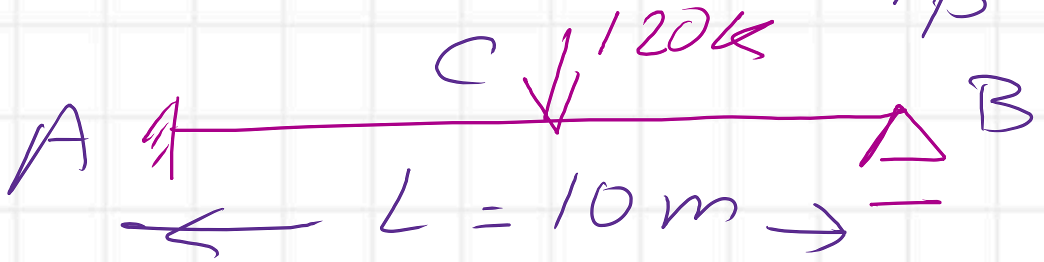
No any point has $M = M_p$

② Point C

if Some Guess that $P_p = 120$ kN

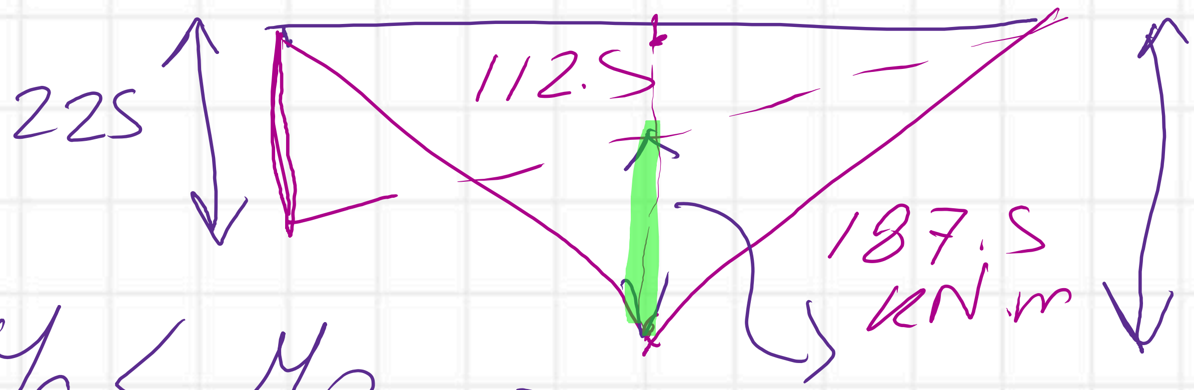
$$M_A = -\frac{3}{16}PL = \frac{3}{16}(120)(10) = -225 \text{ kN.m}$$

$$\frac{PL}{4} = 300 \text{ kN.m}$$



$M_p = 266.67$ kN.m

$$M_C = \frac{5PL}{32} = \frac{5(120)(10)}{32} = 187.5 \text{ kN.m}$$

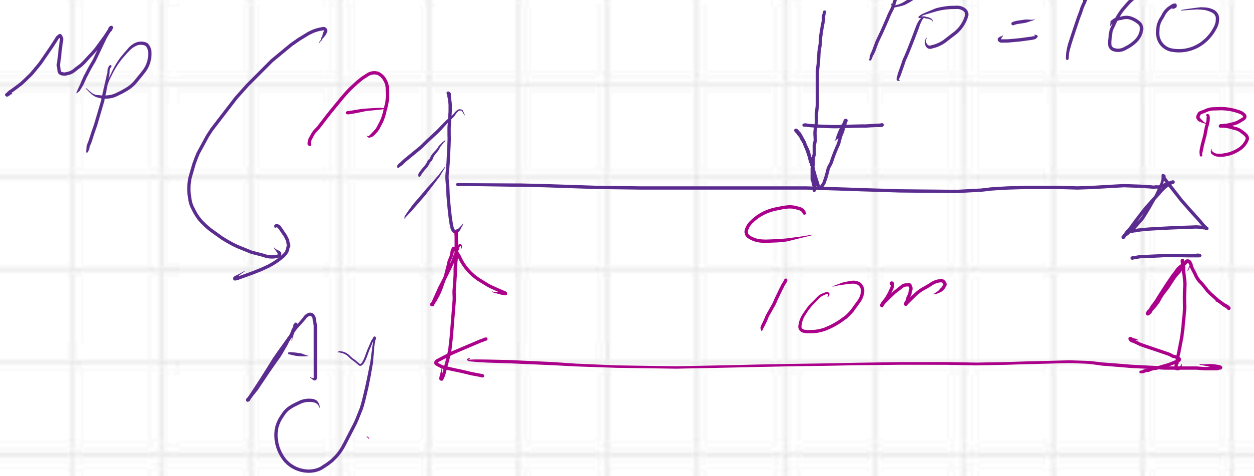


$$\left.\begin{aligned} M_C &< M_p \\ M_A &< M_p \end{aligned}\right\}$$ No any point has $M = M_p$

Lower bound Practice problem

$M_p = 266.67$ kN.m

$\boxed{D}$



$B_y = 80 - \frac{266.67}{10}$

$= 53.333$ kN

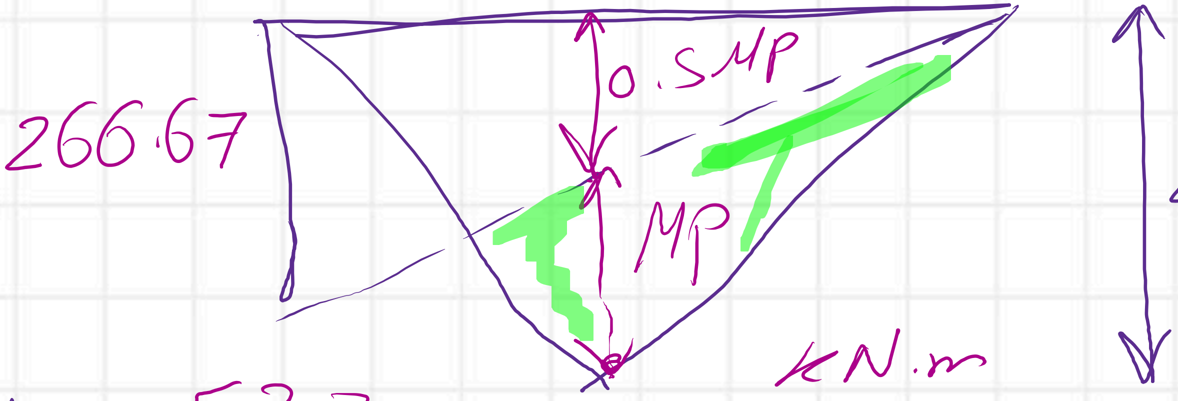
$A_y = 80 + 26.667$

$= 106.667$

$M_{C_{Left}} = -266.67 + 106.667(5) = 266.67$

$\frac{PL}{4} = \frac{160(10)}{4}$

$= 400$ kN.m



$M_{c\,right} = 53.333(5) = 266.67$ kN.m $\Rightarrow M_p$ Correct

$L = 10\ m$ $M_p = 266.67\ kN\cdot m$

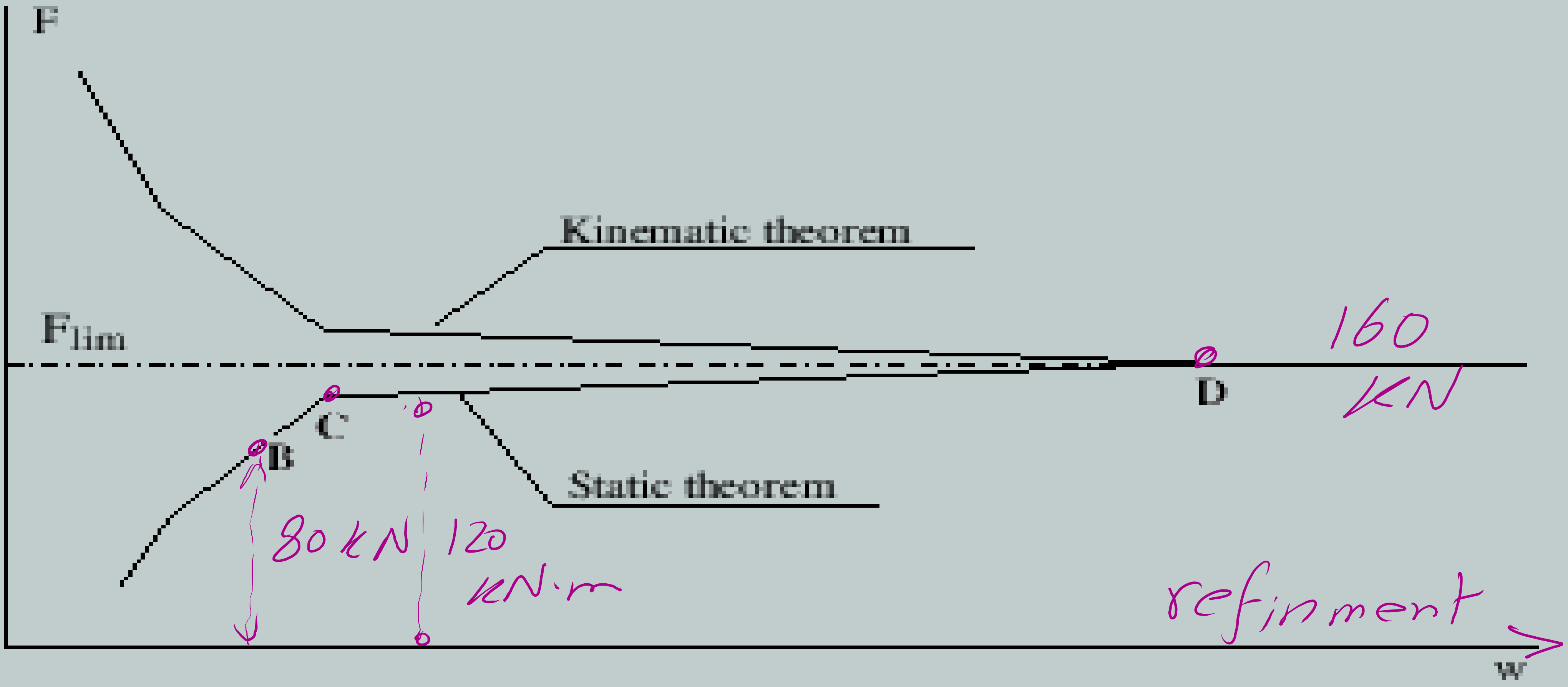


Figure 7.6: Upper and lower bound values