Solve the following equations by ~~Cholesky's~~ triangularisation method. LU

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

Doolittle Algorithm

Solution:

We have $A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$

Let $\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$

$L_{21} = (-1)(-\frac{8}{2}) = +4$

$L_{31} = (-1)(-\frac{4}{2}) = +2$

$A \rightarrow U_1$

$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\rightarrow R_2 - (\frac{8}{2} R_1) \rightarrow R_2$

$\rightarrow R_3 - (\frac{4}{2} R_1) \rightarrow R_3$

$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ +4 & 1 & 0 \\ +2 & ? & 1 \end{bmatrix}$

diagonal = 1

upper = 0

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Our matrix was
$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \xrightarrow{} \begin{matrix} R_2 - \left(\frac{8}{2}\right) \cdot R_1 \rightarrow R_2 \\ R_3 - \left(\frac{4}{2}\right) R_1 \rightarrow R_3 \end{matrix}$$

A → U_1

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 9 & -9 \end{bmatrix}$$

Pivot

to be zero

$$R_3 - \left(\frac{-9}{7} R_2\right) \rightarrow R_3$$

$$\left(\frac{9}{7}\right)(14) - 9 = -27$$

as U_{33}

Remember

L_1

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & ? & 1 \end{bmatrix}$$

L_{32} is Unkown get it from the Convert $U_1 \rightarrow U_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & +1 \end{bmatrix} \leftarrow L_2$$

U_2

$$L_2 \rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & (-27) \end{bmatrix}$$

Since $AX = B$ & $A = L \cdot U$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix}}_{U} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}}_{B}$$

$U \cdot X \rightarrow C$

Let $UX = C$

$LC = B$

To get L^{-1} matrix $(L \mid I)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - R_1(2) \rightarrow R_3]{R_2 - 4R_1 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & -\frac{9}{7} & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & -4 & 1 & 0 \\ 0 & -\frac{9}{7} & 1 & -2 & 0 & 1 \end{array}\right] \xrightarrow[R_3]{R_3 + \frac{9}{7}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & -\frac{50}{7} & \frac{9}{7} & +1 \end{array}\right]$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -\frac{50}{7} & \frac{9}{7} & +1 \end{bmatrix} \rightarrow$$

But

$$LC = B$$

$$L^{-1}LC = L^{-1}B$$

$$B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$C = L^{-1}B$$

get C matrix →

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$C = L^{-1} \cdot B$

$$\begin{bmatrix} C \\ C_2 \\ C_3 \end{bmatrix} = \overset{L^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -\frac{50}{7} & \frac{9}{7} & +1 \end{bmatrix}} \cdot \overset{B}{\begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}}$$

$$B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

check $L^{-1} \cdot B$ is giving the C-Matrix

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 12 + 0 + 0 \\ -48 + 20 + 0 \\ -\frac{600}{7} + \frac{180}{7} + 33 \end{bmatrix} = \overset{C}{\begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}} \Rightarrow \text{Final C matrix}$$

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$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

but $U \cdot X = C$

$$\overset{U}{\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{C}{\begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}}$$

We need to get inverse of U

Use augmented matrix to get U^{-1} matrix.

$$U = \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 4 & 1 & 0 & 0 \\ 0 & -7 & -14 & 0 & 1 & 0 \\ 0 & 0 & -27 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3/-27 \to R_3]{R_1/2 \to R_1,\ R_2/-7 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 1/2 & 2 & 1/2 & 0 & 0 \\ 0 & +1 & +2 & 0 & \frac{-1}{7} & 0 \\ 0 & 0 & +1 & 0 & 0 & \frac{-1}{27} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 2 & 1/2 & 0 & 0 \\ 0 & +1 & +2 & 0 & \frac{-1}{7} & 0 \\ 0 & 0 & +1 & 0 & 0 & \frac{-1}{27} \end{array}\right] \xrightarrow[-2R_3+R_2 \to R_2]{-2R_3+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & \frac{1}{2} & 0 & +\frac{2}{27} \\ 0 & 1 & 0 & 0 & \frac{-1}{7} & +\frac{2}{27} \\ 0 & 0 & +1 & 0 & 0 & \frac{-1}{27} \end{array}\right] \xrightarrow{-\frac{1}{2}R_2 + R_1\ (R_1)}$$

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$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{2}{27} \\ 0 & 1 & 0 & 0 & -\frac{1}{7} & \frac{2}{27} \\ 0 & 0 & +1 & 0 & 0 & -\frac{1}{27} \end{array}\right] \xrightarrow[+R_1\ (R_1)]{-\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{14} & +\frac{1}{27} \\ 0 & 1 & 0 & 0 & -\frac{1}{7} & \frac{2}{27} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{27} \end{array}\right]$$

$$UX = C \rightarrow U^{-1}UX = U^{-1}C$$

$$U^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{14} & \frac{1}{27} \\ 0 & -\frac{1}{7} & \frac{2}{27} \\ 0 & 0 & -\frac{1}{27} \end{bmatrix} \Rightarrow X = U^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{14} & \frac{1}{27} \\ 0 & -\frac{1}{7} & \frac{2}{27} \\ 0 & 0 & -\frac{1}{27} \end{bmatrix} \cdot \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

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$$\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{U^{-1}}{\begin{bmatrix} \frac{1}{2} & \frac{1}{14} & \frac{1}{27} \\ 0 & -\frac{1}{7} & \frac{2}{27} \\ 0 & 0 & -\frac{1}{27} \end{bmatrix}} \cdot \underset{C}{\begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} =6 - 2 - 1 \\ =0 + 4 - 2 \\ =0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Unkown matrix

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Solve the following equations by ~~Cholesky's~~ L.U triangularisation method.

$$2x + y + 4z = 12$$
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original Example ↓

This is the final check

Final check $x = 3$, $y = 2$, $z = +1$ →

$2(3) + (2) + 4(1) = 12$ R.H.S

$8(3) - 3(2) + 2(1) = 20$ R.H.S

$4(3) + 11(2) - (1) = 33$ R.H.S

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