Summary of Post-27 Content - LTR

(a) Find L_p the Limiting effective Unbraced Length

(b) Find L_r : Limiting effective Unbraced length for Elastic LTB

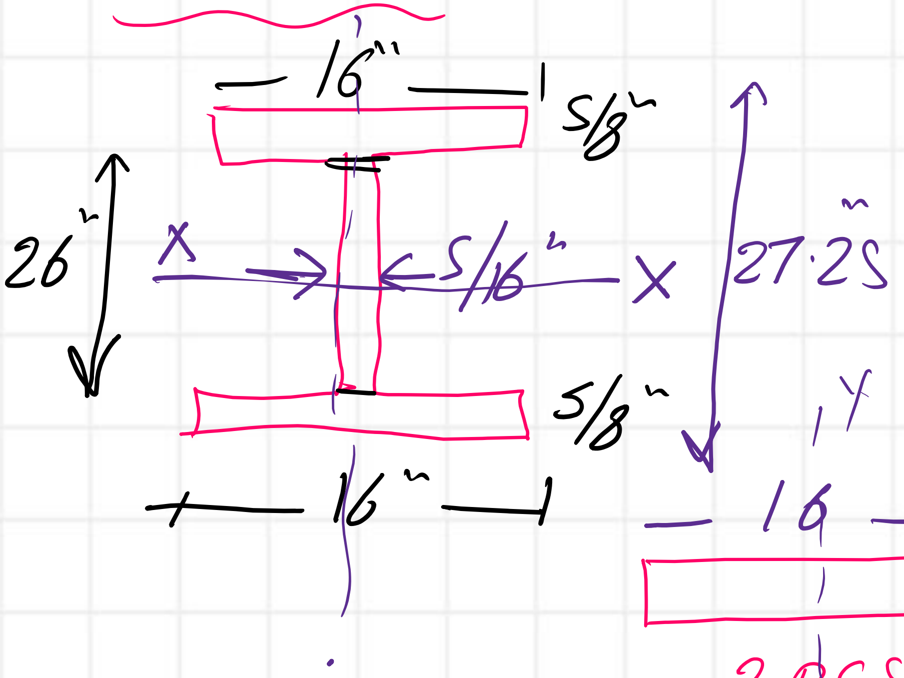
(c) Find C_B, the Coefficient of bending for beam AB → 45' which is Laterally supported at third points

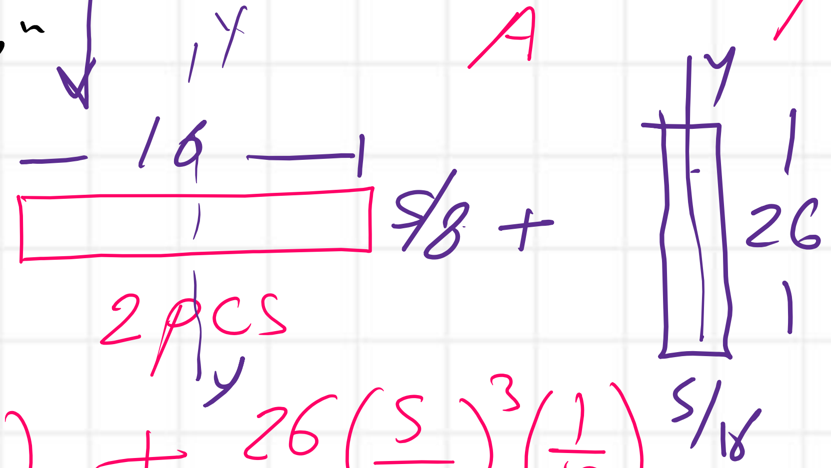
L.T.B

$$L_p = 300\, r_y \sqrt{F_y}$$

We need to Estimate I_y, A

$$\frac{I_y}{A} = r_y^2 \Rightarrow r_y = \sqrt{\frac{I_y}{A}}$$





$$I_y = \frac{5}{8}\frac{(16)^3}{12}(2) + 26\left(\frac{5}{16}\right)^3\left(\frac{1}{12}\right)$$

$$= 426.67 \simeq 427 \text{ inch}^4$$

$$A = 2(16)\left(\frac{5}{8}\right) + 26\left(\frac{5}{16}\right)$$

$$= 28.125 \text{ inch}^2$$

$$r_y = \sqrt{\frac{427}{28.125}} = 3.896 \text{ inch}$$

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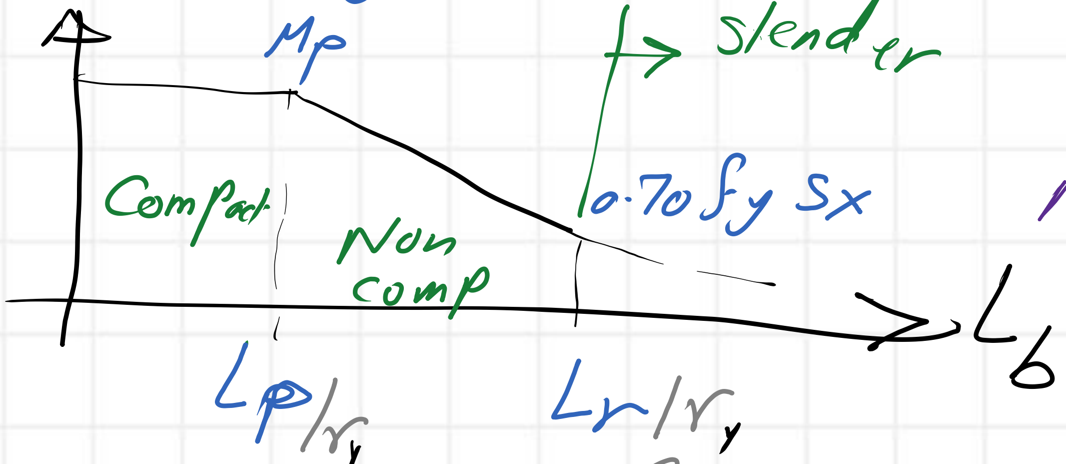
L.T.b $\quad L_b = \frac{45}{3} = 15'$

$E = 29000$ ksi , $f_y = 65$ ksi

$C_b = 1$

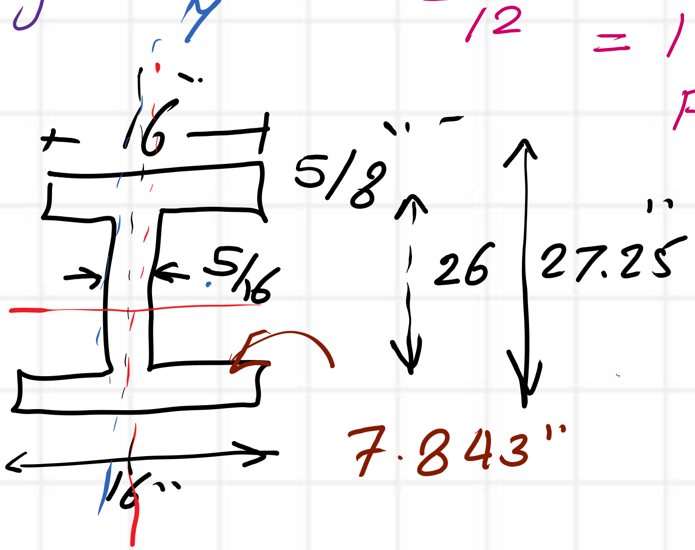


$M_p = 1728$ Ft-kips $\quad$ LTB

$M_r = 1114$ Ft-kips

$M_p = Z_x F_y = 319.06(65)\frac{1}{12} = 1728.24$ FT.kips

$M_r = 0.7 F_y S_x = 0.7(65)\frac{(293.8)}{12} = 1113.99$ FT.kips



$\frac{L_p}{r_y} = \frac{300}{\sqrt{f_y}} = \frac{300}{\sqrt{65}} = 37.21$

$F_y = 65$ ksi

$L_p = \frac{300\, r_y}{\sqrt{F_y}} = \frac{300(3.896)}{\sqrt{65}} = 144.93'' \rightarrow /12 \rightarrow$

$L_p = 12.078'$

$L_b = 15'$ given

$L_b > L_P$

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Beer chapter-3

The determination of the stresses in noncircular members subjected to a torsional loading is beyond the scope of this text. However, results obtained from the mathematical theory of elasticity for straight bars with a *uniform rectangular cross section* are given here for our use.† Denoting by L the length of the bar, by a and b, respectively, the wider and narrower side of its cross section, and by T the magnitude of the torque applied to the bar (Fig. 3.44), the maximum shearing stress occurs along the center line of the *wider* face and is equal to

$$\tau_{\max} = \frac{T}{c_1 a b^2} \tag{3.40}$$

The angle of twist can be expressed as

$$\phi = \frac{TL}{c_2 a b^3 G} \tag{3.41}$$

Coefficients c_1 and c_2 depend only upon the ratio a/b and are given in Table 3.1 for a number of values of that ratio. Note that Eqs. (3.40) and (3.41) are valid only within the elastic range.

Table 3.1 shows that for $a/b \geq 5$, the coefficients c_1 and c_2 are equal. It may be shown that for such values of a/b, we have

$$c_1 = c_2 = \tfrac{1}{3}(1 - 0.630 b/a) \qquad (\textit{for } a/b \geq 5 \textit{ only}) \tag{3.42}$$

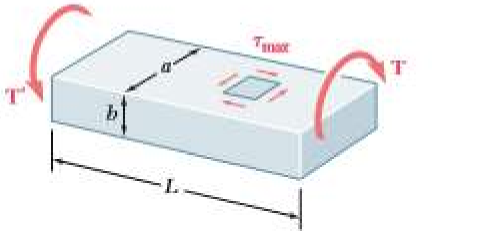


Fig. 3.44 Shaft with rectangular cross section, showing the location of maximum shearing stress.

Table 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

$$\phi = \frac{TL}{J_p G}$$

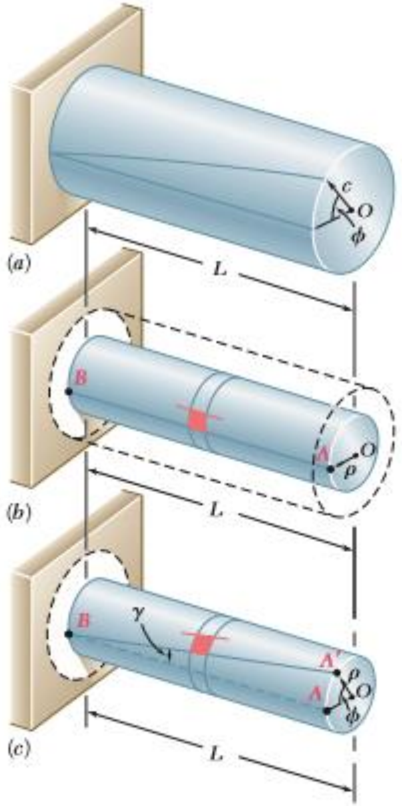
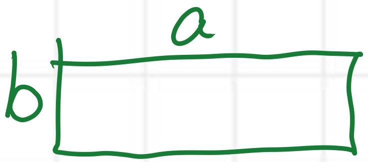


Fig. 3.13 Shearing strain deformation. (a) The angle of twist ϕ. (b) Undeformed portion of shaft of radius ρ. (c) Deformed portion of shaft; angle of twist ϕ and shearing strain γ share the same arc length AA'.

$R\phi$

ϕ



$$f = \frac{M\, y_{max}}{I}$$

$$\tau_{max} = \frac{T \cdot R}{J_p}$$

$$\tau_{max} = \frac{T}{\frac{1}{3} a b^2}$$

$$\phi = \frac{TL}{G(\frac{1}{3} a b^3)}$$

J:

$$\gamma_{max} = \frac{R\phi}{L} = \frac{\tau_m}{G} = \frac{TR}{J_p G}$$

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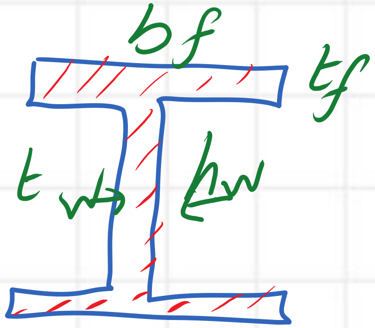
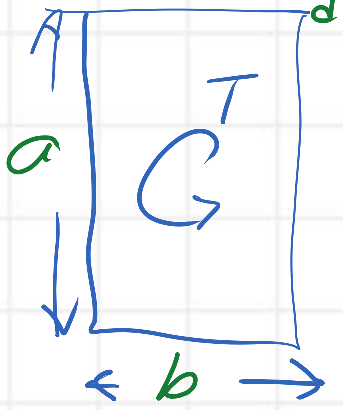




Height more than 5 depth

$$\tau_{max} = \frac{TL}{C_1 ab^2}$$

$$\phi = \frac{TL}{C_2 ab^3 G}$$

$a > 5b$

$C_1 = C_2 = \frac{1}{3}$

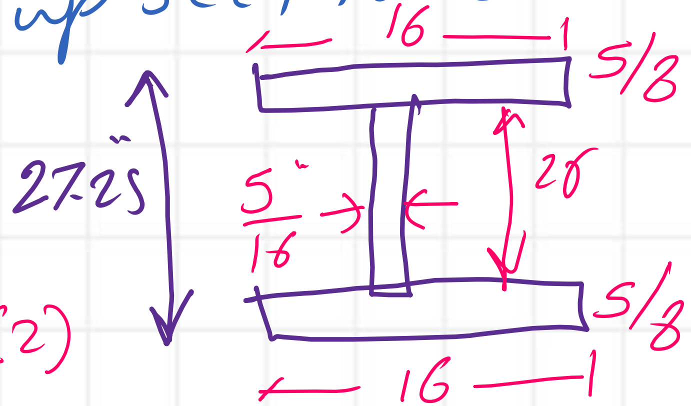
$$\tau_{max} = \frac{TL}{\frac{1}{3} ab^2}$$

$$\phi = \frac{TL}{\frac{1}{3} ab^3 G}$$

J: Torsional Coefficient → AISC $= \frac{1}{3} ab^3$

$J = \sum \frac{1}{3} ab^3$ Case of I beam [Rolled / built up sections] or T sections

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7F_y}{E} \frac{S_x}{J} \frac{h_o}{c} \right)^2}}$$



$a_1 = 16"$
$b_1 = 5/8$
$a_2 = 26"$
$b_2 = 5/16"$

$$J = \sum \frac{1}{3} ab^3 \Rightarrow \frac{1}{3}(16)\left(\frac{5}{8}\right)^3(2) + \frac{1}{3}(26)\left(\frac{5}{16}\right)^3 = 2.8686 \text{ inch}^3$$

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$$S_x = 293.78 \text{ inch}^3 \Rightarrow 294 \text{ inch}^3$$

where

r_y = radius of gyration about y-axis, in. (mm)

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$

r_{ts} = effective radius of gyration, in (provided in AISC Table 1

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I-shapes

h_o = distance between flange centroids, in (AISC Table 1-1)

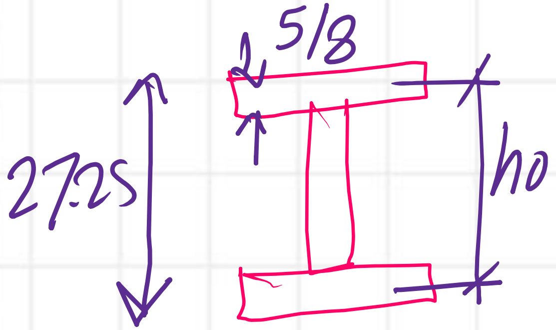
$$I_y = 427 \text{ inch}^4$$

$$C_w = \frac{I_y h^2}{4c^2}$$

$$r_{ts} = \sqrt{\frac{\sqrt{I_y C_w}}{S_x}}$$

$$C_w = \frac{I_y h^2}{4c^2}$$

$$c = \begin{cases} 1 & \text{for doubly - symmetric I - shapes} \\ \frac{h}{2}\sqrt{\frac{I_y}{C_w}} & \text{for channels} \end{cases}$$



$$h_o = h - t_f = 27.25 - \frac{5}{8} = 26.625''$$

$$c = 1$$

$$C_w = \frac{427(26.625)^2}{4} = 75674.07 \text{ inch}^6$$

$$r_{ts} = \sqrt{\frac{\sqrt{427(75674.07)}}{294}} = 4.397 \approx 4.40''$$

$$\frac{E}{0.7F_y} = \frac{29000}{0.7(65)} = 637.36$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7F_y}{E} \frac{S_x}{J} \frac{h_o}{c} \right)^2}}$$

$$L_r = 1.95(4.395)\left(\frac{29\times10^3}{0.7(65)}\right) \sqrt{\frac{J(1)}{294\,(26.625)}} \sqrt{\sqrt{}}$$

$$= 61.7392\sqrt{J} \sqrt{1+\sqrt{1+6.67\left(\frac{0.7(65)}{29000} \cdot \frac{294}{J} \frac{(26.625)}{1}\right)^2}}$$

$S_x = 294 \text{ inch}^3$

$C_w = I_y h^2/4c^2$

where

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$

r_y = radius of gyration about y-axis, in. (mm)

r_{ts} = effective radius of gyration, in (provided in AISC Table 1

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I-shapes

h_o = distance between flange centroids, in (AISC Table 1-1)

$$C_w = \frac{I_y h^2}{4c^2}$$

$$c = \begin{cases} 1 & \text{for doubly-symmetric I-shapes} \\ \dfrac{h}{2}\sqrt{\dfrac{I_y}{C_w}} & \text{for channels} \end{cases}$$

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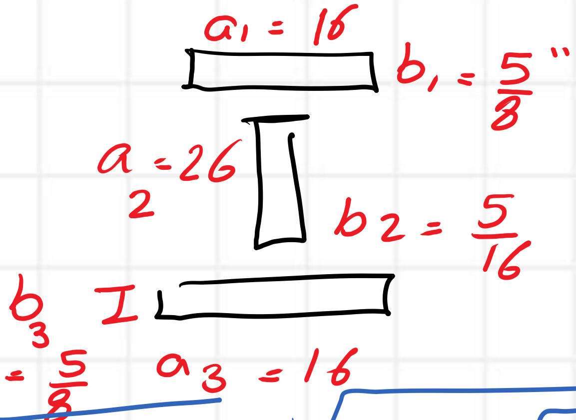
$$L_r = 1.95(4.395)\left(\frac{29\times10^3}{0.7(65)}\right)\sqrt{\frac{J(1)}{294\,(26.625}}$$

$$= 61.7392\sqrt{J}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(65)}{29000}\cdot\frac{294}{J}\left(\frac{26.625}{1}\right)\right)^2}}$$

$$L_r = 61.7392\sqrt{J}\sqrt{1+\sqrt{1+6.76\left(\frac{12.2815}{J^2}\right)}}$$

$$J = \sum \frac{1}{3} a b^3$$



$$J = \frac{1}{3}\sum\left((16)\left(\frac{5}{8}\right)^3(2) + 26\left(\frac{5}{16}\right)^3\right)$$

$$J = \frac{1}{3}\left[7.8125 + 0.79346\right] = 2.868\ \text{inch}^3$$

$$L_r = 61.7392\sqrt{2.868}\sqrt{1+\sqrt{1+123.962}}$$

$$L_r = 104.575\sqrt{1+11.1786} = 104.575(3.4897) \rightarrow \div 12 \rightarrow = 30.41'$$

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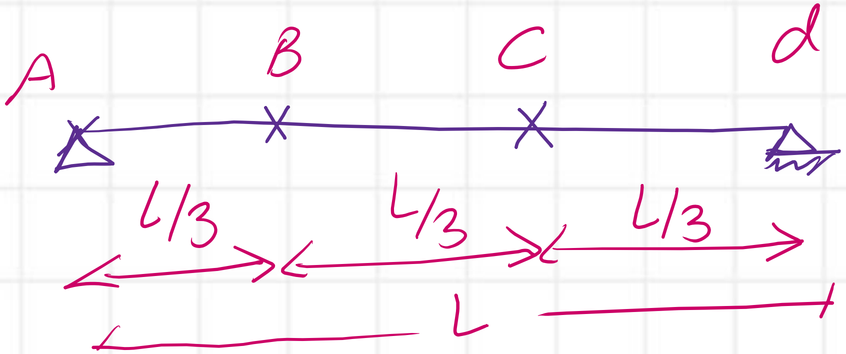
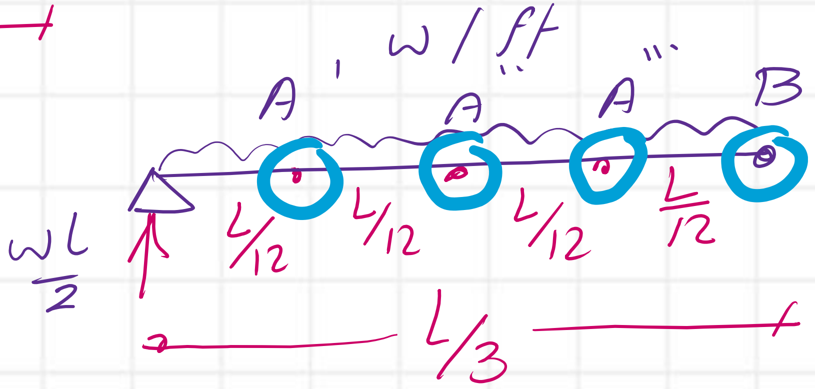


$L = 45'$

For Left part A-B
right part C-d



Divide to Four parts

Divide each part into 4 Quarters, A', A'', A'''

Find moment value at Each point

$$M_{A'} = \frac{wL}{2}\left(\frac{L}{12}\right) - w\left(\frac{L}{12}\right)\left(\frac{L}{24}\right) = \frac{wL^2}{12(24)}(11).$$

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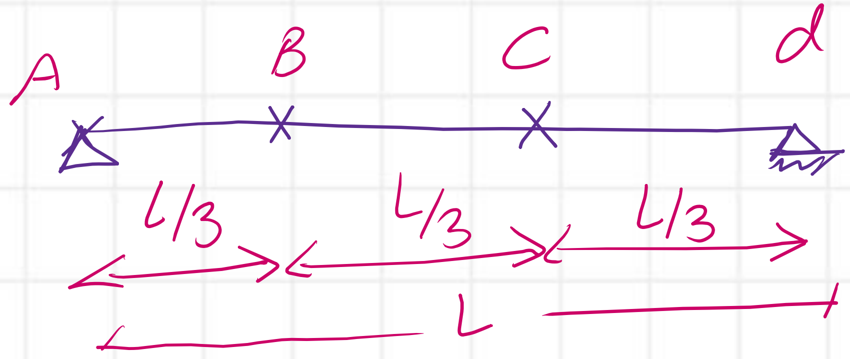
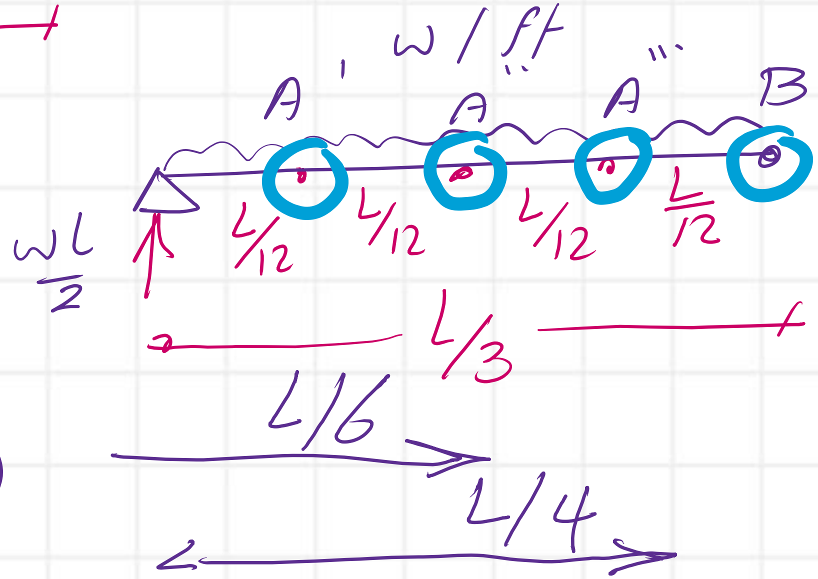


$L = 45'$

For Left part — A - B

right part — C - d



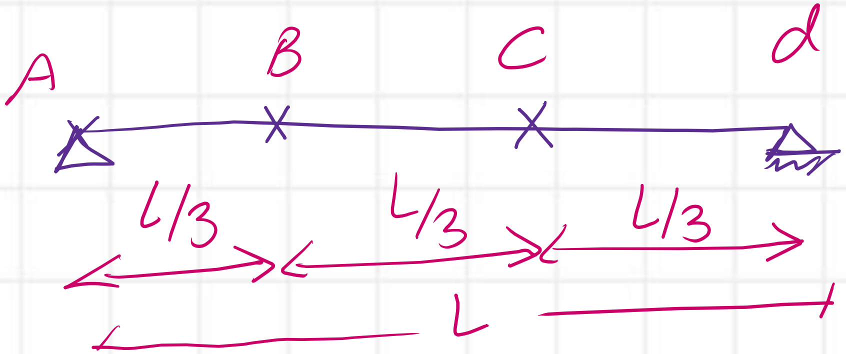
Divide to Four parts

Find moment at A''

$$\frac{wL}{2}\left(\frac{L}{6}\right) - w\left(\frac{L}{6}\right)\left(\frac{L}{12}\right)$$

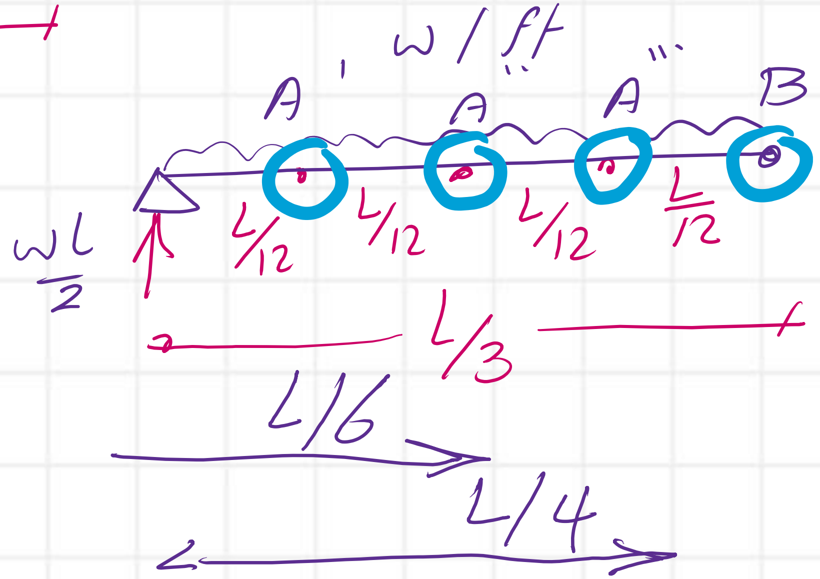
$$M_{A''} = \frac{wL^2}{72}(6-1) = 5\frac{wL^2}{72}$$

$$M_{A}'' = \frac{wL}{2}\left(\frac{L}{4}\right) - wL^2\left(\frac{L}{4}\right)\left(\frac{L}{8}\right) = \frac{wL^2}{32}(4-1) = \frac{3wL^2}{32}$$



$L = 45'$

For Left part A - B
right part C - d



Divide to Four parts

Find moment at B

$$\frac{wL}{2}\left(\frac{L}{3}\right) - w\left(\frac{L}{3}\right)\left(\frac{L}{6}\right)$$

$$M_B = \frac{wL^2}{18}(3-1) = wL^2\left(\frac{2}{18}\right) = \frac{wL^2}{9}$$

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$$C_b \text{ value} = \frac{12.5\, M_{max}}{2.5\, M_{max} + 3M_{A'} + 4M_{A''} + 3M_{A'''}}$$

M_{max} max of

$$M_{A'} = \frac{11}{12(24)} wL^2$$

$$M_{A''} = \frac{5w}{72} L^2$$

$$M_{A'''} = \frac{3}{32} wL^2$$

$$M_B = \frac{wL^2}{9} \Rightarrow M_{max}$$

$$C_b = \frac{12.5\,(wL^2/9)}{wL^2\left[\left(\frac{2.5}{9}\right) + \frac{3(11)}{12(24)} + \frac{4(5)}{72} + 3\left(\frac{3}{32}\right)\right]}$$

$$C_b = \frac{(12.5/9)}{\frac{80+33+80+81}{12(24)}} =$$

$$C_b = \frac{(12.5/9)}{\left(\frac{80+33+80+81}{12(24)}\right)} = \frac{12.5}{9}\,\frac{(12)(24)}{274}$$

$$C_b = 1.4598 \Rightarrow 1.46$$