Second Solved Problem for LU decomposition

Using Crout's method 3x3 matrix

option-1

Turn Matrix A into a Lower matrix L

→ Column operations

Elementary matrices

get U^{-1} → inverse of U matrix

Example # 2

Write the lower and upper matrix.

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

Use Crout's method

Solution

Using 1st option: $A \rightarrow$ Lower matrix $\rightarrow$ option-1

Find determinant

(1) check if matrix A is invertible.

$$|A| = 10 \begin{vmatrix} -10 & 3 \\ 2 & -10 \end{vmatrix} - 3 \begin{vmatrix} 2 & 3 \\ 3 & -10 \end{vmatrix} + 4 \begin{vmatrix} 2 & -10 \\ 3 & 2 \end{vmatrix}$$

$$= 10(100-6) - 3(-20-9) + 4(4+30)$$

$$= 940 + 87 + 136 = 1163 \neq 0$$

matrix is invertible

has L, U

Prepared by Eng.Maged Kamel.

Example : #2

Write the lower and upper matrix for Crout's method

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

Solution : option-1

$A \rightarrow$ Lower matrix

$A \begin{cases} \rightarrow L \\ \rightarrow U \end{cases}$

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \longrightarrow$$

get U_{12} & U_{13}

option-1

Convert $A \rightarrow$ Lower matrix

$$\begin{array}{ccc} & C_2 & \\ \end{array}$$

$$\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix}$$

$$\begin{bmatrix} 10 & 3 & 4 \end{bmatrix} = \begin{bmatrix} L_{11} & 3/10 & 4/10 \end{bmatrix}$$

$$\begin{array}{ccc} & U_{12} & U_{13} \end{array}$$

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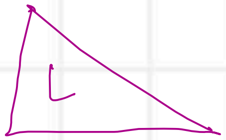
$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \Longrightarrow$$

$$= \frac{a_{12}}{a_{11}} = \frac{3}{10} \rightarrow (-1)\,C_1 + C_2 = -0.3\,C_1 + C_2$$

$$= \frac{a_{13}}{a_{11}} = \frac{4}{10} = (-1)\,C_1 + C_3 = -0.4\,C_1 + C_3$$



$$\begin{bmatrix} 10 & -0.3(10)+3 & -0.4(10)+4 \\ 2 & -0.3(2)-10 & -0.4(2)+3 \\ 3 & -0.3(3)+2 & -0.4(3)-10 \end{bmatrix}$$

$$>> \begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 2.20 \\ 3 & 1.1 & -11.20 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & L_{22}U_{23} \\ L_{31} & L_{32} & L_{32}U_{23} + L_{33} \end{bmatrix}$$

$$\left.\begin{aligned} L_{11} &= 10 \\ L_{21} &= 2 \\ L_{31} &= 3 \end{aligned}\right\} \quad \left.\begin{aligned} L_{22} &= -10.6 \\ L_{32} &= 1.10 \end{aligned}\right\}$$

$$U_{23} = \frac{2.20}{(-10.60)} = -\frac{11}{53}$$

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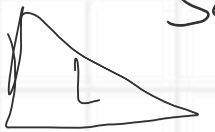
pivot

$$L_1 \rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 2 & \boxed{-10.6} & 2.20 \\ 3 & 1.10 & -11.20 \end{bmatrix}$$

Column one

same



Colum 2

$5\ C_2 + C_3$

Consider -10.60 as a pivot

divide $\frac{2.20}{-10.60} = -\frac{11}{53}$ $(-1)\,C_2 + C_3 = \frac{11}{53} C_2 + C_3$

$U_{23} = \frac{-11}{53}$

$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & -10.60\left(\frac{11}{53}\right) + 2.20 \\ 3 & 1.10 & 1.10\left(\frac{11}{53}\right) - 11.20 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.10 & -\frac{1163}{106} \end{bmatrix}$$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{12} & L_{22} & 0 \\ L_{13} & L_{32} & L_{33} \end{bmatrix}$$

$L_{33} = -\frac{1163}{106}$

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$$L_{11} \quad A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \Rightarrow$$

L_{21}, L_{31}

$U_{12} = \frac{3}{10}$, $U_{13} = \frac{4}{10}$

$L_{21} = 2$, $L_{11} = 10$

$L_{31} = 3$, $U_{23} = -\frac{11}{53}$

$U_{11} = 1$

$U_{22} = 1$

$U_{33} = 1$

$L_{33} = -\frac{1163}{106}$ $\quad L_{22} = -10.60$, $L_{32} = 1.10$

Check $L \cdot U = A$

$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.60 & 0 \\ 3 & 1.10 & -\frac{1163}{106} \end{bmatrix} \begin{bmatrix} 1 & \frac{3}{10} & \frac{4}{10} \\ 0 & 1 & -\frac{11}{53} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & \frac{8}{10} + \frac{11}{5} \\ 3 & 0.9 + 1.1 & 1.2 - \frac{121}{530} - \frac{1163}{106} \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} = A$$

OK

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if we wish to use Elementary matrices

$$\underset{A}{\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}} \rightarrow \underset{E_1}{\begin{bmatrix} 1 & -\frac{3}{10} & -\frac{4}{10} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\underset{AE_1}{\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 2.2 \\ 3 & 1.1 & -11.2 \end{bmatrix}} \underset{E_2}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{2.2}{10.6} \\ 0 & 0 & 1 \end{bmatrix}} \Rightarrow \underset{L}{\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.1 & -\frac{1163}{106} \end{bmatrix}}$$

$$\left.\begin{aligned} L_{11} &= 10 \\ L_{21} &= 2 \\ L_{31} &= 3 \end{aligned}\right\} \rightarrow$$

$$A \cdot E_1 \cdot E_2 = L$$

$$L U E_1 E_2 = L \quad / L$$

$$I U (E_1 E_2) = I$$

$$E_1 E_2 = U^{-1}$$

$$\underbrace{\begin{bmatrix} 1 & -\frac{3}{10} & -\frac{4}{10} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{2.2}{10.6} \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} \Rightarrow \overbrace{\begin{bmatrix} 1 & -\frac{3}{10} & -\frac{49}{106} \\ 0 & 1 & \frac{2.2}{10.6} \\ 0 & 0 & 1 \end{bmatrix}}^{U^{-1}}$$

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} >$$

$$U_{11} = U_{22} = U_{33} = 1$$

$$U_{12} = 3/10 \quad \& \quad U_{23} = -\frac{11}{53}$$

$$U_{13} = 4/10$$

$$\underset{U}{\begin{bmatrix} 1 & \frac{3}{10} & \frac{4}{10} \\ 0 & 1 & -\frac{11}{53} \\ 0 & 0 & 1 \end{bmatrix}} \underset{U^{-1}}{\begin{bmatrix} 1 & -\frac{3}{10} & -\frac{49}{106} \\ 0 & 1 & \frac{2.2}{10.6} \\ 0 & 0 & 1 \end{bmatrix}} \Rightarrow \underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$