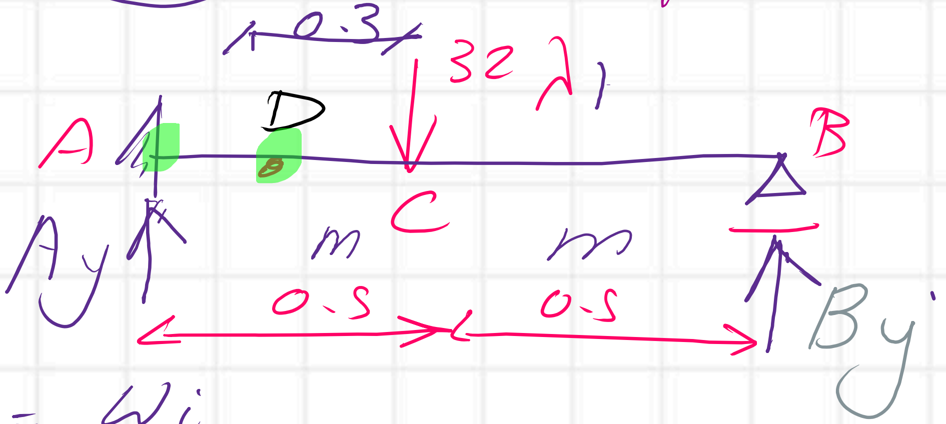
How does hinge Location affect λ Factor ?

1. Assume Location of P.H at point D to left of P Load by 0.30 m
   Find λ_1 value $\Rightarrow$ verify by general Expression
2. Assume Location of P.H at Point E to the right of P Load by 0.30m $\Rightarrow$
   Find λ_2 $\Rightarrow$ Verify by general Expression
3. Draw a graph between Location and λ Values $\Rightarrow$ Compare with correct λ

Find λ Value (upper bound) → P. Hinge at D

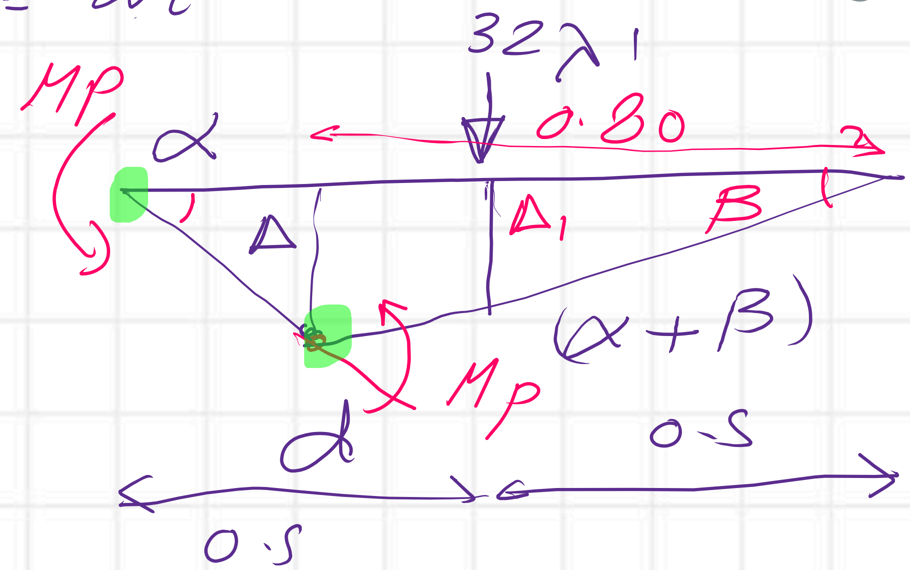


deflection at $d = \Delta$

upper bound

$We = Wi$

$We = Wi$



$Mp = 9\ kN.m$

$We = 32\lambda_1\Delta_1$

$Wi = Mp(\alpha+B) + Mp\alpha$

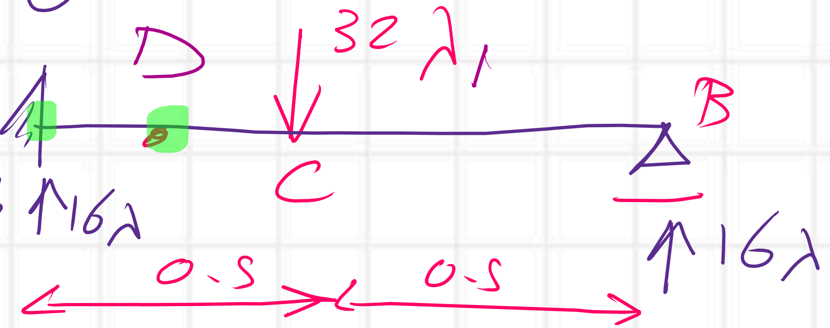
$\frac{\Delta_1}{\Delta} = \frac{0.5}{0.80}$

$\alpha = \frac{\Delta}{0.2}$

$B = \Delta / 0.80$

Correct
$\lambda = 1.685$
hinge at C

Verify λ value for Plastic hinge at D



$A_d = 0.3\ m$

deflection at $d = \Delta$

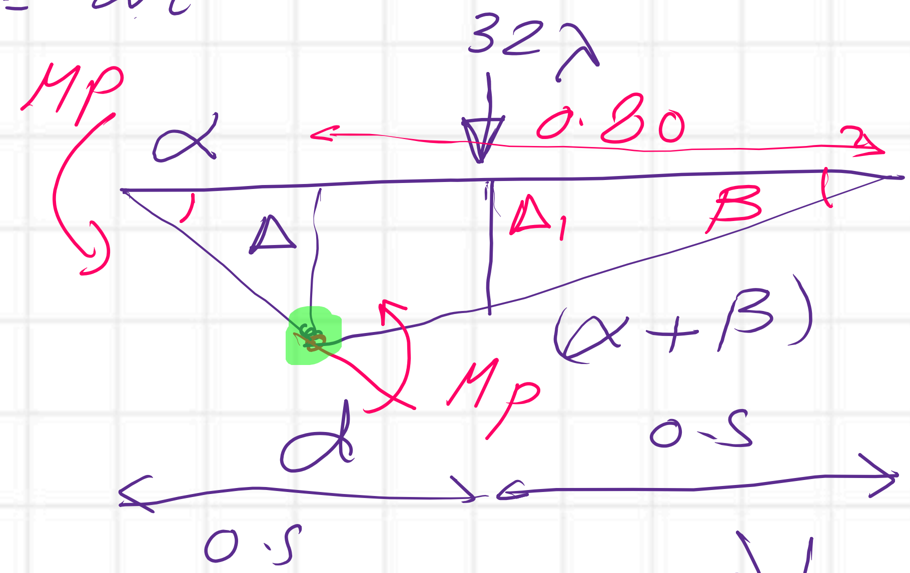
$We = Wi$

$\Delta_1 = \frac{0.5}{0.8}\Delta$

$\alpha = \frac{\Delta}{0.2}$

$\beta = \frac{\Delta}{0.80}$

$We = Wi$



$$\Rightarrow \alpha + \beta = \frac{\Delta}{0.2} + \frac{\Delta}{0.80} = \frac{\Delta}{0.2(0.80)}$$

$$We = 32\lambda_1 \Delta_1 = 32\lambda_1\left(\frac{0.5}{0.80}\right)\Delta = \frac{16\lambda_1\Delta}{0.80}$$

$$W_e = \frac{16\lambda_1\Delta}{0.80} = 20\Delta\lambda_1$$

$$W_i = M_p\left(\frac{\Delta}{0.2}\right) + M_p\left(\frac{1.0\Delta}{(0.2)(0.8)}\right)$$

α $\alpha + B$

$$W_i = \frac{M_p(1.8)\Delta}{(0.2)(0.80)} = \frac{45}{4} M_p\Delta$$

Equate

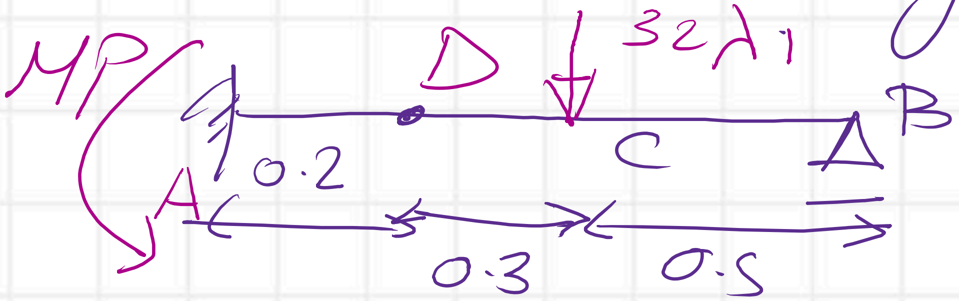
$$20\Delta\lambda_1 = \frac{45}{4} M_p \Delta$$

$$\lambda_1 = \frac{45 M_p}{4(20)} = \frac{9}{16}(M_p) \Rightarrow M_p = 9 \text{ kN.m}$$

$$\lambda_1 = \frac{9}{16}(9) = 5.0625 \Rightarrow \;>>\; 1.685 \text{ actual } \lambda$$

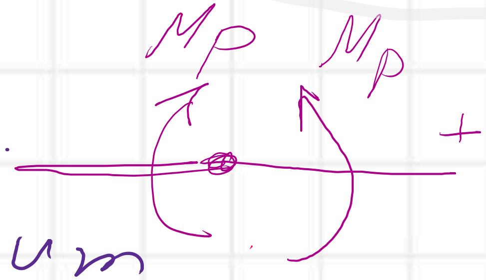
Solution not Correct

Examine the System based on Plastic hinge at D



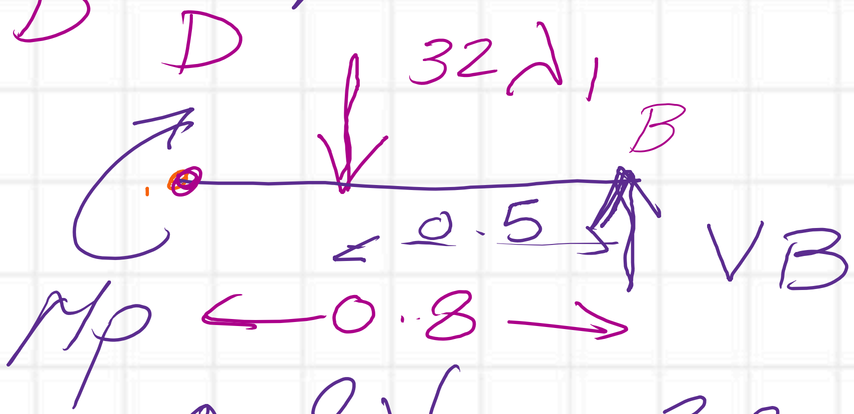
Find VB based on Location of the plastic hinge

VA

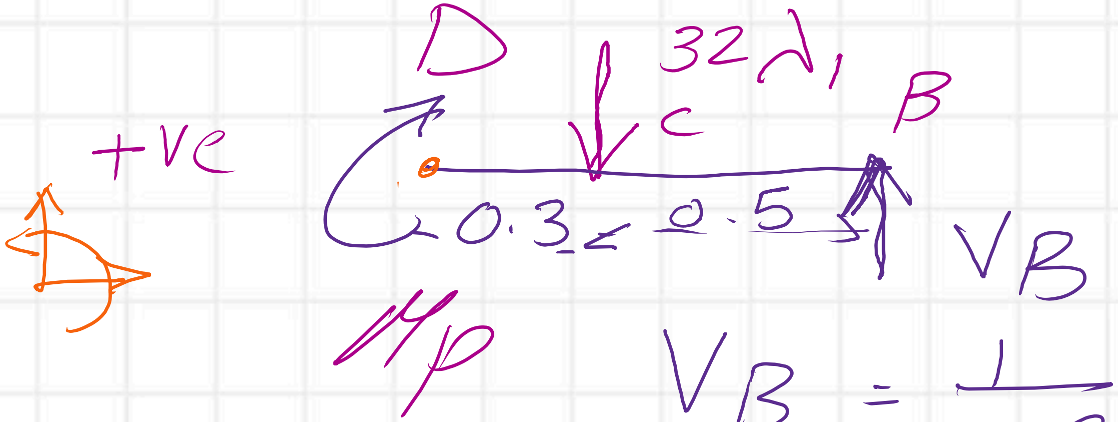


Examine Equilibrium at D

$DB = 0.80\ m$



$$M_p = 0.8V_B - 32\lambda_1(0.30)$$
$$= 0.8V_B - 9.6\lambda_1$$



$$M_p = 0.8\,V_B - 9.6\,\lambda_1$$

$$V_B = \frac{1}{0.80}\left(9.6\lambda_1 + M_p\right)$$

$$\lambda = 5.0625$$

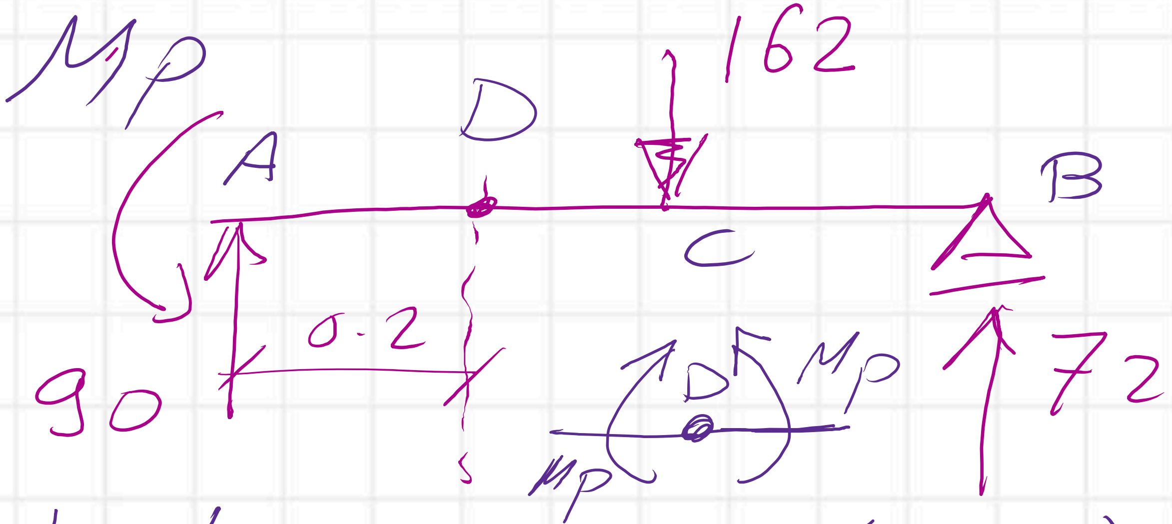
$$M_p = 9 \text{ kN.m}$$

$$V_B = (9.6\lambda_1 + 9)/0.80 = 72 \text{ kN}$$

Check $M_C = \left[\dfrac{9.6\lambda_1 + 9}{0.80}\right](0.50)$ → Uniqness

$$M_C = V_B(0.5) = 72(0.5) = 36 \text{ kN.m} > 9 \text{ kN.m}\ (M_p)$$

Solution is not correct



$32\lambda_1 = 32(5.0625)$

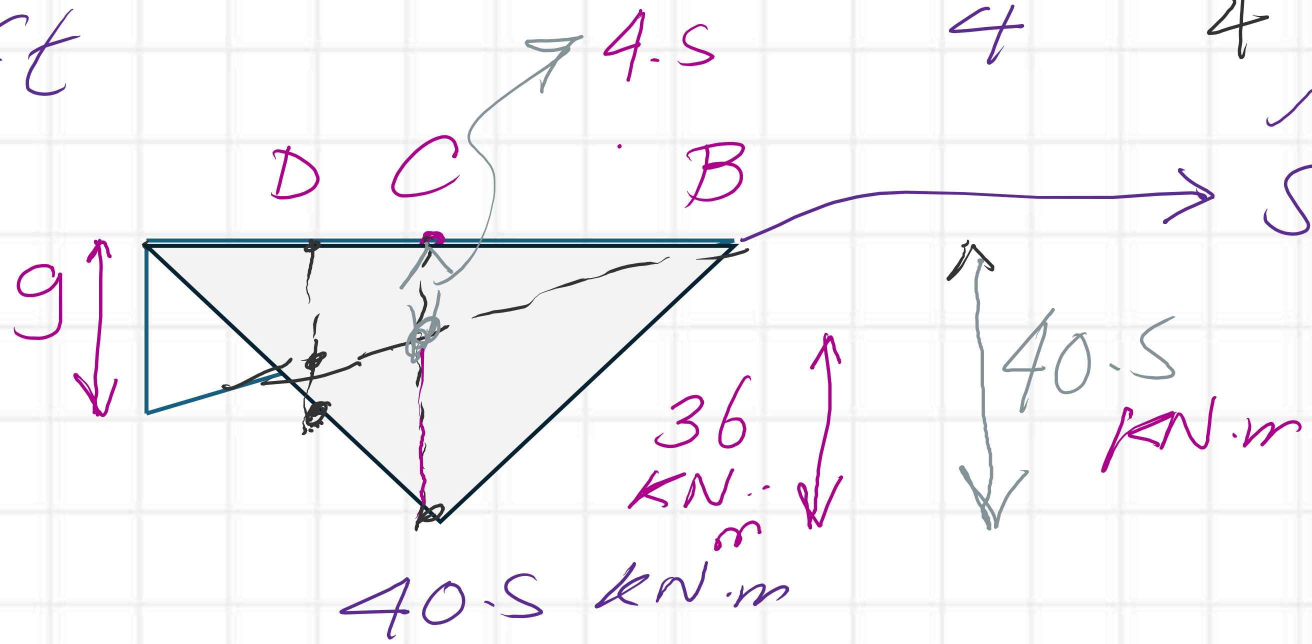
$= 162 \text{ kN}$

$M_p = 9 \text{ kN.m}$

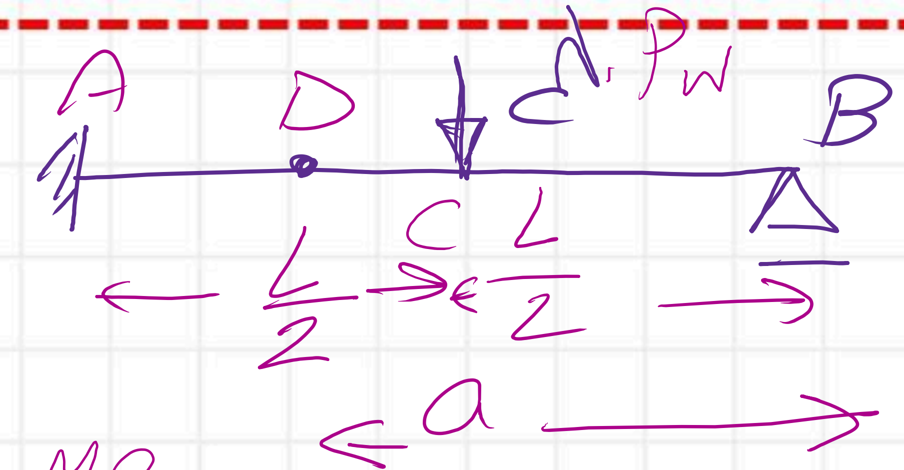
Check $M_D = 90(0.20) - \underset{M_p}{9} = 18 - 9 = 9 \text{ kN.m}$

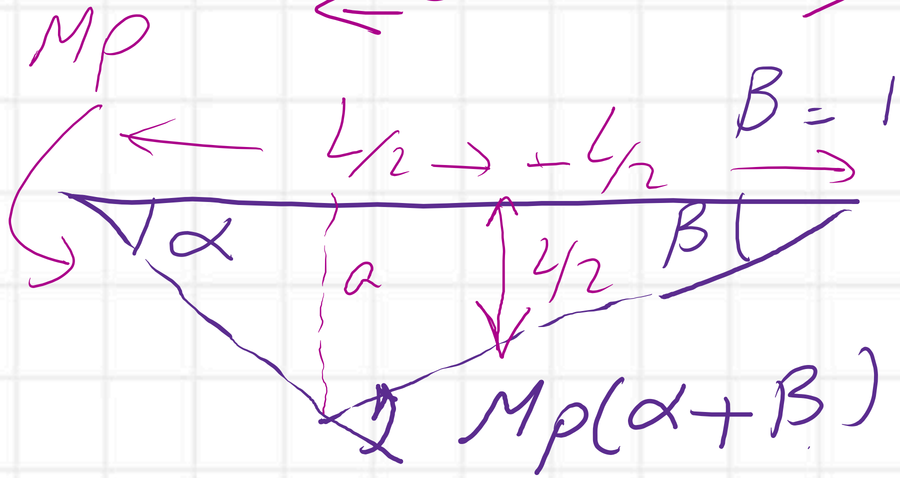
From left

$\frac{PL}{4} = \frac{162(1)}{4} = 40.5 \text{ kN.m}$



Use Statical method





Derive a general Expression

Virtual rotation

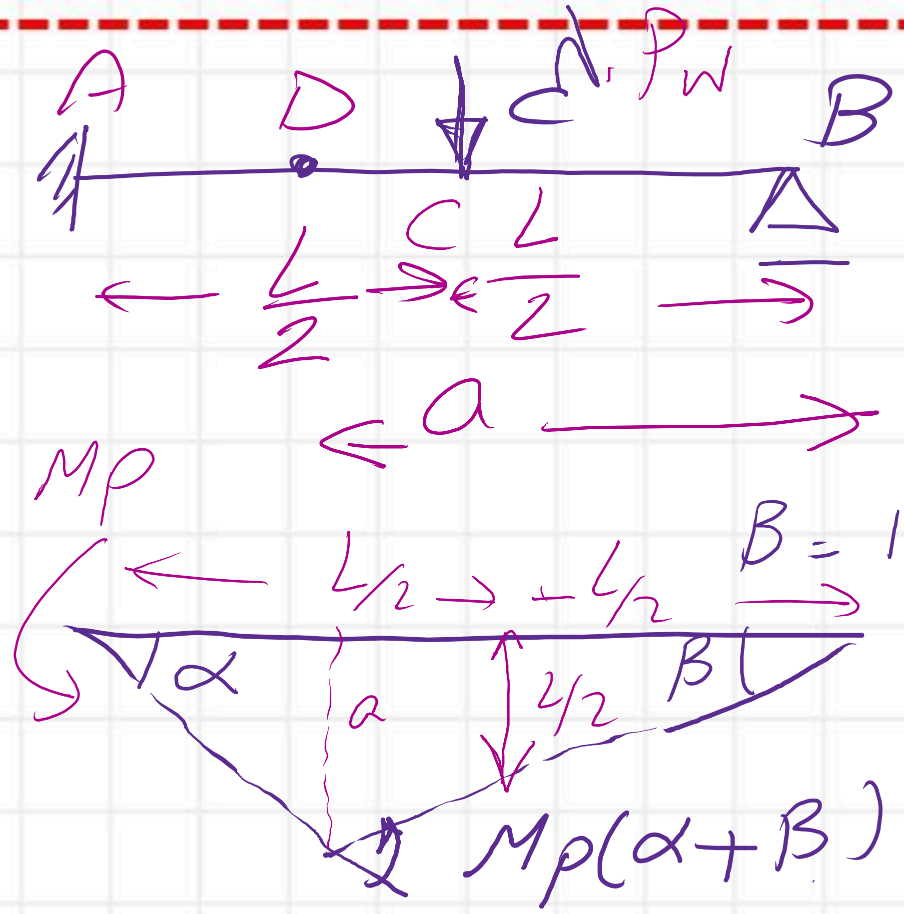
$\theta_B = 1 \Rightarrow$

$\Delta_D = a \qquad \alpha = \frac{\Delta_D}{L-a} = \frac{a}{L-a}$

$\Delta_C = \ell/2$

$W_e = \lambda_1 P_w \Delta_C = \lambda_1 P_w \left(\frac{L}{2}\right)$

$W_i = M_p(2\alpha + \beta) = M_p\left(\frac{2a}{L-a} + 1\right)$



Derive a general Expression

Virtual rotation

$\theta_B = 1 \Rightarrow$

$W_e = W_i$

$$\lambda_1 P_w \frac{L}{2} = M_p\left(\frac{L+a}{L-a}\right)$$

$$\lambda_1 = \frac{M_p}{P_w \frac{L}{2}} \frac{(L+a)}{(L-a)} \quad \Big\} \; a > \frac{L}{2}, \; < L$$

Find λ_1 value $\frac{(P_W)}{\lambda_1}\left(\frac{L}{2}\right) = \frac{M_p(L+a)}{(L-a)}$ $\frac{\ell}{2} = 0.5$

$a = 0.80$

$$\lambda_1 = \frac{M_p(L+a)}{P_W \frac{L}{2}(L-a)}$$

$a > L/2$
$< L$
$\Rightarrow$ general Expression $\rightarrow$ I

$P_W = 32$ KN
$M_p = 9$ KN.m

$$\lambda_1 = \frac{2(9)}{32}\left(\frac{1+a}{1-a}\right)$$

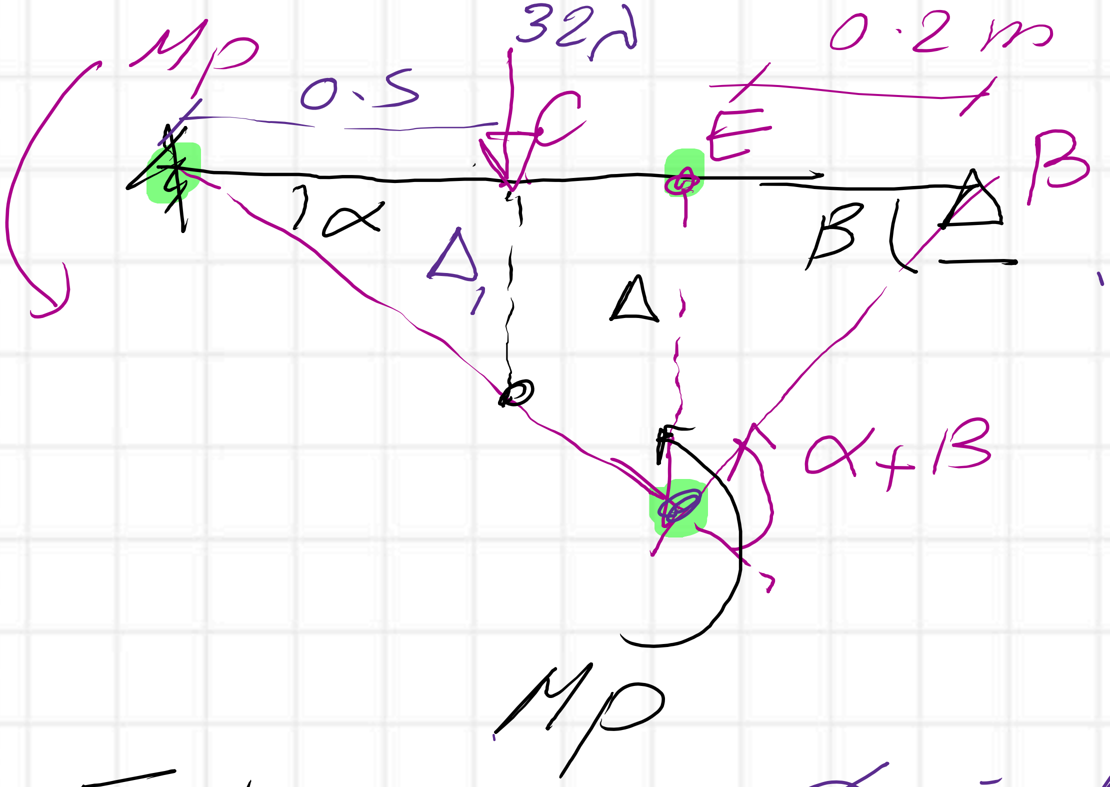
$$\lambda_1 = \frac{9}{16}\left(\frac{1+a}{1-a}\right) \Rightarrow a = 0.80$$

$$\lambda_1 = \frac{9}{16}(1.8)/(0.20) = 5.0625$$

when $a = 0.5$

$$\lambda = \frac{9}{16}\left(\frac{1.5}{0.5}\right) = \frac{27}{16} \Rightarrow 1.6875$$

Find λ_2 Value (upper bound) $\rightarrow$ P. Hinge at E



deflection at E = Δ

upper bound

$W_e = W_i$

$W_e = 32\lambda_2\Delta_1$

$W_i = M_p(\alpha+\beta) + M_p\alpha$

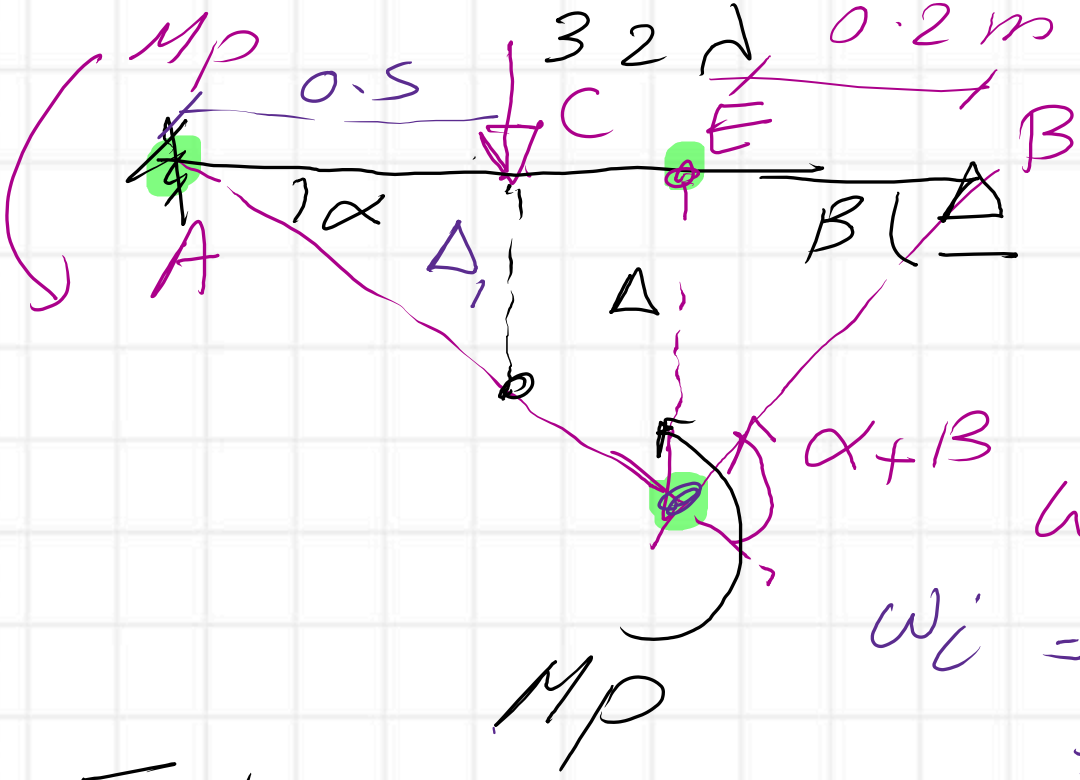
Find λ_2

$\alpha = \dfrac{\Delta}{0.8}$ & $\beta = \dfrac{\Delta}{0.2}$ & $\dfrac{\Delta_1}{\Delta} = \dfrac{0.5}{0.80}$

$\alpha+\beta = \dfrac{5\Delta}{0.80} = 25\Delta/4$

$W_e = 32\lambda\Delta_1 = 32\lambda_2\left(\dfrac{0.5}{0.80}\right)\Delta = 20\lambda_2\Delta$

Find λ Value (upper bound) $\rightarrow$ P. Hinge at E



deflection at $E = \Delta$

$$\alpha + \beta = \frac{25}{4}\Delta \quad \& \quad \alpha = \frac{\Delta}{0.80}$$

$$W_e = 20\lambda_2 \Delta$$

$$W_i = M_p \alpha + M_p(\alpha+\beta)$$

$$= M_p\left(\frac{\Delta}{0.80} + \frac{25\,\Delta}{4}\right) = M_p\left(\frac{30\,\Delta}{4}\right)$$

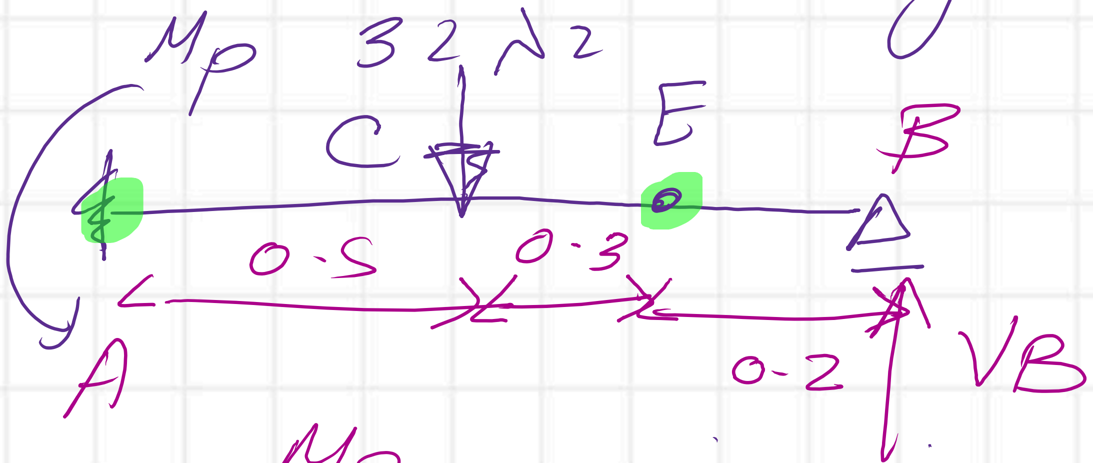
Find λ_2

$$20\lambda_2 \Delta = M_p\left(\frac{30}{4}\right)\Delta$$

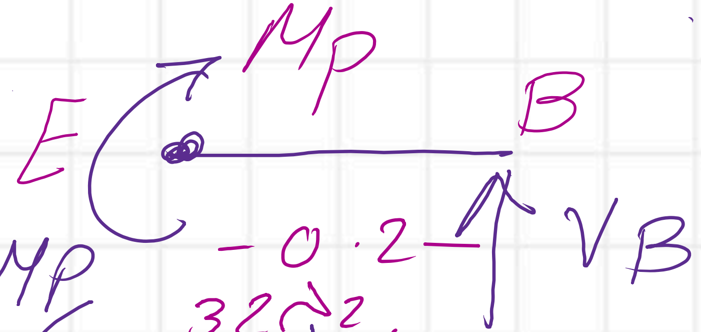
$$\lambda_2 = \frac{30}{80} M_p = \frac{3}{8} M_p \rightarrow M_p = 9$$

$$\lambda_2 = \frac{27}{8} = 3.375 \gg \lambda_{act}^{1.685}$$

Examine the System based on Plastic hinge at E

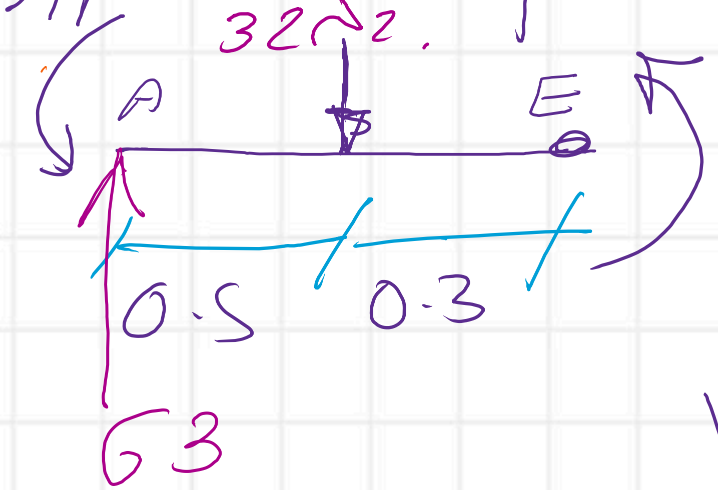


Find VB, VA $\Big\}$ $\lambda_2 = 3.375$



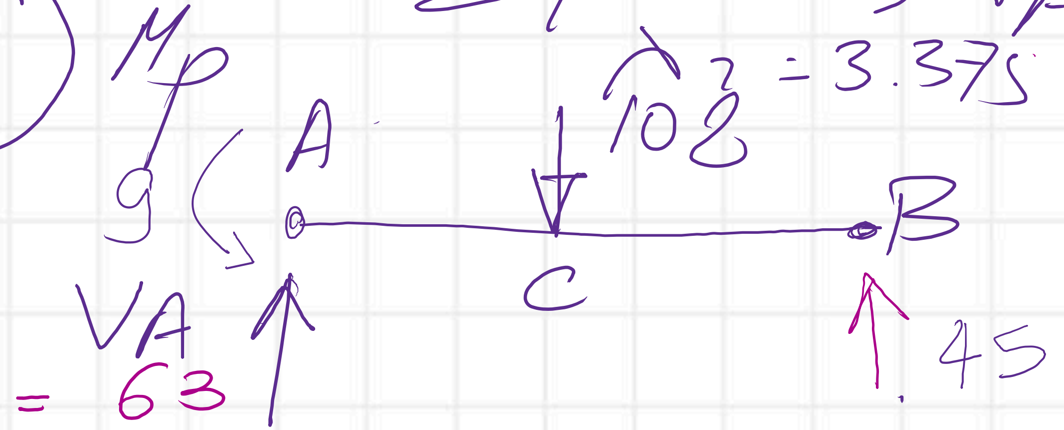
$V_B(0.20) = M_p \Rightarrow V_B = 5M_p$

$M_p = 9\ kN.m \Rightarrow V_B = 45\ kN$



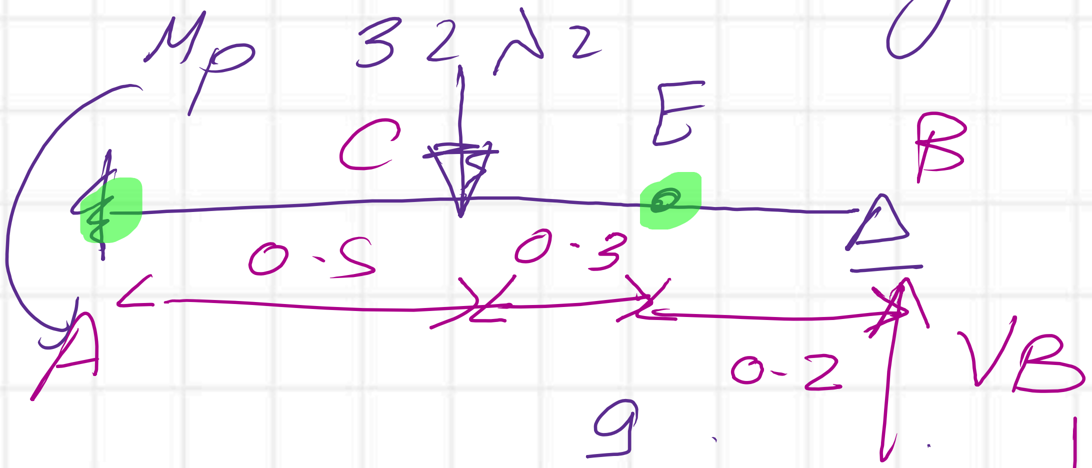
$\Sigma Y = 0 \rightarrow V_A = 108 - 45 = 63\ kN$

$\lambda_2 = 3.375 \Rightarrow 32\lambda_2$



$32\lambda_2 = 108$

Examine the System based on Plastic hinge at E

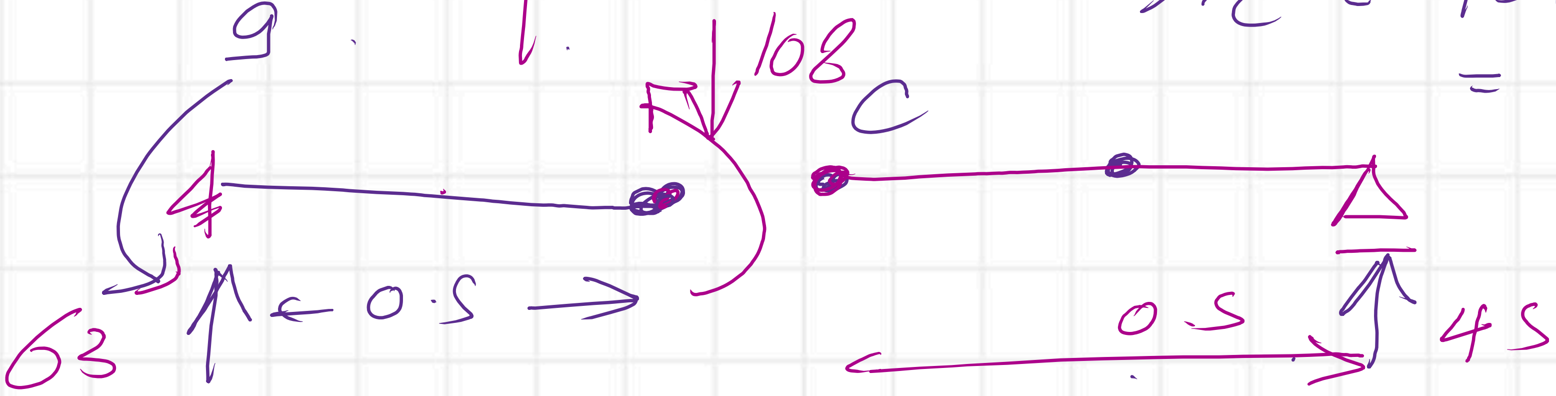


$V_B = 45$ kN

$V_A = 63$ kN

$M_C = 45(0.5)$

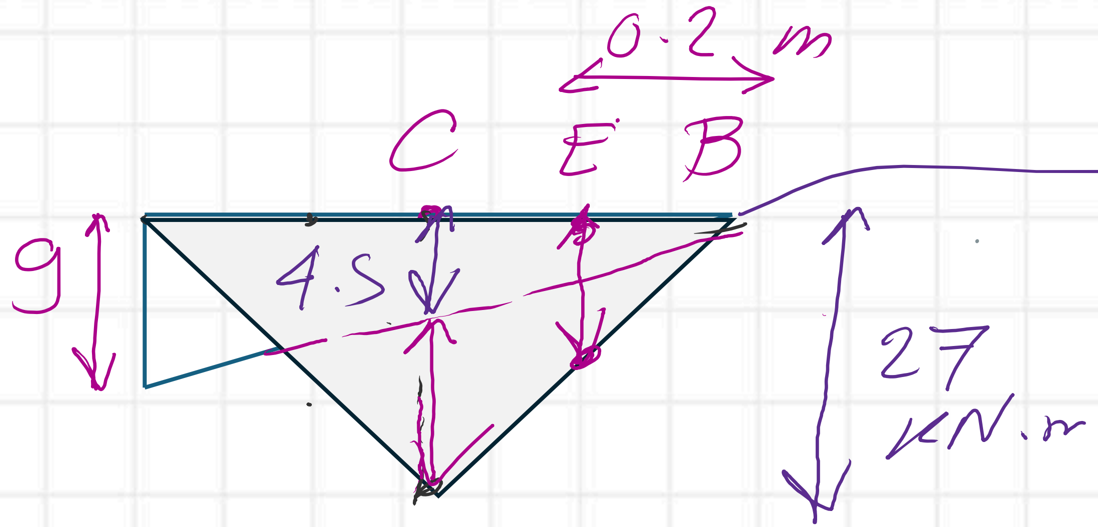
$= 22.5$ kN.m



$M_C = 63(0.5) - 9 = 22.5$ kN.m $\gg$ 9 kN.m

$\lambda_2 = 3.375 \gg 1.685$

λ Correct



$P = 108$ kN

Statical method

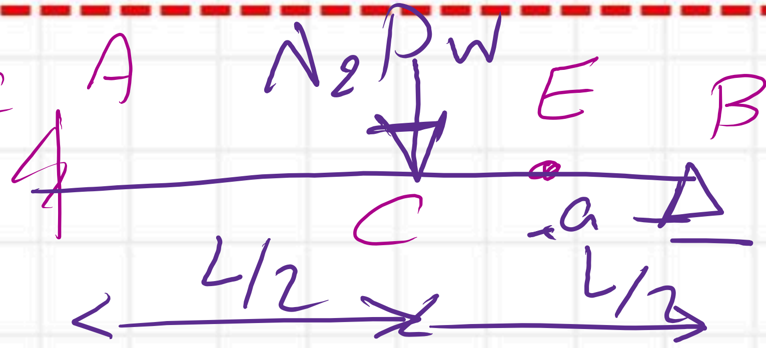
$\lambda_2 = 3.375$

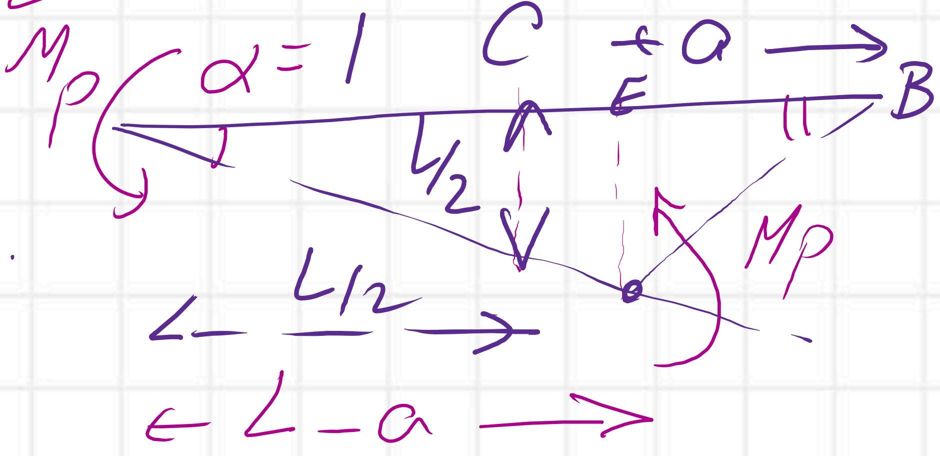
$\frac{PL}{4} = 108\left(\frac{1}{4}\right) = 27$ kN.m

$M_C = 27 - 0.5(0.9)$

$M_C = 22.50$ kN.m

Plastic hinge at E





For Plastic hinge at E

general Expression

Virtual rotation

$\alpha = 1$

$\Delta_C = \frac{L}{2}$

$\Delta_E = (L-a)$

$\beta = \frac{\Delta_E}{a} = \frac{L-a}{a}$

$W_e = (\lambda_2 P_W)\Delta_C = \lambda_2 P_W \frac{L}{2}$

$W_i = M_p(2\alpha + \beta) = M_p\left(2 + \frac{L-a}{a}\right)$

$$W_e = (\lambda_2 P_w)\underset{C}{\Delta} = \lambda_2 P_w \frac{L}{2}$$

$$W_i = M_p(2\alpha + \beta) = M_p\left(2 + \frac{L-a}{a}\right)$$

$$\lambda_2 = \frac{M_p}{P_w \frac{L}{2}}\left(\frac{L+a}{a}\right) \quad \text{For } a < l/2$$
$$> 0$$

For $M_p = 9$ kN.m
$P_w = 32$ kN
$L = 1$
$a = 0.20$ m

$$\lambda_2 = \frac{9}{32\left(\frac{1}{2}\right)}\left(\frac{1+0.2}{0.20}\right) = \frac{54}{16} = 3.375$$

For $a = 0.5 \longrightarrow \frac{9}{16}\left(\frac{1.5}{0.5}\right) = 1.6875$

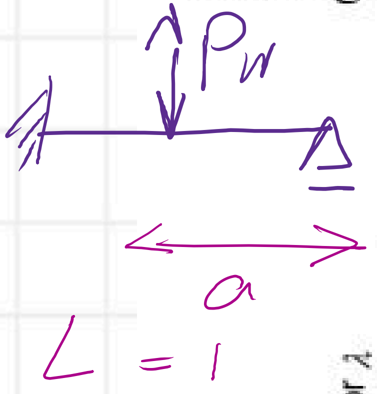
$a = 0.5$ $\qquad$ $a = 0.5$

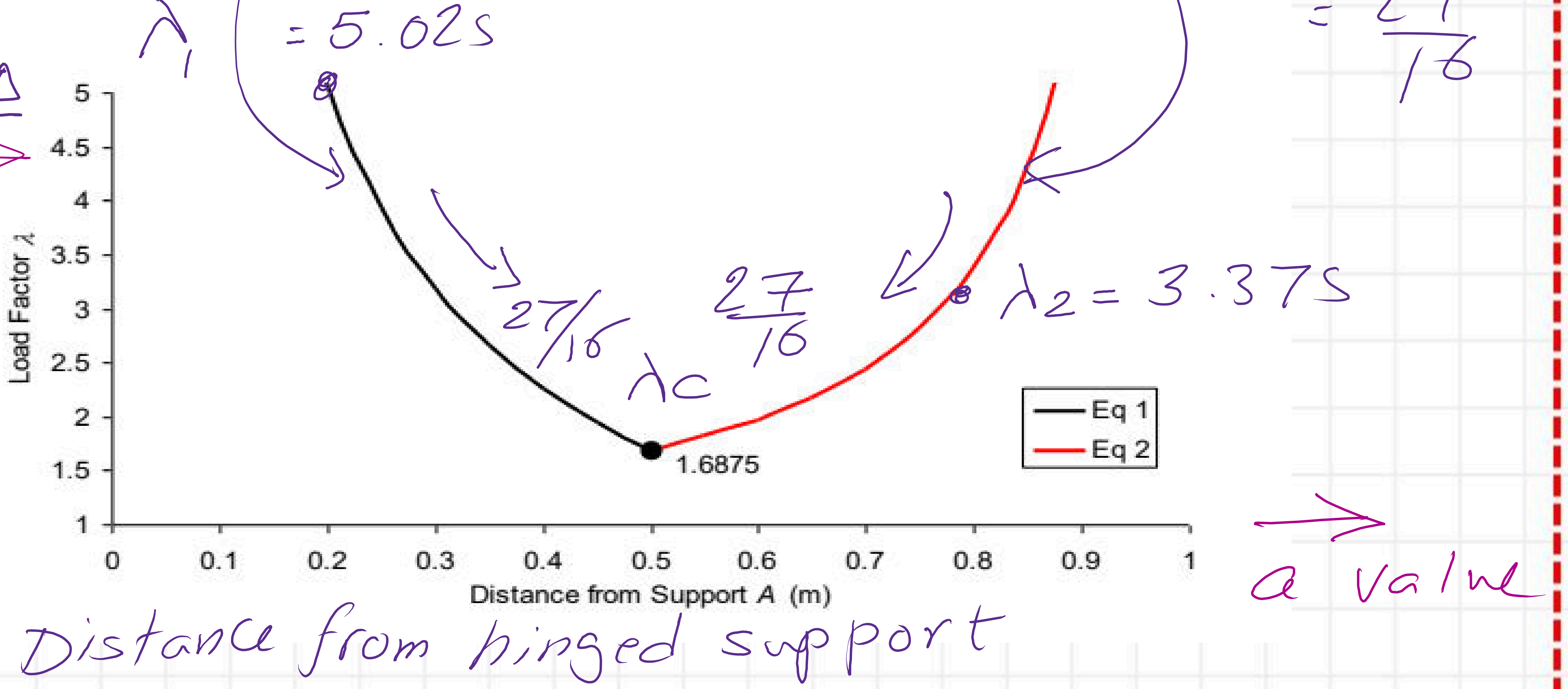
$$\lambda_{1<a\le 0.5} = \frac{9}{16}\left[\frac{a+1}{1-a}\right] \qquad \lambda = \frac{9}{16}\left(\frac{1.5}{0.5}\right)$$

$$\lambda_{0.5<a\le 0} = \frac{9}{16}\left[\frac{1+a}{a}\right] \qquad \lambda = \frac{9}{16}\,\frac{1.5}{0.5} = \frac{27}{16}$$

Summary

Plotting how the collapse load factor changes with the position of the hinge, we get:



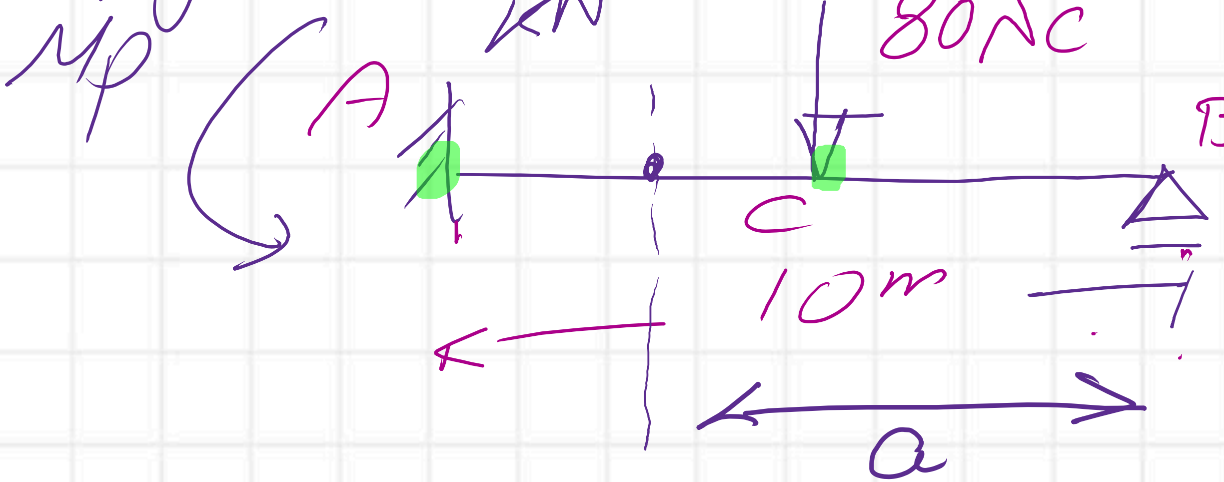


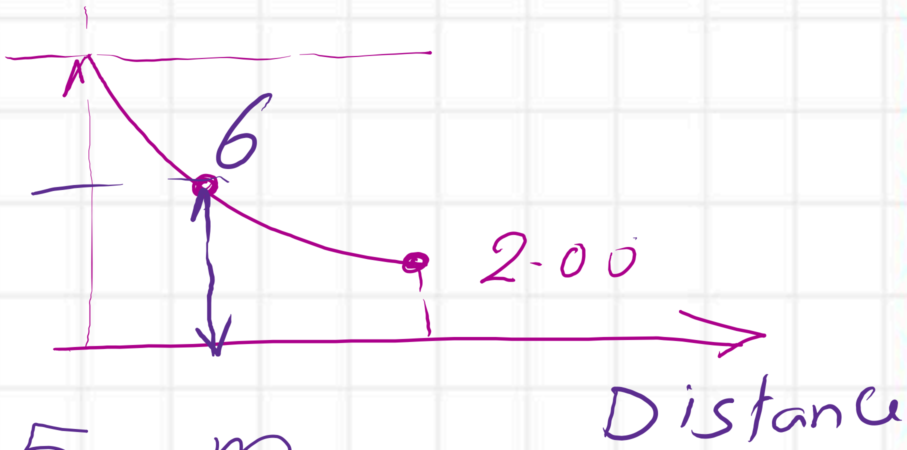
$P_W = 80\,kN$

$P_y = 90.66\,kN$

$M_p = 266.67\,kN.m$

$M_y = 170\,kN.m$





$a > 5\,m$, $< 10\,m$

$$\lambda_1 = \frac{M_p(L+a)}{P_W \frac{L}{2}(L-a)} \qquad a > L/2,\ < L$$

$a = 8\,m$

$L = 10$

$L + a = 18\,m$

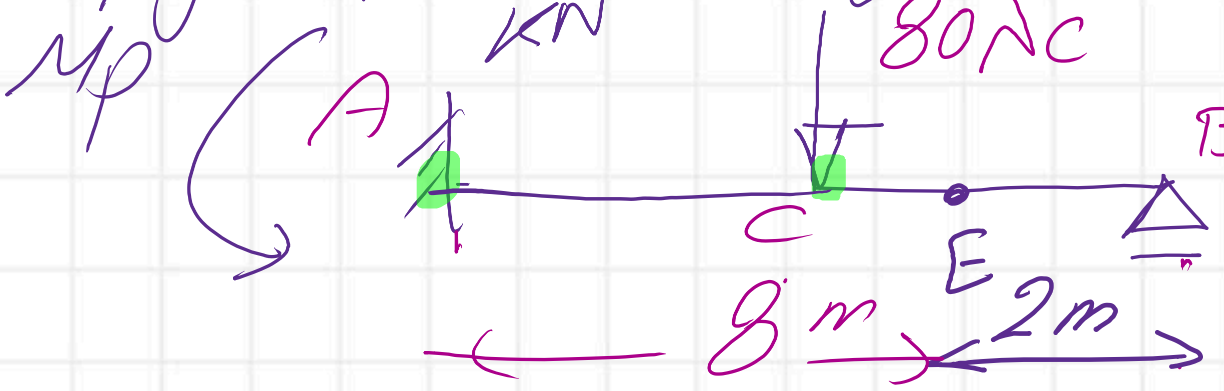
$L - a = 2\,m$

$$= \frac{(266.67)\left(\frac{18}{2}\right)}{80(5)} = 6.00$$

$a = 5m \Rightarrow \lambda = \frac{266.67(15)}{400\,(5)} = 2.00$

$P_w = 80\,kN$ $\quad M_p = 266.67\ kN.m$

$P_y = 90.66\ kN$ $\quad M_y = 170\ kN.m$



$l = 10\,m \Rightarrow l/2 = 5$

$a = 2\,m$

$L + a = 12\,m$

$$\lambda_2 = \frac{M_p}{P_w \frac{L}{2}}\left(\frac{L+a}{a}\right) \qquad \left.\begin{matrix} a > 0 \\ < 5m \end{matrix}\right\} \qquad \lambda_2 = \frac{266.67}{80(5)}\left(\frac{12}{2}\right) = 4.00$$

For $a = 5m$

$L + a = 15$

$$\lambda_2 = \frac{266.67}{400}\left(\frac{15}{5}\right) = 2.00$$

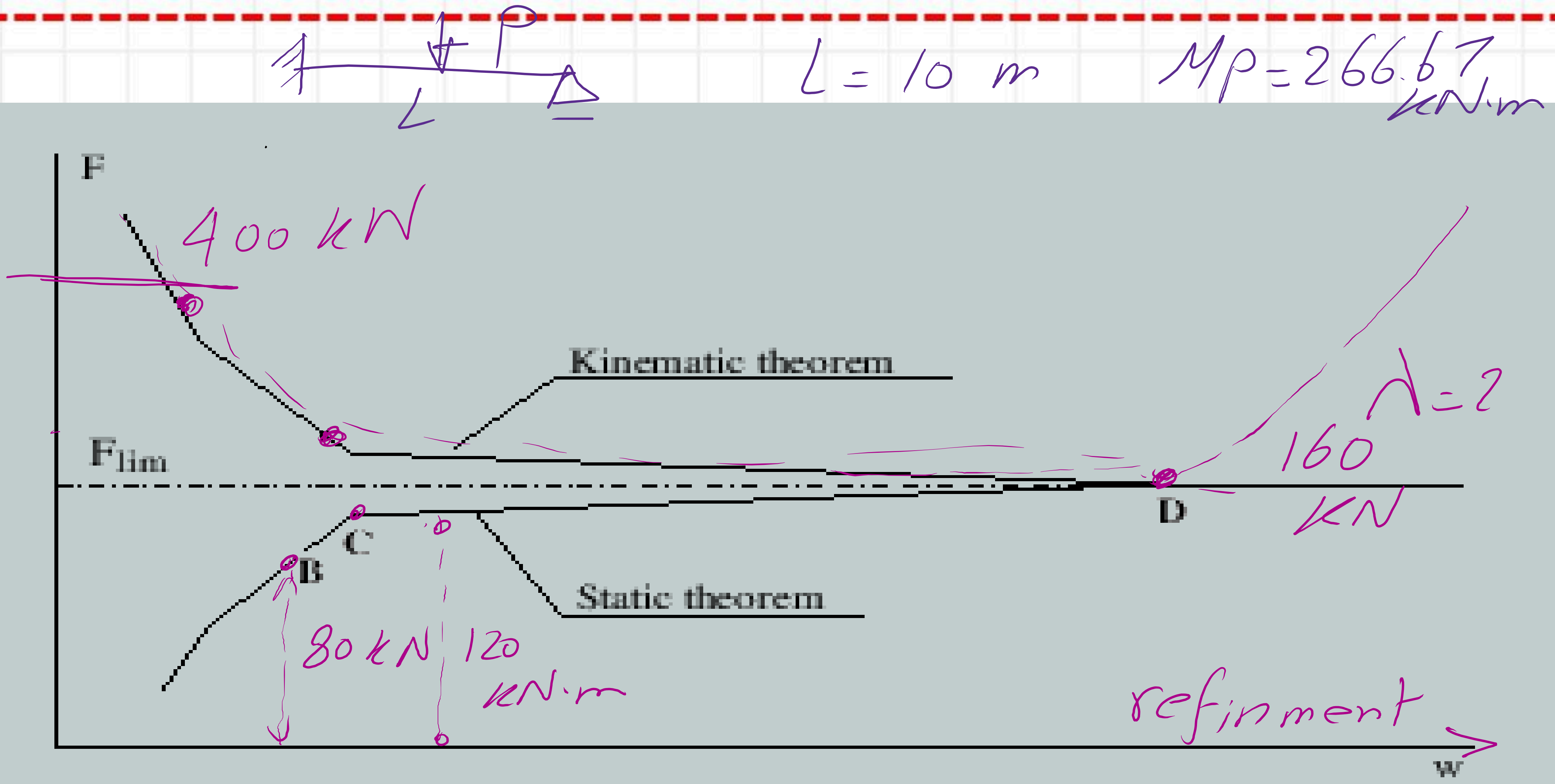


Figure 7.6: Upper and lower bound values