The Equilibrium (or Statical) Method

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In this method, free and reactant bending moment diagrams are drawn. These diagrams are overlaid to identify the likely locations of plastic hinges. This method therefore satisfies the equilibrium criterion first leaving the two remaining criterion to derived therefrom.

https://www.colincaprani.com/

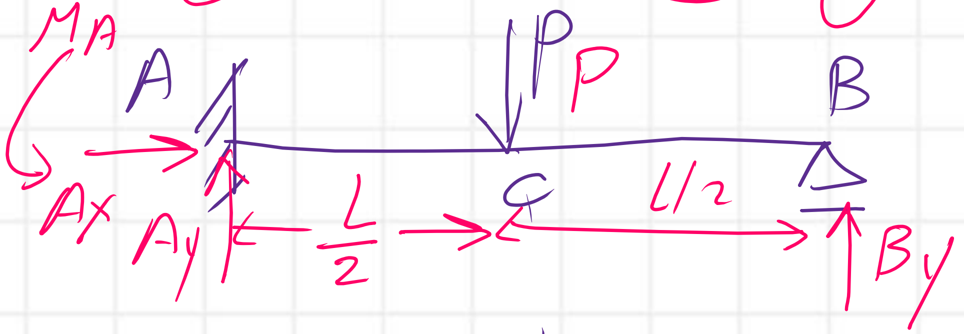
Structural Analysis III

The Kinematic (or Mechanism) Method

In this method, a collapse mechanism is first postulated. Virtual work equations are then written for this collapse state, allowing the calculations of the collapse bending moment diagram. This method satisfies the mechanism condition first, leaving the remaining two criteria to be derived therefrom.

Prepared by Eng.Maged Kamel.

Consider a statically indeterminate Beam M_p at $\boxed{A}$



Convert to a statically determinate

No of Reaction = 4

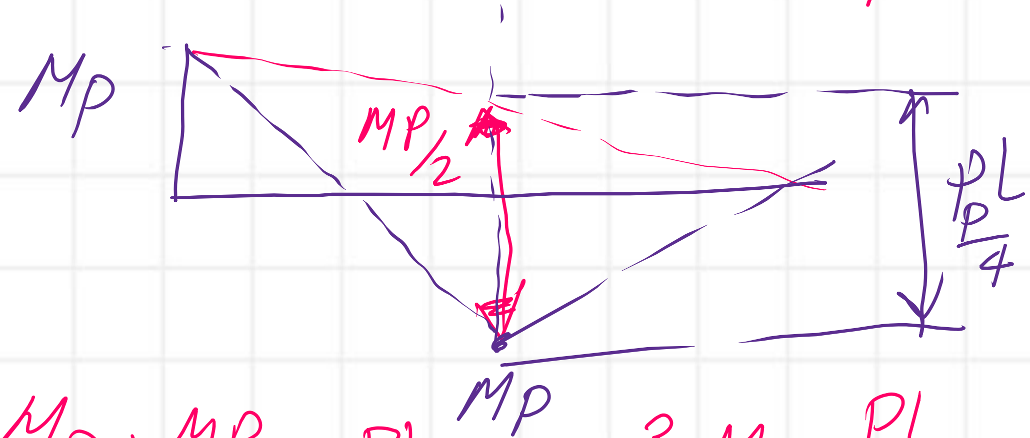
No of Equations = 3

$\Sigma X = 0$

$\Sigma Y = 0$

$\Sigma M = 0$

Find P_p M_p



introduce a hinge at A ⇒ system is determinate.

AT failure add another hinge at C

$4-3-2=\boxed{-1}$

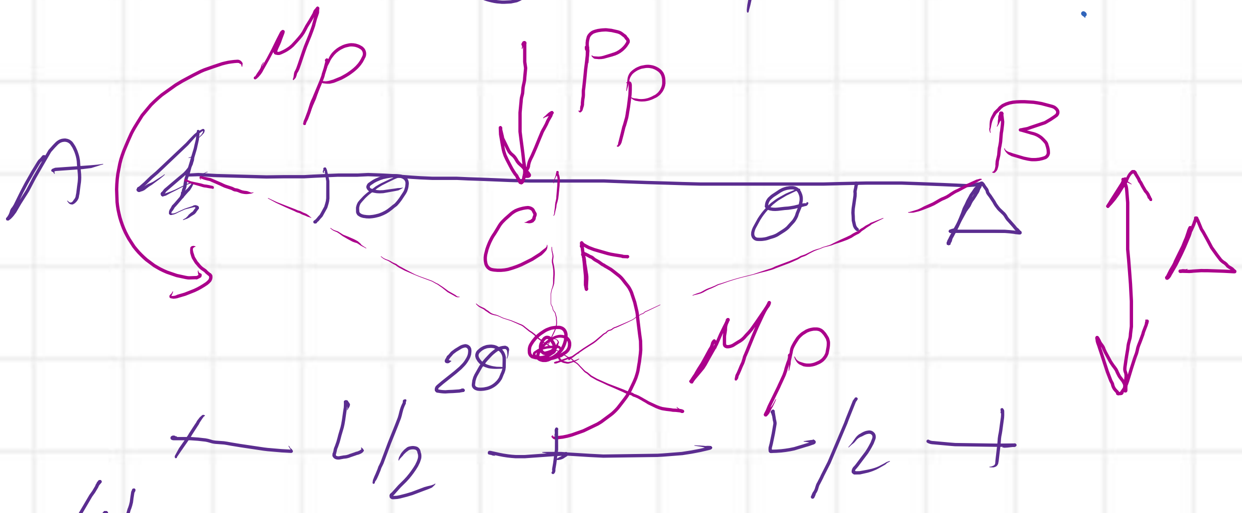
$$M_p+\frac{M_p}{2}=\frac{PL}{4} \Rightarrow \frac{3}{2}M_p=\frac{PL}{4}$$

$$M_p=\frac{1}{6}P_pL \Rightarrow \frac{M_p}{2}=P_pL/12$$

$$M_C=\frac{P_pL}{4}-\frac{P_pL}{12}=\frac{P_pL}{6}$$

$$\Rightarrow M_p=P_bL/6 \Rightarrow P_b=6M_p/L$$

Solving by Kinematic Method



$$\theta = \tan\theta = \frac{\Delta}{L/2}$$

$$\Delta = \frac{L\theta}{2}$$

$$W_i = W_E$$

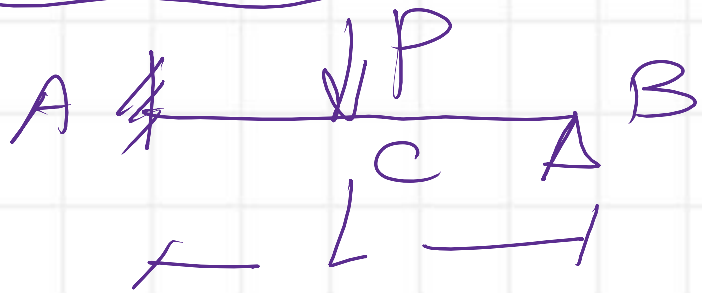
$$\Rightarrow P_p \cdot \Delta = M_p\theta + 2M_p(\theta)$$

$$P_p\left(\frac{L\theta}{2}\right) = \theta(3M_p)$$

$$P_p = \frac{6M_p}{L} \rightarrow M_p = \frac{P_p L}{6} \Rightarrow \text{Same result}$$

We Use the incremental method

A given Beam with l



$l = 1\,m$

with given M_y : yielding moment $= 7.5\ kN.m$

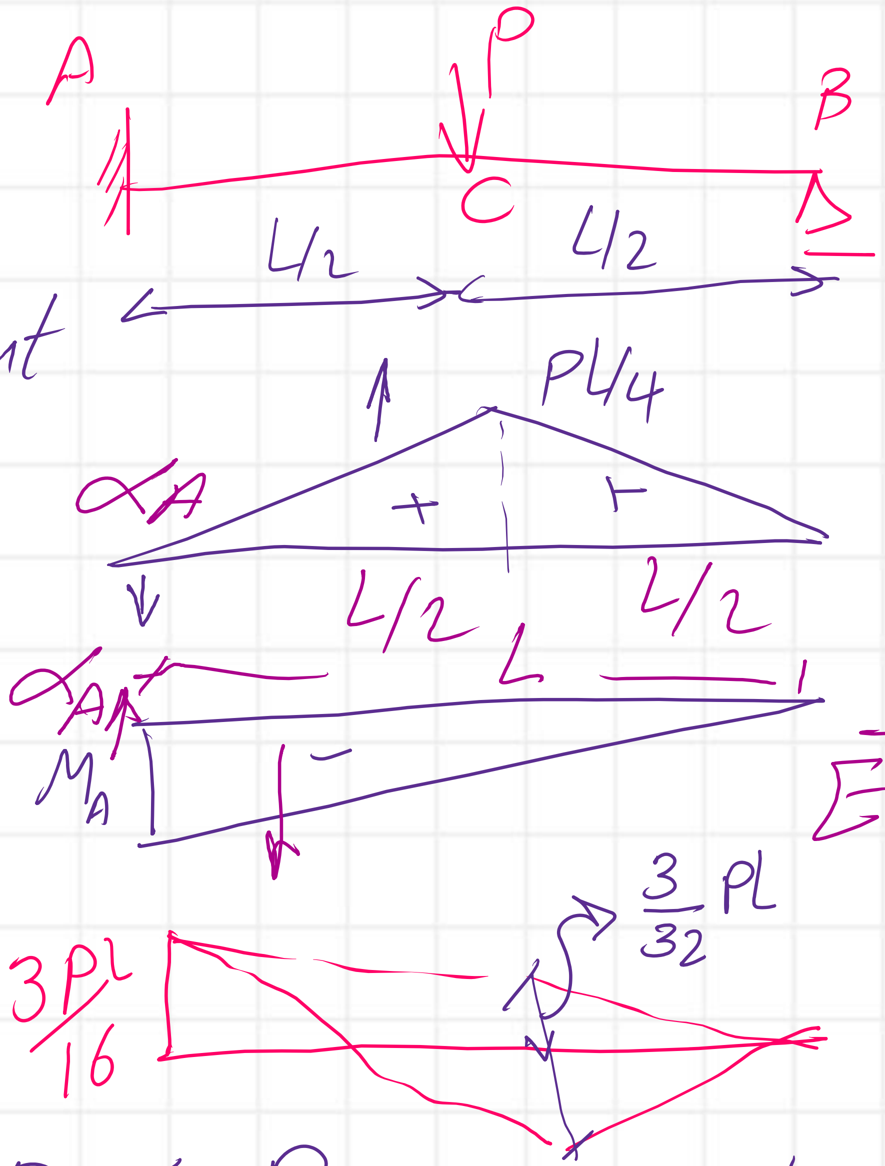
$M_p =$ Plastic moment $= 9\ kN.m$

We will start to apply increments

We need to Estimate M_A, M_C value from statics

What is the value of M_A?

slope at $A = \alpha$



$\frac{3PL}{16}$

$\alpha_A = \frac{1}{2}\frac{PL}{4}\cdot\frac{L}{2}\left(\frac{1}{EI}\right)$ +ve Load

$\alpha_A = \frac{1}{2}\frac{M_A l}{EI}\left(\frac{2}{3}\right) = \frac{M_A L}{3EI}$ −ve Load

$\Sigma\alpha = 0$

$\frac{1}{EI}\times\frac{PL^2}{16} = \frac{1}{3}\frac{M_A L}{EI} \Rightarrow M_A = \frac{3P}{16}\cdot L$

$M_C = \frac{PL}{4} - \frac{3PL}{16(2)} = \frac{PL}{32}(8-3)$

$M_C = \frac{5PL}{32}$

For $P < P_p$ No plastic Hinge

$EI = 10\ kN.m^2$

$P_{working} = 32\ kN$ $\quad M_A = \frac{3PL}{16}$ $\quad l = 1m$, $P = 32$

Load of 32 kN

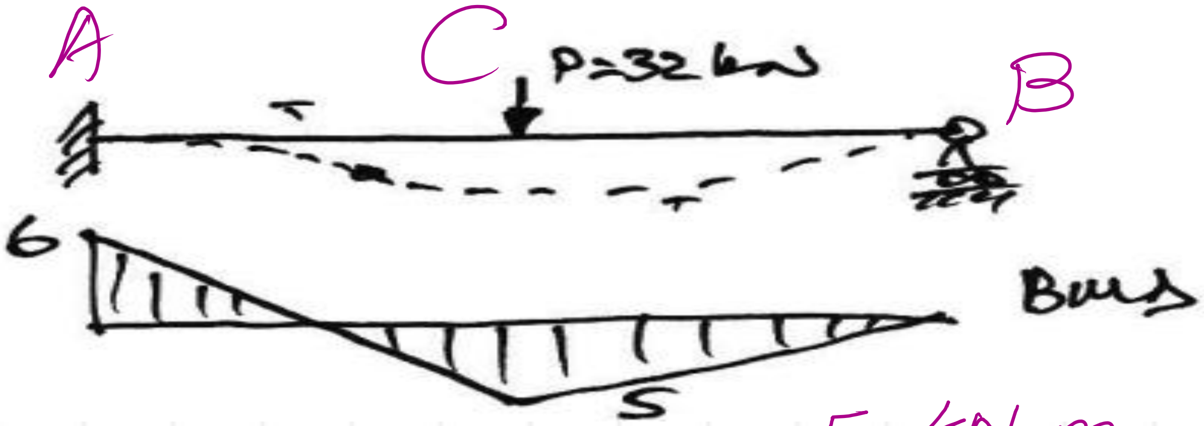
At this value of load the BMD is as shown, with:

$$M_A = \frac{3(32)(1)}{16} = 6\,\text{kNm} \qquad M_C = \frac{5(32)(1)}{32} = 5\ \text{kNm}$$

$M_C = \frac{5PL}{32}$

Since the peak moments are less than the yield moments, we know that yield stress has not been reached at any point in the beam. Also, the maximum moment occurs at *A* and so this point will first reach the yield moment.

$M_y = 7.50\ kN.m > M_C > M_A$



$6\ kN.m$

$\to 5\ kN.m$

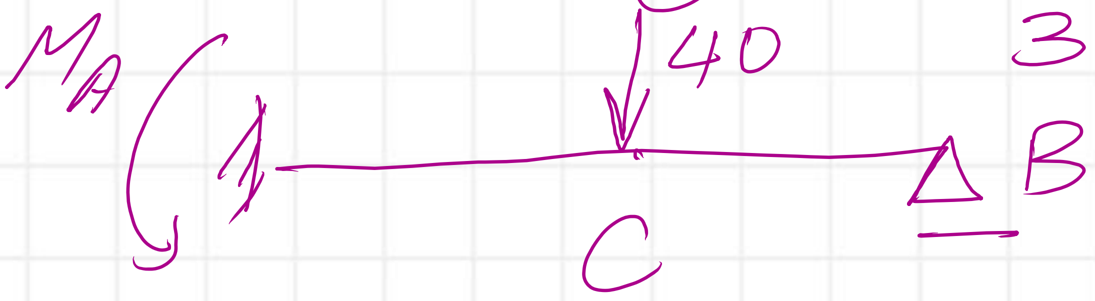
$P < P_y$

For yielding

$\lambda = \frac{40}{32} = 1.25$

$M_{A_{max}} = M_y = 7.50$ kN.m $= 3 P_y \frac{L}{16}$

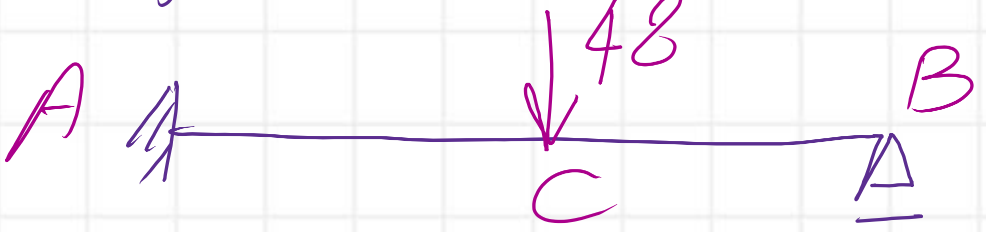
$P_y = \frac{16(7.5)}{3} = 40$ kN $\Longrightarrow$



$M_A = M_y$

$M_C = \frac{5 P_y L}{32} = \frac{5(40)(1)}{32} = 6.25$ kN.m $< M_y$

Load of 48 kN



9 kN.m = M_p

B.M

First Plastic Hinge is develbped at A

No more moment to be added

only one hinge

$M_y = 7.50$ kN.m

$M_p = 9$ kN.m

$M_A = \frac{3}{16} PL$

$M_C = \frac{5}{32} PL$

$M_A = \frac{3}{16}(48)(1) = 9$ kN.m $= M_p$

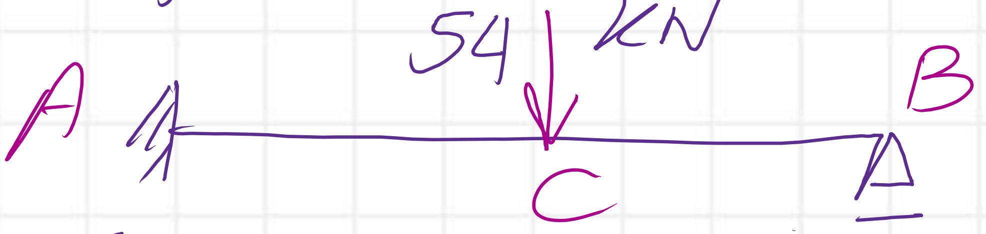
$M_C = \frac{5}{32}(48)(1) = 7.5$ kN.m

$\lambda = \frac{48}{32} = 1.5$

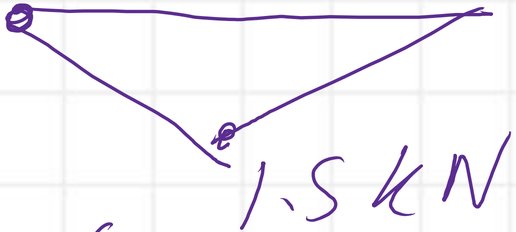
Load of 54 kN

$M_y = 7.50$ kN.m

$M_p = 9$ kN.m $= M_A$



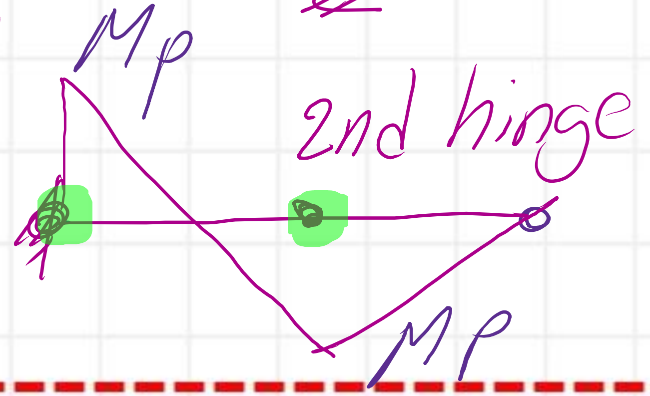
$P = 54$ kN



$\frac{PL}{4} = \frac{6}{4}$

$= 1.5$

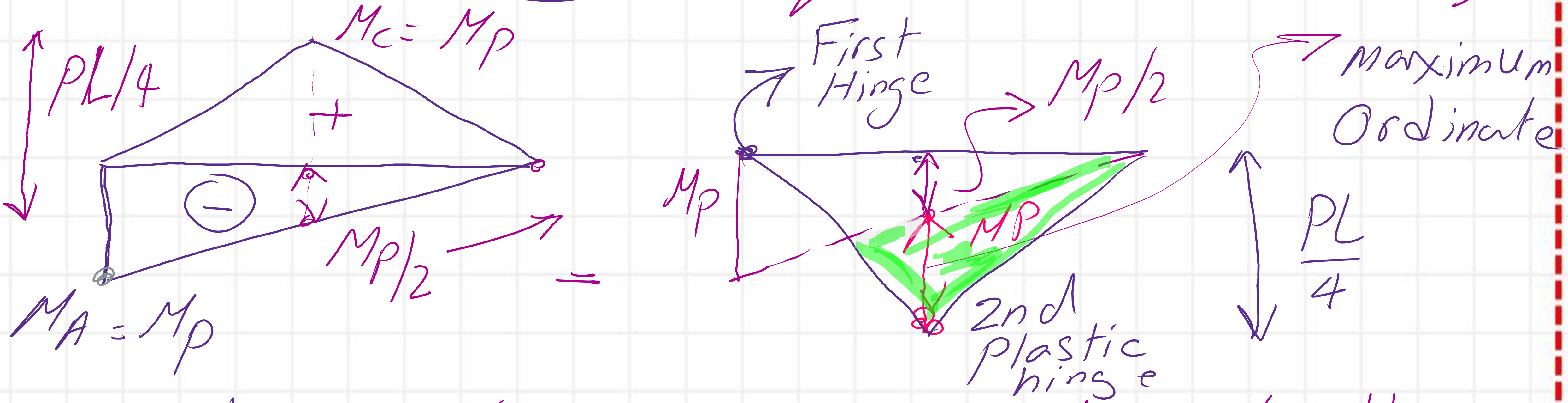
add



$\Rightarrow$ Check $P_p = 6\frac{M_p}{L}$

$= 6(9)/1 = 54$ kN

Statical method (Equilibrium method)



Two diagrams (+ve) & (−ve) are Laid together

$$\frac{PL}{4} - \frac{M_p}{2} = M_p \Rightarrow \frac{3}{2} M_p = \frac{PL}{4}$$

$$P = \frac{6 M_p}{L}$$

$$\lambda_c = \frac{\text{Collapse Load}}{\text{Working Load}} = \frac{54}{32} = 1.6875$$

$=$ $P_p = 54\ kN$

$P_w = 32\ kN$

3.3 *Important Definitions*

Load Factor

The load factor for a possible collapse mechanism i, denoted λ_i, is of prime importance in plastic analysis:

$$\lambda_i = \frac{\text{Collapse Load for Mechanism } i}{\text{Working Load}}$$

The working load is the load which the structure is expected to carry in the course of its lifetime.

Factor of Safety

This is defined as:

$$\text{FoS} = \frac{\text{First yield load}}{\text{Working Load}}$$

$$\frac{P_y}{P_w} = \frac{40}{32} = 1.25$$

The FoS is an elastic analysis measure of the safety of a design. For our example:

$$\text{FoS} = \frac{40}{32} = 1.25$$