First Solved Problem for LU decomposition

Using Crout's method 3x3 matrix

option-1

Turn Matrix A into a Lower matrix L

→ Column operations

Elementary matrices

get U^{-1} → inverse of U matrix

Example # 1

Write the lower and upper matrix for Crout 's method

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow |A| = +10$$

$|A| \neq 0$

Solution : option-1

$A \to$ Lower matrix

$A \begin{cases} \to L \\ \to U \end{cases}$

$U_{12} = 1 = U_{13}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

option -1

Convert $A \to$ Lower matrix

$$\begin{array}{ccc} & C_2 & \\ \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & \frac{1}{1} & \frac{1}{1} \end{bmatrix}$$

$$\begin{array}{ccc} & U_{12} & U_{13} \end{array}$$

A

$$\begin{bmatrix} \boxed{1} & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \underset{\Downarrow}{-\frac{a_{12}}{a_{11}} \cdot C_1 + C_2} & -\frac{a_{13}}{a_{11}} \cdot C_1 + C_3 \\ & 0 & 0 \\ 4 & -1(4)+3 & -1(4)+(-1) \\ 3 & -1(3)+5 & -1(3)+3 \end{bmatrix}$$

$\Rightarrow$ Convert to an Lower matrix

$L \rightarrow L_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & -5 \\ 3 & +2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & L_{22} U_{23} \\ L_{31} & L_{32} & L_{32} U_{23} + L_{33} \end{bmatrix}$$

$L_{11} = 1$

$L_{21} = 4$

$L_{31} = 3$

$L_{22} = -1$

$L_{32} = +2$

get $\Rightarrow$

$$U_{23} = \frac{-5}{-1} = +5$$

Prepared by Eng.Maged Kamel.

$\frac{-5}{-1}(-C_2)+C_3$

Column one same

$5C_2+C_3$

Column 2

$$L_1 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 4 & \boxed{-1} & -5 \\ 3 & +2 & 0 \end{bmatrix}$$

Consider -1 as a pivot

$-\frac{5C_3}{C_2}C_2+C_3 \rightarrow C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & -5 \\ 3 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & -5(-1)-5 \\ 3 & 2 & -5(2)+0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} L_{11} & 0 & 0 \\ L_{12} & L_{22} & 0 \\ L_{13} & L_{32} & L_{33} \end{bmatrix} \Rightarrow L_{33} = -10$$

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L_{11} A

$$L_{21} \quad \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \Rightarrow$$

$U_{12} = +1, \quad U_{13} = 1$

$L_{21} = +4, \quad L_{22} = -1,$

$L_{32} = +2, \quad L_{33} = -10$

$U_{11} = 1$

$U_{22} = 1$

$U_{33} = 1$

$U_{23} = +5$

Check $L \cdot U = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = A$$

Check $\rightarrow$ Back to A

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if we wish to use Elementary matrices

Pivot

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}}_{A} \rightarrow \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} \rightarrow \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & -5 \\ 3 & 2 & 0 \end{bmatrix}}_{AE_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} \Rightarrow \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix}}_{L}$$

$$\left.\begin{matrix} L_{11} = 1, & L_{31} = 3 \\ L_{21} = 4 & L_{32} = 2 \end{matrix}\right\} \quad L_{33} = -10$$

$$A \cdot E_1 \cdot E_2 = L$$

$$L \cdot U \, E_1 E_2 = L \quad /L$$

$$I \, U \cdot (E_1 \cdot E_2) = I$$

$$E_1 E_2 = U^{-1}$$

$$\underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} \Rightarrow \overbrace{\begin{bmatrix} 1 & -1 & +4 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}}^{U^{-1}}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \longrightarrow$$

$U_{11} = U_{22} = U_{33} = 1$

$U_{12} = \frac{1}{1} = 1$ & $U_{23} = 5$

$U_{13} = \frac{1}{1} = 1$

$$\underset{U}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}} \underset{U^{-1}}{\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}} \Rightarrow \underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$