Modified Newton Raphson Method

A simple modification to the standard Newton method for approximating the root of a *Multiple* root function is described and analyzed. For the same number of function and derivative evaluations, the modified method converges faster.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'[x_i + a(x_i) + f(x_i)]}$$

where $$a(x_i) = -\frac{1}{2f'(x_i)}$$

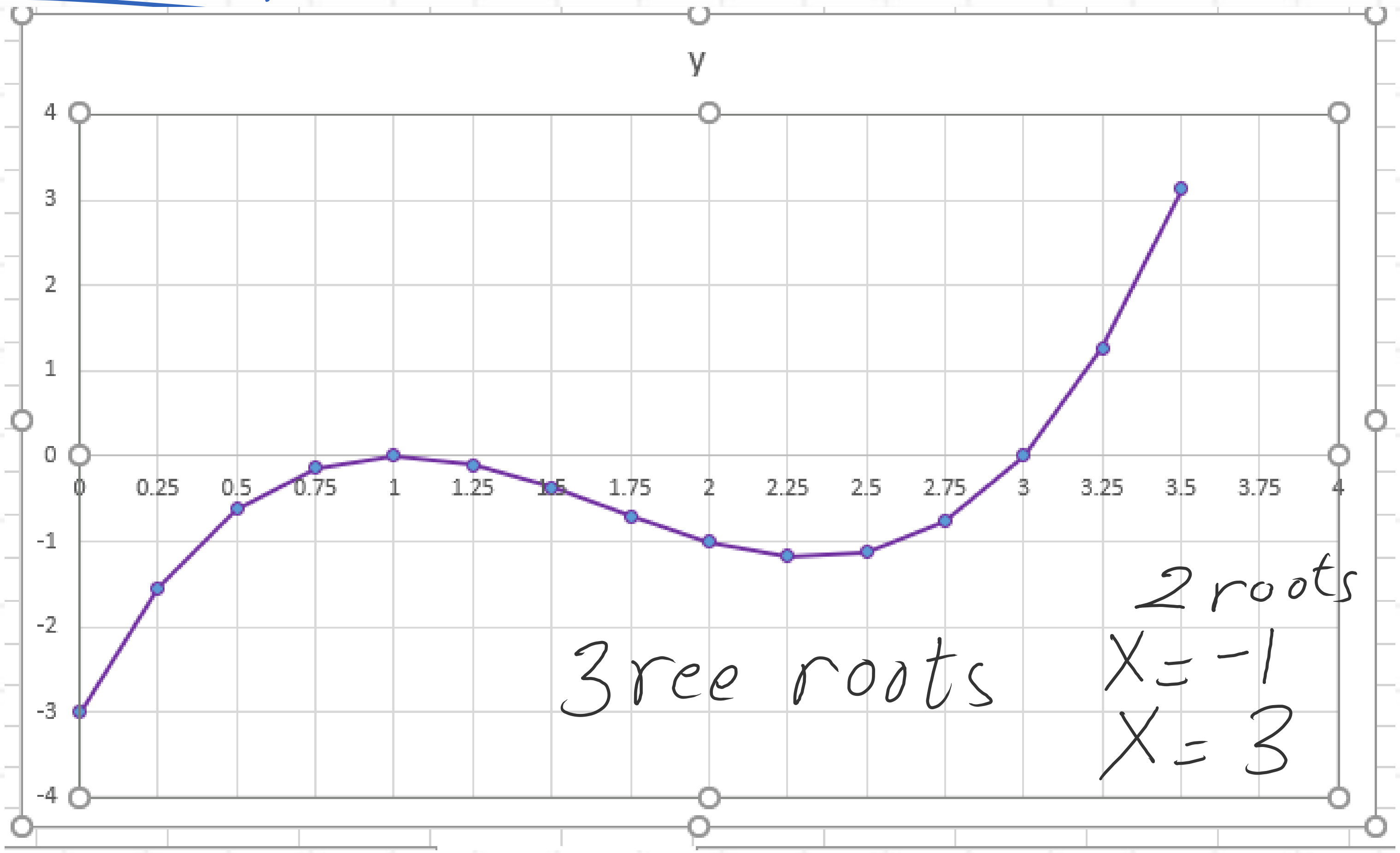
$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Example# 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=0.

Find the roots of the given function.

2nd Choice $\longrightarrow$ $X_0 = 4$

Solution



3ree roots

2 roots

$X = -1$

$X = 3$

Analytical Solution

$f(x) = (x-3)(x-1)$
$(x-1)$

Please refer to the excell graph.

$(x-3)(x^2 - 2x + 1)$

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Example# 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=0.
Find the roots of the given function.

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

modified Newton - Raphson

$f'(x) = (3x^2 - 10x + 7)$

$f''(x) = (6x - 10)$

Try → $x_0 = 0$

$f(0) = 0 - 0 + 0 - 3 = -3$

$f'(0) = 0 - 0 + 7 = 7$

$f''(0) = 6(0) - 10 = -10$

find $f(0)$, $f'(0)$, $f''(0)$

$X_1 = \overset{x_0}{0} - \frac{(-3)(7)}{(7)^2 - (-3)(-10)} = 0 + \frac{21}{19}$

$X_1 = 1.1052631$

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Example# 8 let f(x) $= X^3 - 5X^2 + 7x - 3$ with first choice x0=0

Use $x_1 = 1.105263$

$$x_{i+1} = x_i - \frac{f(x_i) f'(x_i)}{[f'(x_i)]^2 - f(x_i) f''(x_i)}$$

$f'(x) = 3x^2 - 10x + 7$

$f''(x) = 6x - 10$

We have $x_1 = +1.105263$

$f(1.105263) = (1.105263)^3 - 5(1.105263)^2 + 7(1.105263) - 3$

$= -0.02094$

$f'(1.105263) = 3(1.105263)^2 - 10(1.105263) + 7 = -0.3878$

$f''(1.105263) = 6(1.105263) - 10 = -3.368$

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Example# 8-Let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=0

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

$f'(x) = 3x^2 - 10x + 7$

$f''(x) = 6x - 10$

Get X_2 value

We have $x_1 = +1.105263$

$$X_2 = +1.105263 - \frac{(-0.02094)(-0.3878)}{[-0.3878]^2 - [-0.02094)(-3.368)]}$$

$X_2 = 1.0000$ $\Rightarrow$

and So on

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After 4 iteration

$x_0 \to 1$ $f(1)=0$ $\to$ get $x_3=1$

modified Newton

	xi	f(x)	f'(x)	f''(x)	Numerator	Denominator	n/d	Xi
x0	0	-3	7	-10	-21	19	-1.10526316	1.105263158
x1	1.1053	-0.020994	-0.3878	-3.368421	0.008142	0.079680174	0.10218149	1.003081664
x2	1.0031	-1.9E-05	-0.0123	-3.98151	2.33E-07	7.57394E-05	0.00307928	1.000002381
x3	1	-8E-12	-8E-06	-3.999988	6.4E-17	3.20006E-11	1.9999E-06	1
x4	1	0	-3E-10	-4	0	1.16925E-19	0	1

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

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Example# 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with second choice x0=4

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Modified Newton

$f'(x) = 3x^2 - 10x + 7$

$f''(x) = 6x - 10$

Now try $\boxed{x_0 = 4}$ Change intial Point

$f(4) = 4^3 - 5(4)^2 + 7(4)^2 - 3 = 9$

$f'(4) = 3(4)^2 - 10(4) + 7 = 15$

$f''(4) = 6(4) - 10 = 14$

$X_1 = 4 - \frac{(9)(15)}{(15)^2 - [(9)(14)]}$

$X_1 = 2.6363$

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Example# 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=0

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Modified Newton

$f'(x) = (3x^2 - 10x + 7)$

$f''(x) = (6x - 10)$

We have $x_1 = 2.6363$

$f(2.6363) = (2.6363)^3 - 5(2.6363)^2 + 7(2.6363) - 3 = -0.9737$

$f'(2.6363) = 3(2.6363)^2 - 10(2.6363) + 7 = 1.4876$

$f''(2.6363) = 6(2.6363) - 10 = 5.8184$

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Example# 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=0

Modified Newton

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

$f'(x) = 3x^2 - 10x + 7$

$f''(x) = 6x - 10$

We have $x_1 = 2.6363$

$$X_2 = 2.6363 - \frac{(-0.9737)(1.4876)}{(1.4876)^2 - (-0.9737)(5.8184)} = \underbrace{2.8202}_{x_2}$$

↓ Continue

it will Converge → $X = 3$

For $X_0 = 4$

After 5 iteration $\rightarrow$ $X_5 = 3.00$

	modified Newton							
	xi	f(x)	f'(x)	f''(x)	Numerator	Denominator	n/d	Xi
x0	4	9	15	14	135	99	1.36363636	2.636363636
x1	2.6364	-0.973704	1.4876	5.818182	-1.448485	7.8781504	-0.18386108	2.820224719
x2	2.8202	-0.595635	2.6588	6.921348	-1.583647	11.19157449	-0.14150349	2.96172821
x3	2.9617	-0.147285	3.6982	7.770368	-0.544692	14.82127737	-0.0367507	2.998478702
x4	2.9985	-0.006076	3.9878	7.990872	-0.02423	15.95139247	-0.00151898	2.999997682
x5	3.000	-9.27E-06	4	7.999986	-3.71E-05	15.99992583	-2.3179E-06	3

Example # 8 let f(x) =$X^3 - 5X^2 + 7x - 3$ with first choice x0=4

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Comparison

Newton-Raphson

	xi	f(x)	f'(x)	Numerator	Denominat	n/d	Xi
x0	4	9	15	9	15	0.6	3.4
x1	3.4	2.304	7.68	2.304	7.68	0.3	3.1
x2	3.1	0.441	4.83	0.441	4.83	0.091304348	3.00869565
x3	3.0087	0.0350857	4.0698	0.035086	4.069792	0.008621011	3.00007464
x4	3.0001	0.0002986	4.0006	0.000299	4.000597	7.46352E-05	3.00000001
x5	3	2.228E-08	4	2.23E-08	4	5.57062E-09	3
x6	3	0	4	0	4	0	3

modified Newton

	xi	f(x)	f'(x)	f''(x)	Numerator	Denominator	Numerator / denominator	Xi
x0	4	9	15	14	135	99	1.36363636	2.636363636
x1	2.6364	-0.973704	1.4876	5.818182	-1.448485	7.8781504	-0.18386108	2.820224719
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x4	2.9985	-0.006076	3.9878	7.990872	-0.02423	15.95139247	-0.00151898	2.999997682
x5	3.000	-9.27E-06	4	7.999986	-3.71E-05	15.99992583	-2.3179E-06	3

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Let us compare with Newton-Raphson method

$$X_i = X_0 - \frac{f(x)}{f'(x}$$

$$f(x) = x^3 - 5x^2 + 7x - 3$$

$$f'(x) = 3x^2 - 10x + 7$$

$$f''(x) = 6x - 10$$

For $\boxed{x_0 = 0}$

$f(0) = -3$

$f'(0) = 7$

$$x_1 = x_0 - \frac{f(x)}{f'(x)} = 0 - \left(\frac{-3}{7}\right) = +\frac{3}{7} = 0.4285$$

$$x_2 = 0.4285 - [-0.8396/3.2653] = 0.6857$$

$f(0.4285) = -0.8396$

$f'(0.4285) = 3.2653$

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$X_i = X_0 - \frac{f(x)}{f'(x)}$ For $X_0 = 0$

Newton-Raphson

Newton-Raphson

	xi	f(x)	f'(x)	Numerator	Denominat	n/d	Xi
x0	0	-3	7	-3	7	-0.428571429	0.42857143
x1	0.4286	-0.83965	3.2653	-0.83965	3.265306	-0.257142857	0.68571429
x2	0.6857	-0.228595	1.5535	-0.228595	1.553469	-0.147151115	0.8328654
x3	0.8329	-0.060537	0.7523	-0.060537	0.75234	-0.080464493	0.91332989
x4	0.9133	-0.015674	0.3692	-0.015674	0.369216	-0.0424534	0.95578329

Let us Compare with Newton-Raphson method

$$X_i = X_0 - \frac{f(x)}{f'(x)}$$

$f(x) = x^3 - 5x^2 + 7x - 3$

$f'(x) = 3x^2 - 10x + 7$

$f''(x) = 6x - 10$

For $\boxed{X_0 = 4}$

$f(4) = 9$

$f'(4) = 15$

$X_1 = X_0 - \frac{f(x)}{f'(x)} = 4 - (9/15) = 4 - 0.60 = 3.40$

$f(3.40) = 2.304$

$f'(3.40) = 7.68$

$x_2 = 3.40 - (2.304)/7.68$

$X_2 = 3.10$

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$x_i = x_0 - f(x)/f'(x)$

Newton-Raphson

$X_0 = 4 \rightarrow X_6 = 3$

	xi	f(x)	f'(x)	Numerator	Denominat	n/d	Xi
x0	4	9	15	9	15	0.6	3.4
x1	3.4	2.304	7.68	2.304	7.68	0.3	3.1
x2	3.1	0.441	4.83	0.441	4.83	0.091304348	3.00869565
x3	3.0087	0.035086	4.0698	0.035086	4.069792	0.008621011	3.00007464
x4	3.0001	0.000299	4.0006	0.000299	4.000597	7.46352E-05	3.00000001
x5	3	2.23E-08	4	2.23E-08	4	5.57062E-09	3
x6	3	0	4	0	4	0	3

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