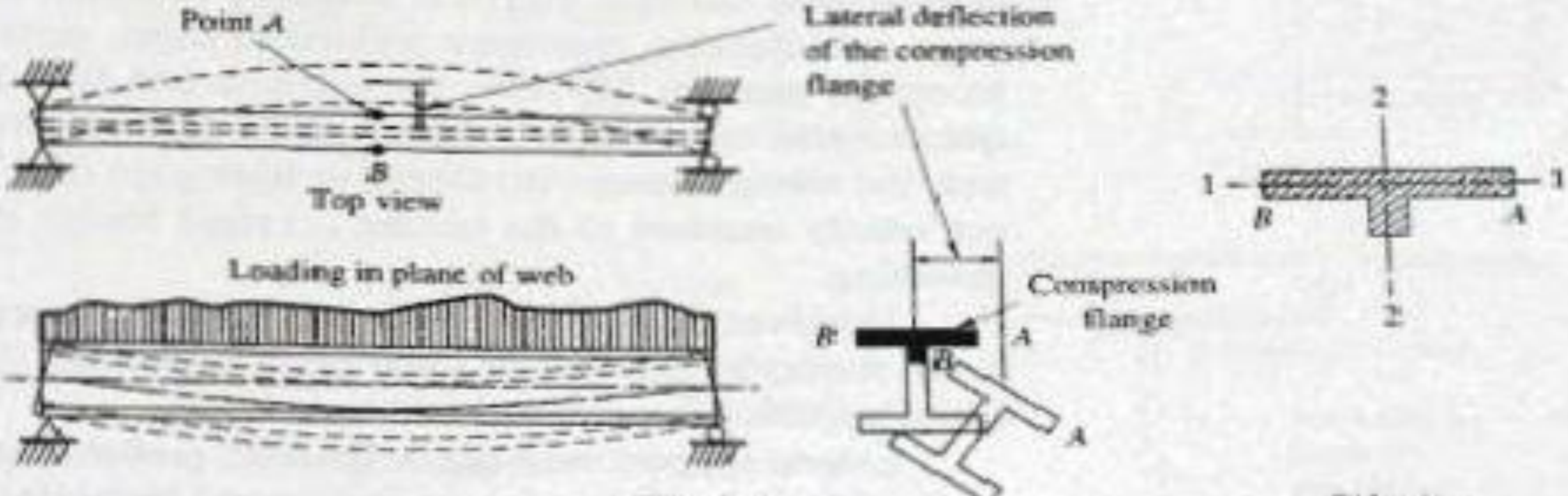
Topics

1. Lateral-Torsional Buckling of Beams
2. Design and analysis For beams, How do we Find L_p? L_r?
3. Solved problems.

Prepared by Eng.Maged Kamel.

Consider the compression zone of the laterally unsupported beam of Fig. 9.1.1. With the loading in the plane of the web, according to ordinary beam theory, points A and B are equally stressed. Imperfections in the beam and accidental eccentricity in loading actually result in different stresses at A and B. Furthermore, residual stresses as discussed in Chapter 6 contribute to unequal stresses across the flange width at any distance from the neutral axis.

Dr. SALmon chapter 9



Side view
(a)

Side view
(b)

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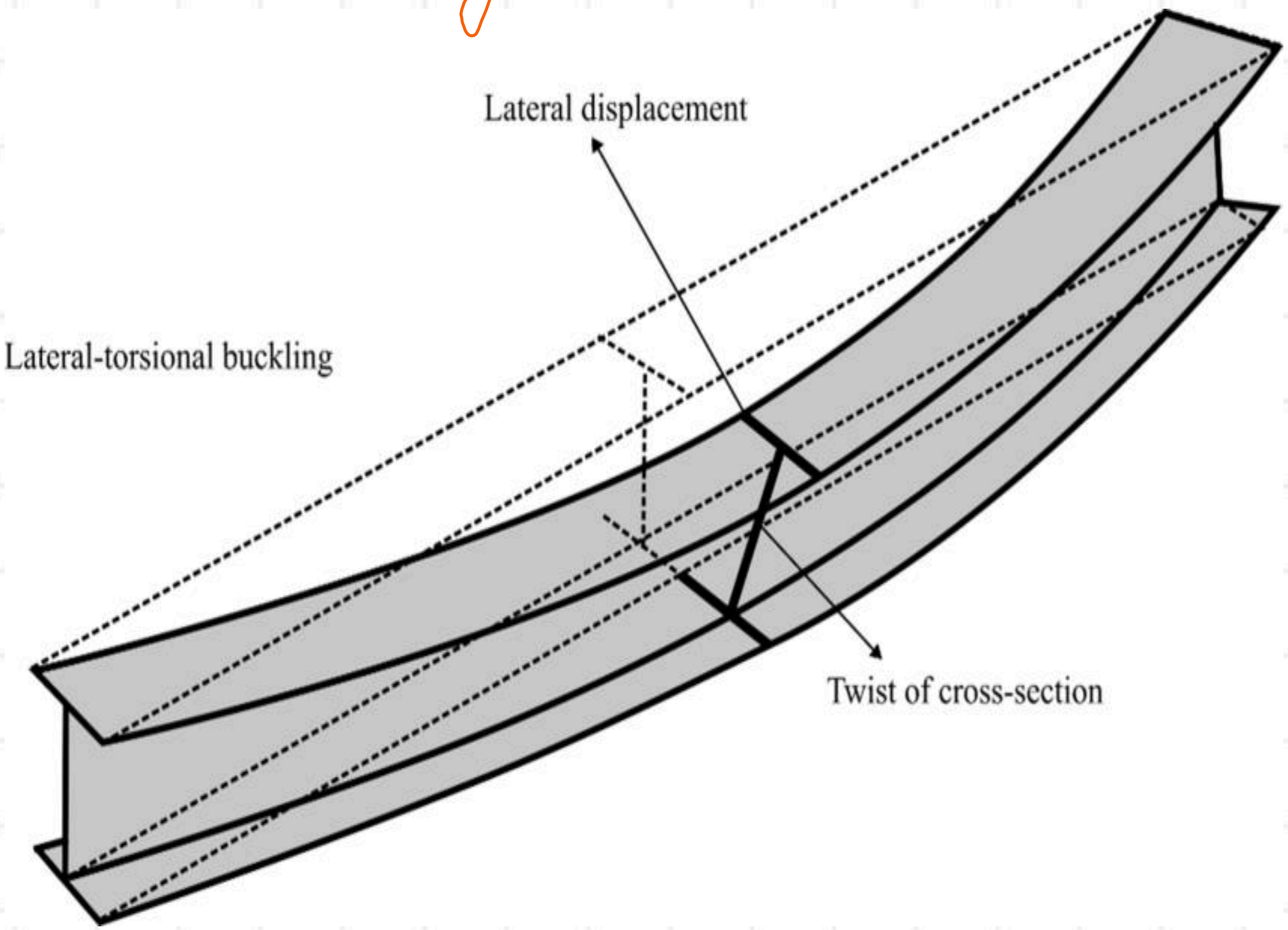
Lateral-Torsional Buckling of Beams

Definition from Schaum Book: Structural steel design

As the name implies, lateral-torsional buckling is an overall instability condition of a beam involving the simultaneous twisting of the member and lateral buckling of the compression flange.

To prevent lateral-torsional buckling, a beam must be braced at certain intervals against either twisting of the cross section or lateral displacement of the compression flange. Unlike column bracing (which requires another structural member framing into the column), beam bracing to prevent lateral-torsional buckling can be minimal. Even the intermittent welding of a metal (floor or roof) deck to the beam may be sufficient bracing for this purpose.

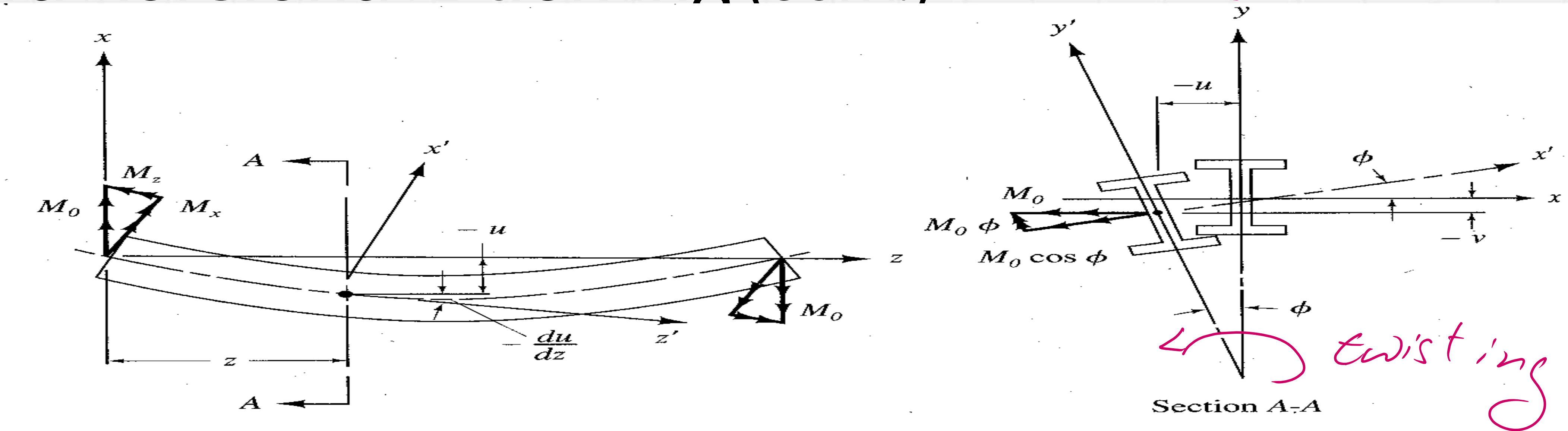
From Research gate



Lateral Torsional Buckling (cont.)

Lateral buckling

I-Beam in a Buckled Position

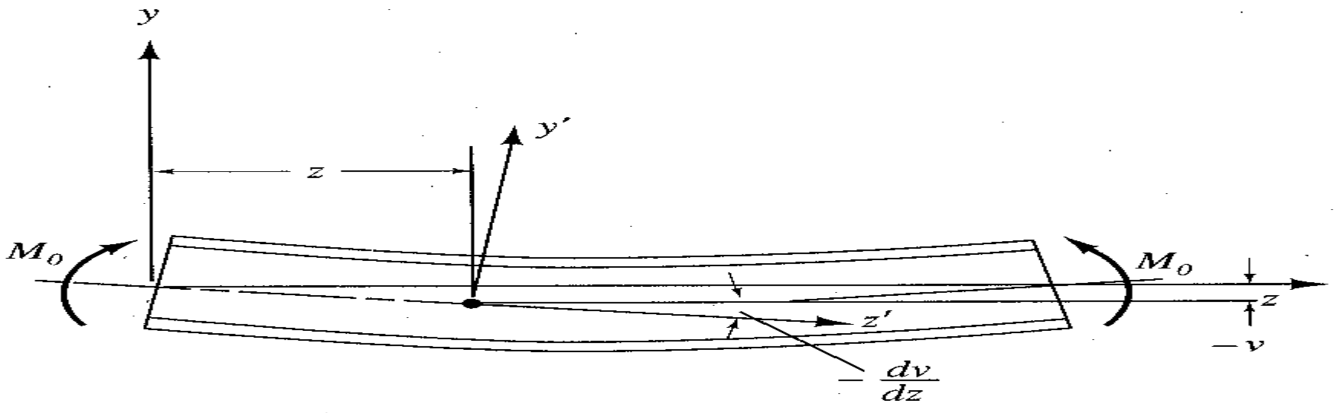


(a) Top view

twisting

Section A-A

(c)

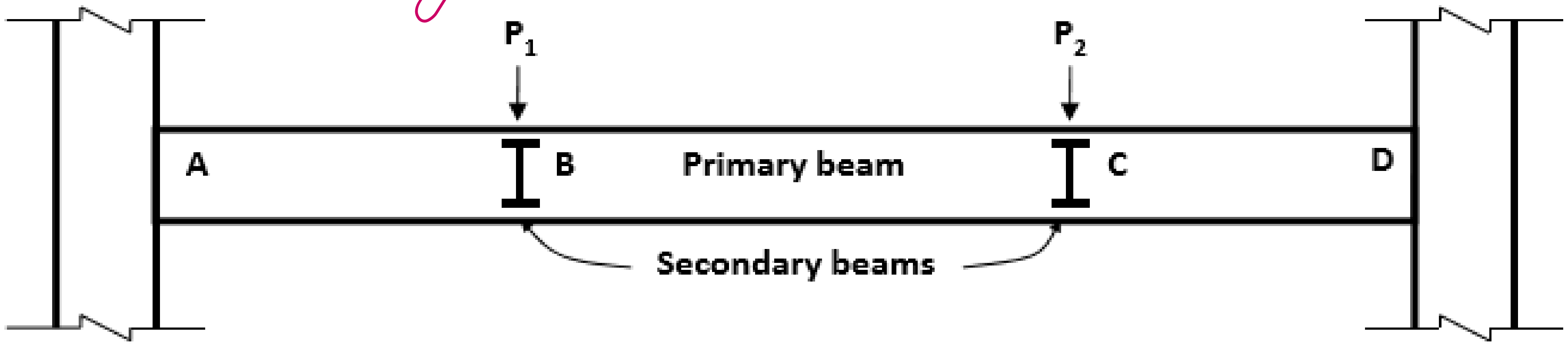


(b) Side view

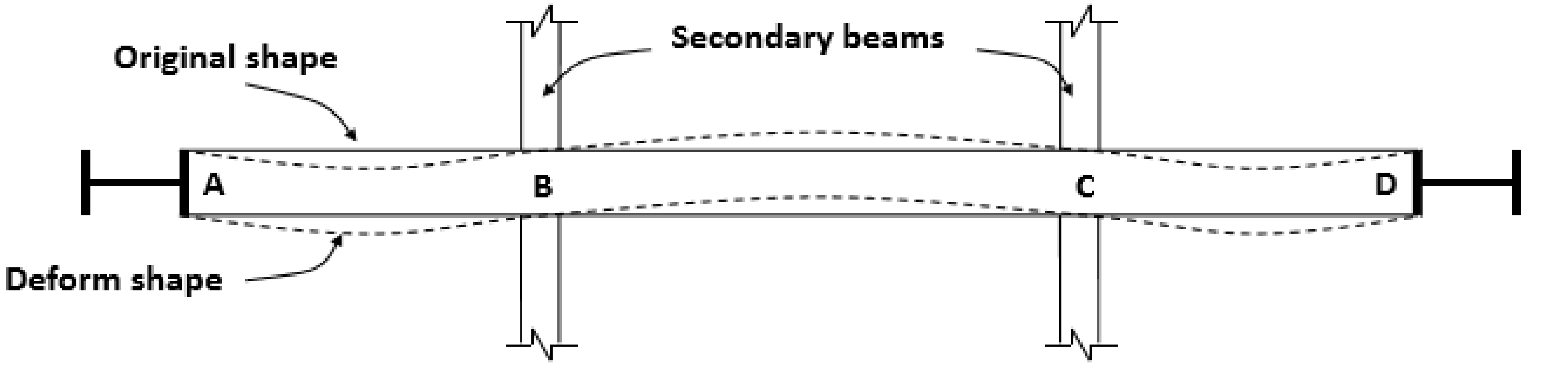
Direction Cosines

	x	y	z
x'	1	ϕ	$-\frac{du}{dz}$
y'	$-\phi$	1	$-\frac{dv}{dz}$
z'	$\frac{du}{dz}$	$\frac{dv}{dz}$	1

Effect of secondary beams



Front view of the primary beam



Plan view

Points A, B, C and D are restrained from deform laterally by the secondary beams and the connection at column

Restrained points

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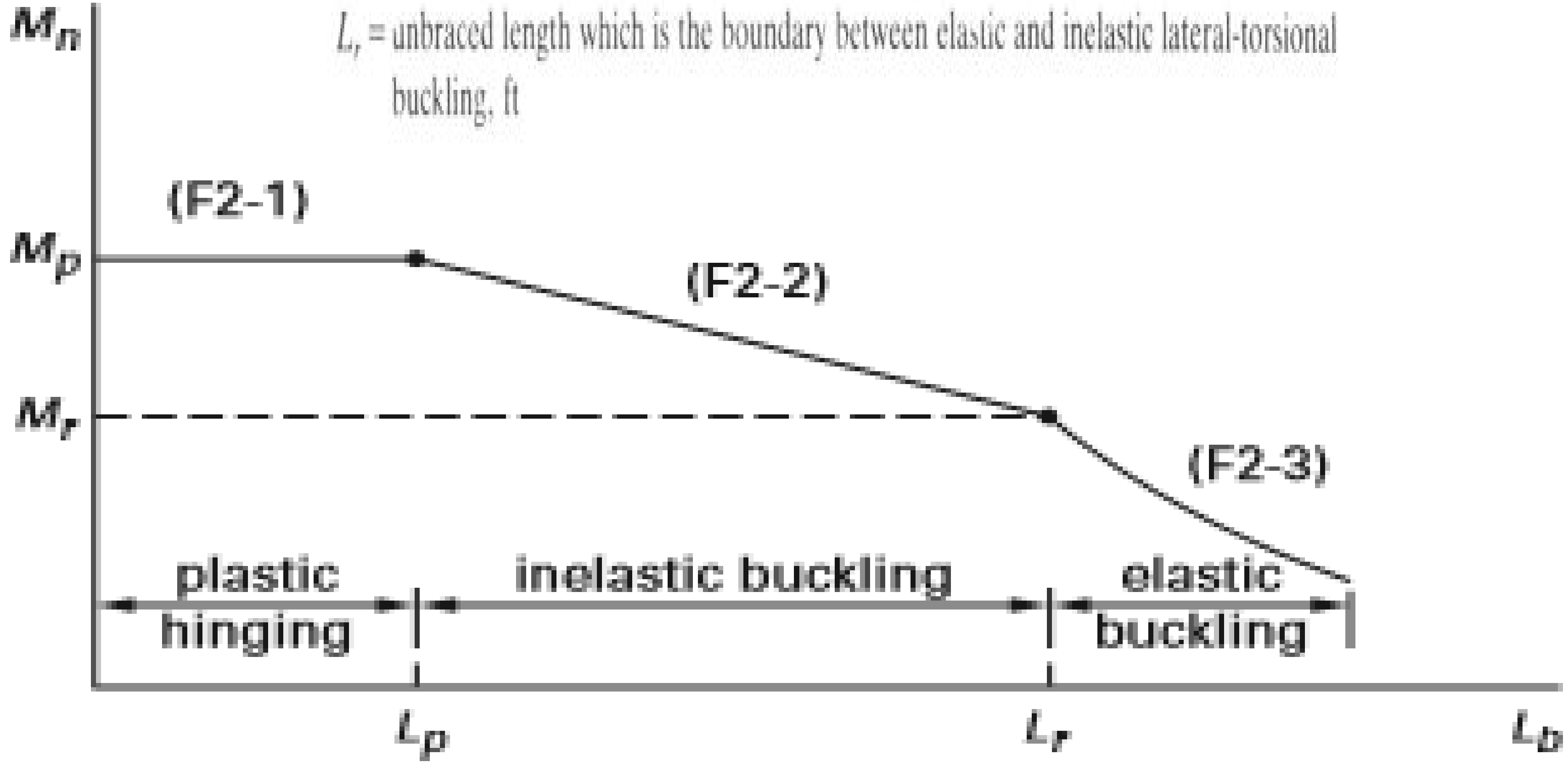
L_r = unbraced length which is the boundary between elastic and inelastic lateral-torsional buckling, ft



Prepared by Eng.Maged Kamel.

1. **Yielding**

$$M_n = M_p = F_y Z_x \quad \text{(F2-1)}$$

where

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

Z_x = plastic section modulus about the x-axis, in.^{3} (mm^3)

2. **Lateral-Torsional Buckling**

(a) When $L_b \le L_p$, the *limit state* of *lateral-torsional buckling* does not apply.

(b) When $L_p < L_b \le L_r$

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(F2-2)}$$

(c) When $L_b > L_r$

$$M_n = F_{cr}S_x \le M_p \quad \text{(F2-3)}$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section, in. (mm)

$$F_{cr} = \frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(F2-4)}$$

and where

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

J = torsional constant, in.^{4} (mm^4)

S_x = elastic section modulus taken about the x-axis, in.^{3} (mm^3)

h_o = distance between the flange centroids, in. (mm)

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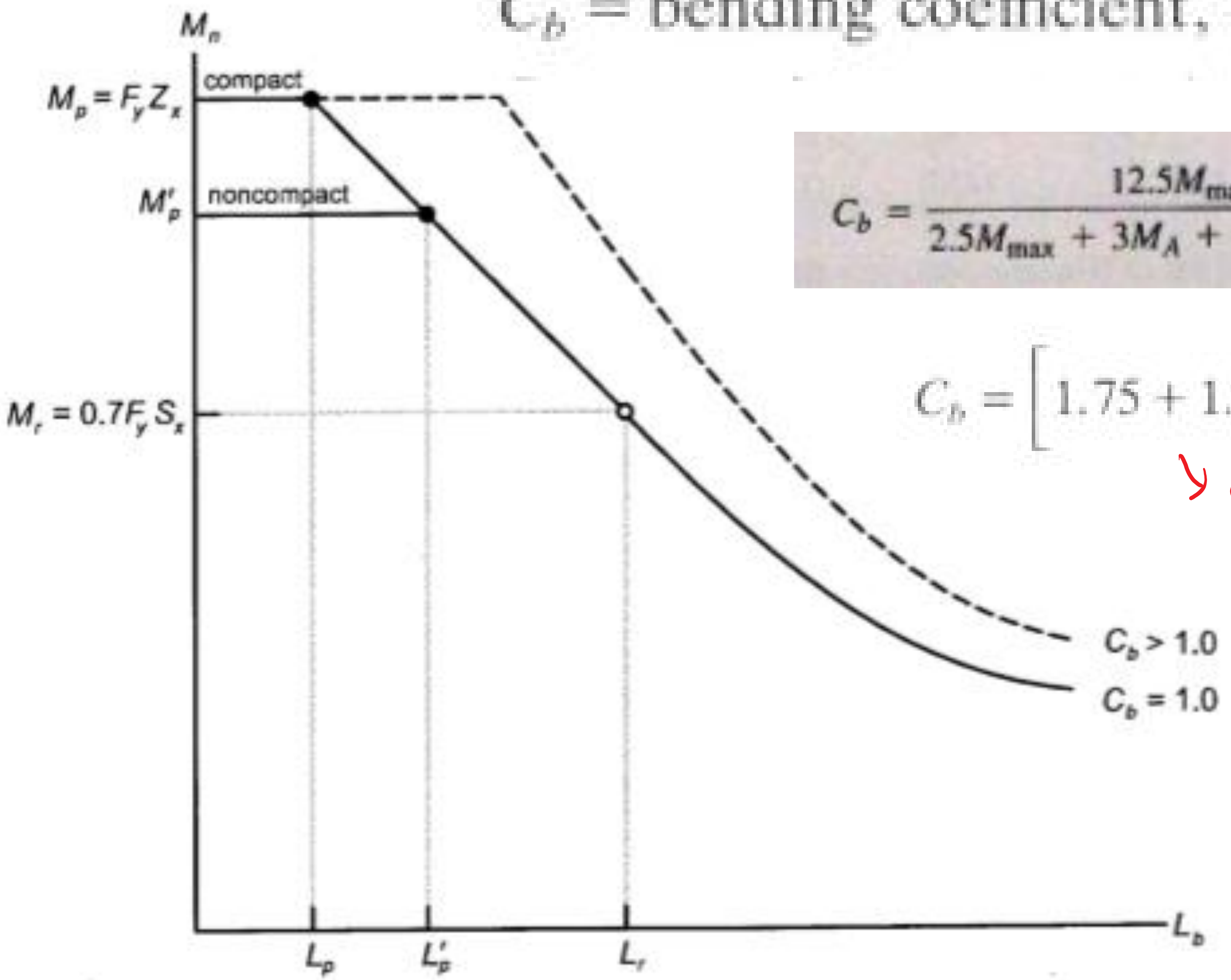
C_b = bending coefficient, defined in Eq. [5.10]



$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C} R_m \le 3.0$$

new

$$C_b = \left[1.75 + 1.05 \frac{M_1}{M_2} + 0.3 \left(\frac{M_1}{M_2}\right)^2\right] \le 2.3$$

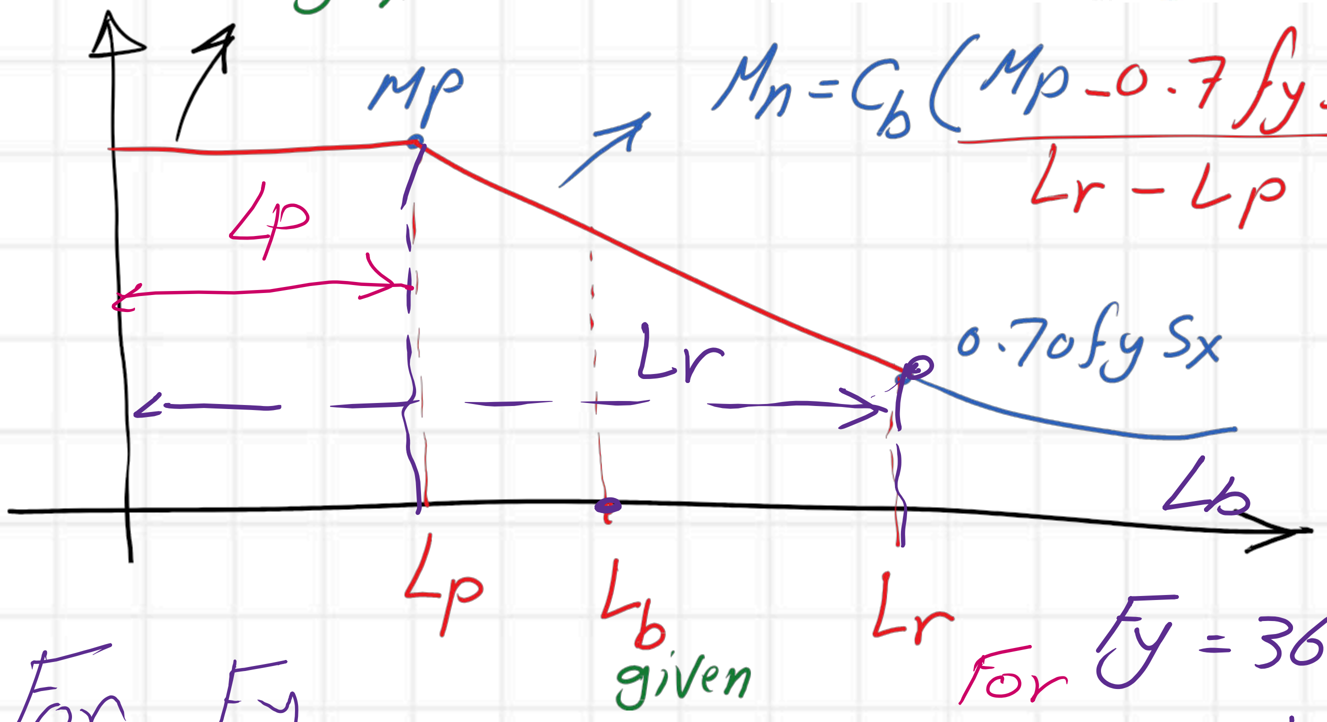
old

Cb Eqnatio

$M_n \quad M_n = f_y Z_x \qquad C_b = 1$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad \text{(Spec. Eq. F2-5)}$$

$$M_n = C_b \left(\frac{M_p - 0.7 f_y S_x}{L_r - L_p} \right) (L_b - L_p)$$



$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

L_b unbraced length

For $F_y = 36$ ksi $\Rightarrow L_p = 49.953\, r_y$

For $F_y = 42$ ksi $\rightarrow L_p = 46.247\, r_y$

For F_y

$F_y = 45$ ksi

$F_y = 50$ ksi

$F_y = 65$ ksi

For $F_y = 45$ ksi $\Longrightarrow L_p = 44.68\, r_y$

$F_y = 50$ ksi $\quad L_p = 42.386\, r_y$

$F_y = 65$ ksi $\quad L_p = 37.175\, r_y$

Example

For W12 x 30 A992 steel where $F_y = 50$ ksi & $F_u = 65$ ksi

Find L_p and L_r

A992 $\Rightarrow F_y = 50$ ksi

$L_p = 42.386\, r_y$

Use this relation For L_p

Table 2-4
Applicable ASTM Specifications for Various Structural Shapes

Steel Type	ASTM Designation		F_y Yield Stress[a] (ksi)	F_u Tensile Stress[a] (ksi)	Applicable Shape Series							HSS		
					W	M	S	HP	C	MC	L	Rect.	Round	Pipe
High-Strength Low-Alloy	A618[f]	[illegible]	[illegible]	[illegible]	▒	■	■	▒	■	■	■	□	□	□
		Gr. III	50	65	□	□	□	□	□	□	□	▒	▒	□
	A709	50	50	65	▒	▒	▒	▒	▒	▒	▒	□	□	□
		50S	50–65	65	▒	▒	▒	▒	▒	▒	▒	□	□	□
		50W	50	70	▒	▒	▒	▒	▒	▒	▒	□	□	□
	A913	50	50[h]	65[h]	▒	▒	▒	▒	▒	▒	▒	□	□	□
		60	60	75	▒	▒	▒	▒	▒	▒	▒	□	□	□
		65	65	80	▒	▒	▒	▒	▒	▒	▒	□	□	□
		70	70	90	▒	▒	▒	▒	▒	▒	▒	□	□	□
	A992		50[i]	65[i]	■	▒	▒	▒	▒	▒	▒	□	□	□
	A1065[k]	Gr. 50[j]	50	60	□	□	□	□	□	□	□	▒	□	□

■ = Preferred material specification.
▒ = Other applicable material specification, the availability of which should be confirmed prior to specification.
□ = Material specification does not apply.

Footnotes on facing page.

W12 X 30   Second Part of Table 1-1

Table 1-1 (continued)

W-Shapes

Properties

W14–W12

Nominal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
35	6.31	36.2	285	45.6	5.25	51.2	24.5	7.47	1.54	11.5	1.79	12.0	0.741	879
30	7.41	41.8	238	38.6	5.21	43.1	20.3	6.24	1.52	9.56	1.77	11.9	0.457	720
26	8.54	47.2	204	33.4	5.17	37.2	17.3	5.34	1.51	8.17	1.75	11.8	0.300	607

$r_y = 1.52$

$L_p = 42.386\,(1.52) = 64.427 \Rightarrow /12 \Rightarrow 5.37'$

$J = 0.457,\ C = 1,\ r_{ts} = 1.77,\ h_o = 11.90$

W12X30

L_r, the limiting unbraced length for the limit state of inelastic lateral-torsional buckling, in. (mm), is:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(F2-6)}$$

where

r_y = radius of gyration about y-axis, in. (mm)

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \quad \text{(F2-7)}$$

and the coefficient c is determined as follows:

(1) For doubly symmetric I-shapes

$$c = 1 \quad \text{(F2-8a)}$$

ry	1.52
rts	1.77
Zx	43.1
Sx	38.6
Cw	720
c	1
J	0.457
H0	11.9
Lp	5.37
lr	15.7

1.95*rts	E/0.70FY	main SQrt	(J*c/sx*h0)	Sqrt	(J*c/sx*h0)^2	6.67*(0.7Fy/E)^2
3.4515	828.5714	0.06547352	0.000994906	0.0032919	9.89837E-07	9.84661E-06

$$L_r = 3.4515(828.5714)\sqrt{\quad + \sqrt{\quad + \quad}}$$

$$L_r = 187.242'' / 12 \Rightarrow 15.604 \text{ FT}$$

$$\approx 15.6 \text{ FT}$$

Confirm your answer from Table 3-2

Z_x

Table 3-2 (continued)
W-Shapes
Selection by Z_x

$F_y = 50$ ksi

W12 X30

$F_y = 50$ ksi

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W16×26^v	**44.2**	**110**	**166**	**67.1**	**101**	**5.93**	**8.98**	**3.96**	**11.2**	**301**	**70.5**	**106**
W12×30	43.1	108	162	67.4	101	3.97	5.96	5.37	15.6	238	64.0	95.9

$L_p = 5.37'$

$L_r = 15.60'$