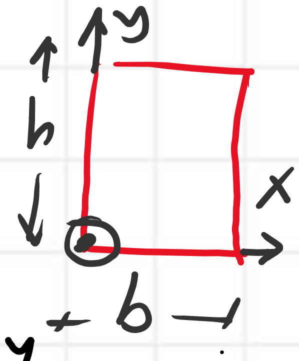
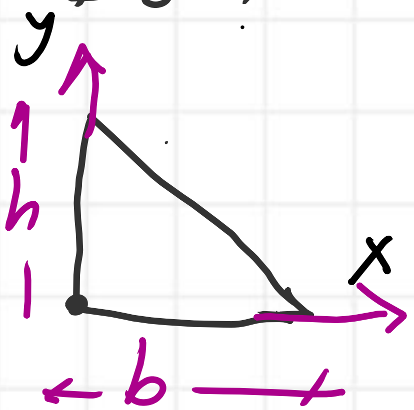
The First Case : $I_x > I_y$

I_{xy} is positive

Some Examples for this Case



$I_x = \frac{bh^3}{3}$, $I_y = \frac{hb^3}{3}$, $I_{xy} = \frac{b^2h^2}{4}$



$I_x = \frac{bh^3}{12}$ & $I_y = \frac{hb^3}{12}$

$I_{xy} = +\frac{b^2h^2}{24}$

$h > b$

Prepared by Eng.Maged Kamel.

Case 1 : $I_x > I_y$, I_{xy} +ve

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

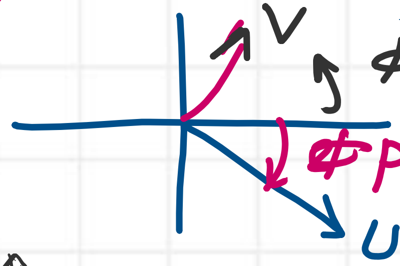
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

Check For $I_{x'}$ is max when Cos 2θ: +ve & sin 2θ: -ve)

$$: +ve + \overset{I_x > I_y}{(+ve)}\overset{Cos}{(+)} - \overset{I_{xy}}{(+)}(-) = I_{max} \rightarrow$$

4rth quarter

For $2\phi P_1$ ↪ Cos : +ve, sin : - ve $\Rightarrow$ $2\phi P_1$: $2\theta_{p_1}$ From : $(0 \rightarrow -\pi/2)$



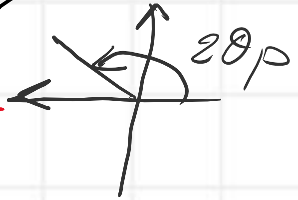
$\Rightarrow$ Normal view

For $I_{x'}$ min : Cos $2\theta_{p_2}$ = -ve, Sin 2θ = +ve

$$= (+) + (+)(-) - (+)(+)$$

$\Rightarrow$ I_{min} $(\pi/2 \rightarrow \pi)$

This happens when $2\theta_{p_2}$ in the 2nd quarter



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Check conditions for x & y axes

$\theta = 0 \rightarrow 2\theta = 0$

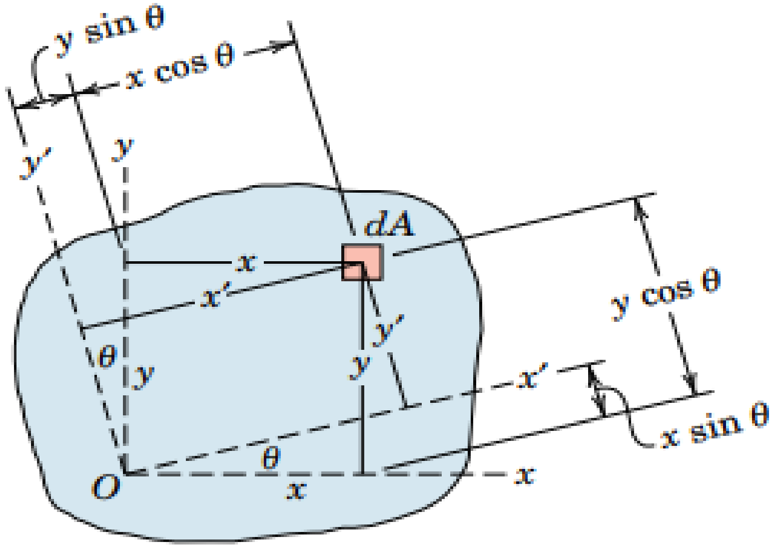


Figure A/6

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

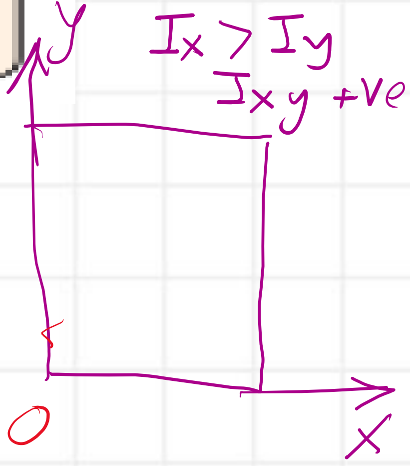
$$\tan \overset{2\theta}{\cancel{2\alpha}} = \frac{2I_{xy}}{I_y - I_x}$$

For $\theta = 0$ This is x-axis situation

$$I_{x'} = \left(\frac{I_x + I_y}{2}\right) + \frac{I_x - I_y}{2}\cos(0) - I_{xy}\sin(0) = I_x$$

$$I_{y'} = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(0) + I_{xy}\sin(0) = I_y$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin(0) + I_{xy}\cos(0) = I_{xy}$$



$I_x > I_y$

I_{xy} +ve

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Check conditions for x & y axes $\theta = \pi/2$

$2\theta = \pi$

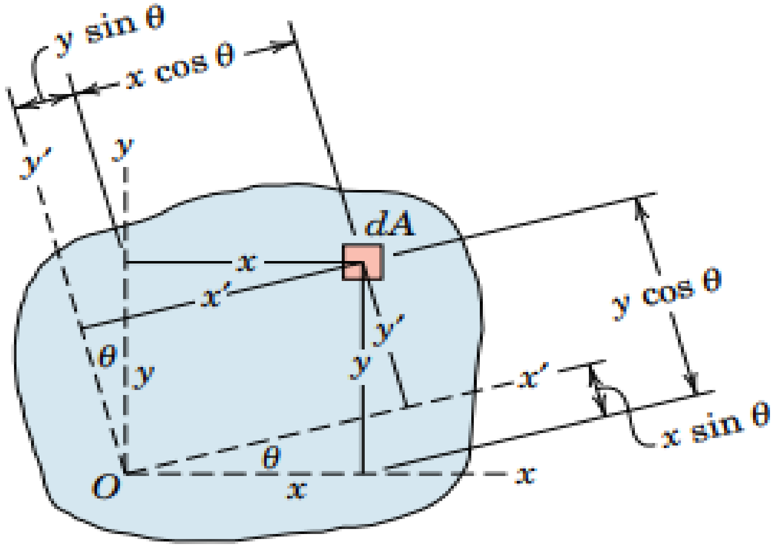


Figure A/6

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$\tan \overset{2\theta}{\cancel{2\alpha}} = \frac{2I_{xy}}{I_y - I_x}$$

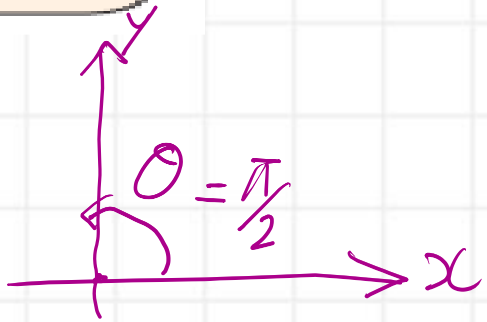
For $\theta = +\frac{\pi}{2}$ This is y-axis situation

$I_{x'} = \left(\frac{I_x + I_y}{2}\right) + \frac{I_x - I_y}{2}\cos(\pi) - I_{xy}\sin(\pi) = I_y$

$I_{y'} = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(\pi) + I_{xy}\sin(\pi) = I_x$

$I_{x'y'} = \frac{I_x - I_y}{2}\sin(\pi) + I_{xy}\cos(\pi) = -I_{xy}$

2θ 2θ



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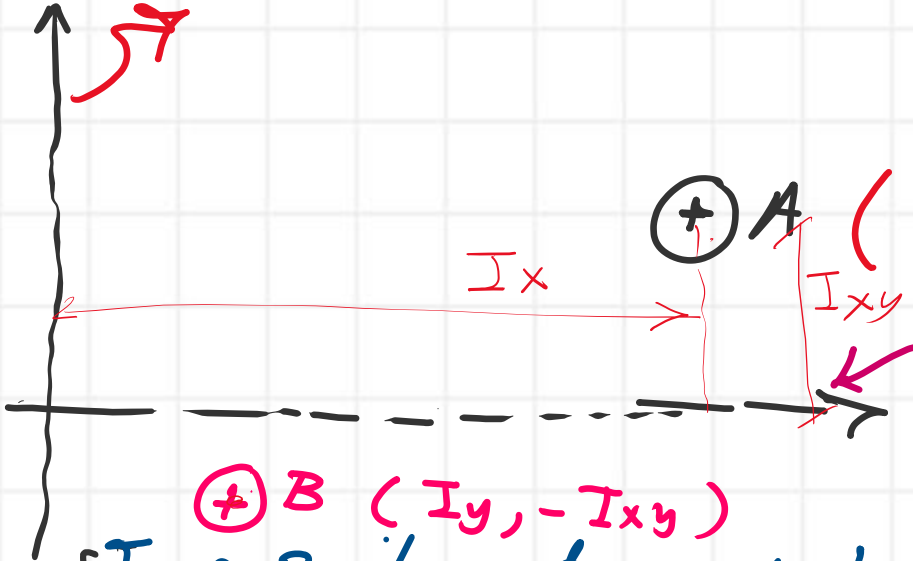
Case #1 Mohr circle : given I_x, I_y, I_{xy} all are positive

+ve I_{xy}

step 2 Draw a vertical axis as I_{xy}

from the previous discussion

$I_{x'y'}$ for $\theta = 90^\circ$ = $-I_{xy}$



step 1 : Draw a horizontal axis -x as I_{max}, I_{min}, I_x, I_y

Step 3: Locate point A with coordinate $(I_x, +I_{xy})$

step4: Locate point B with Coordinate $(I_y, -I_{xy})$

step 5 : Join the Two Points - get point O

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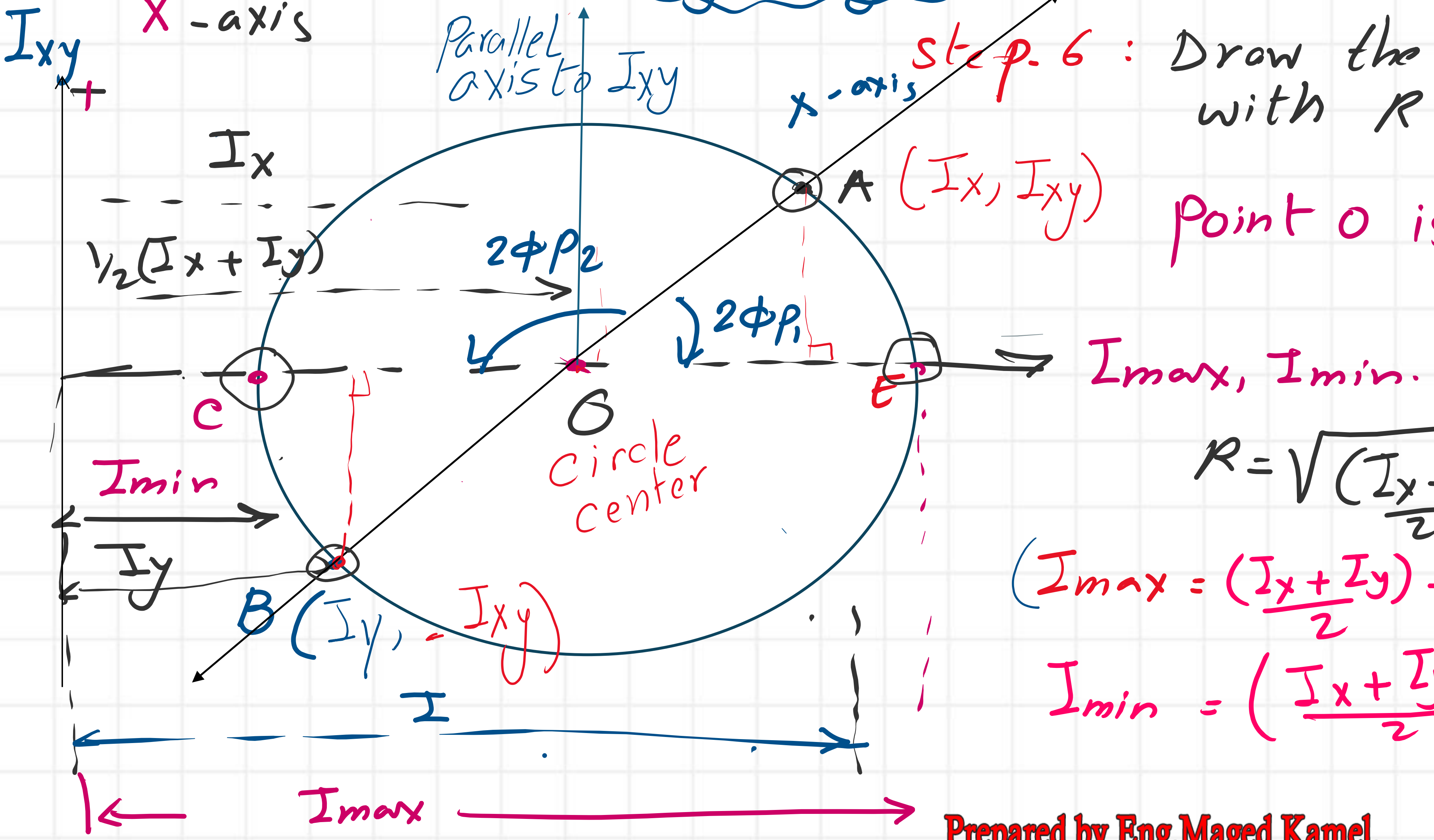
Case # 1 $I_x > I_y$, I_{xy} is positive.



Step 6: Draw the circle with R is $= OA$

Point O is $\left(\frac{I_x+I_y}{2}, 0\right)$

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$\left(I_{max} = \left(\frac{I_x+I_y}{2}\right) + R\right) \Rightarrow E$$

$$I_{min} = \left(\frac{I_x+I_y}{2}\right) - R \Rightarrow C$$

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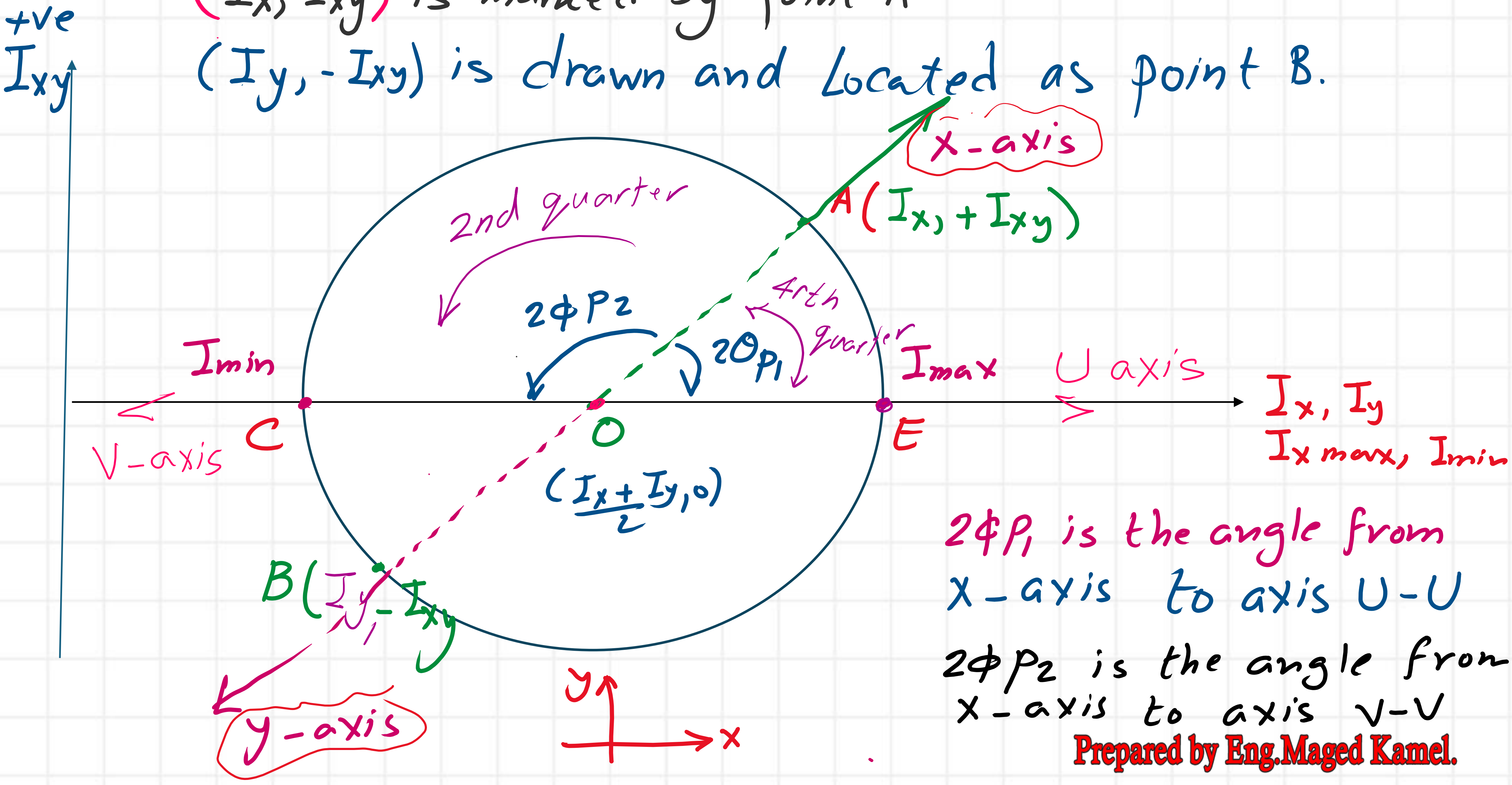
(I_x, I_{xy}) is marked by point A

$(I_y, -I_{xy})$ is drawn and Located as point B.



$2\phi P_1$ is the angle from x-axis to axis U-U

$2\phi P_2$ is the angle from x-axis to axis V-V

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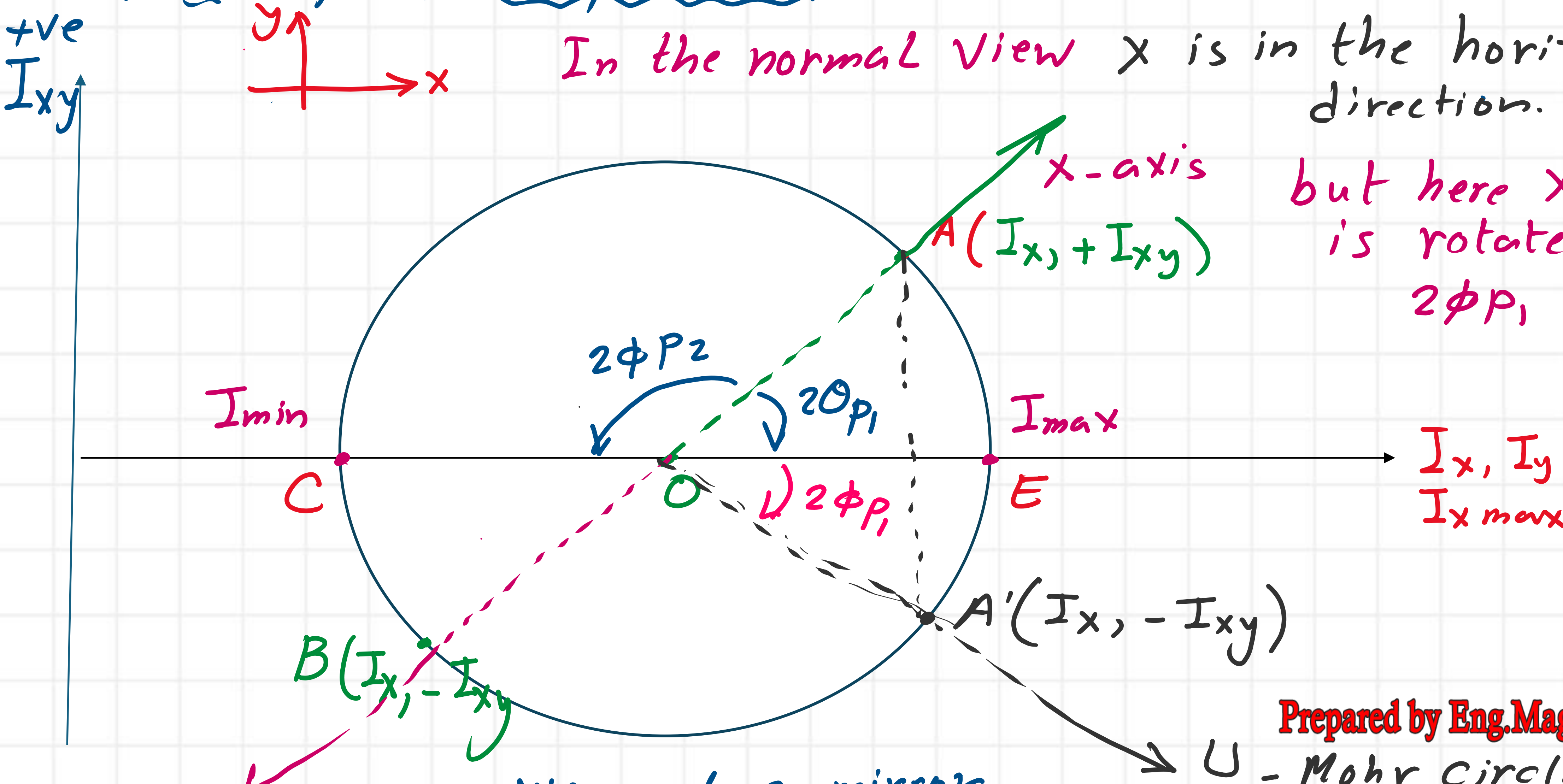
Need of mirror point A'

In the normal view x is in the horizontal direction.

but here x-axis is rotated by $2\phi p_1$



We need a mirror point to let x-axis return to hL. position

U-Mohr circle after rotating 2019

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Need of mirror point A' : I_x, I_y are +ve , I_{xy} is +ve

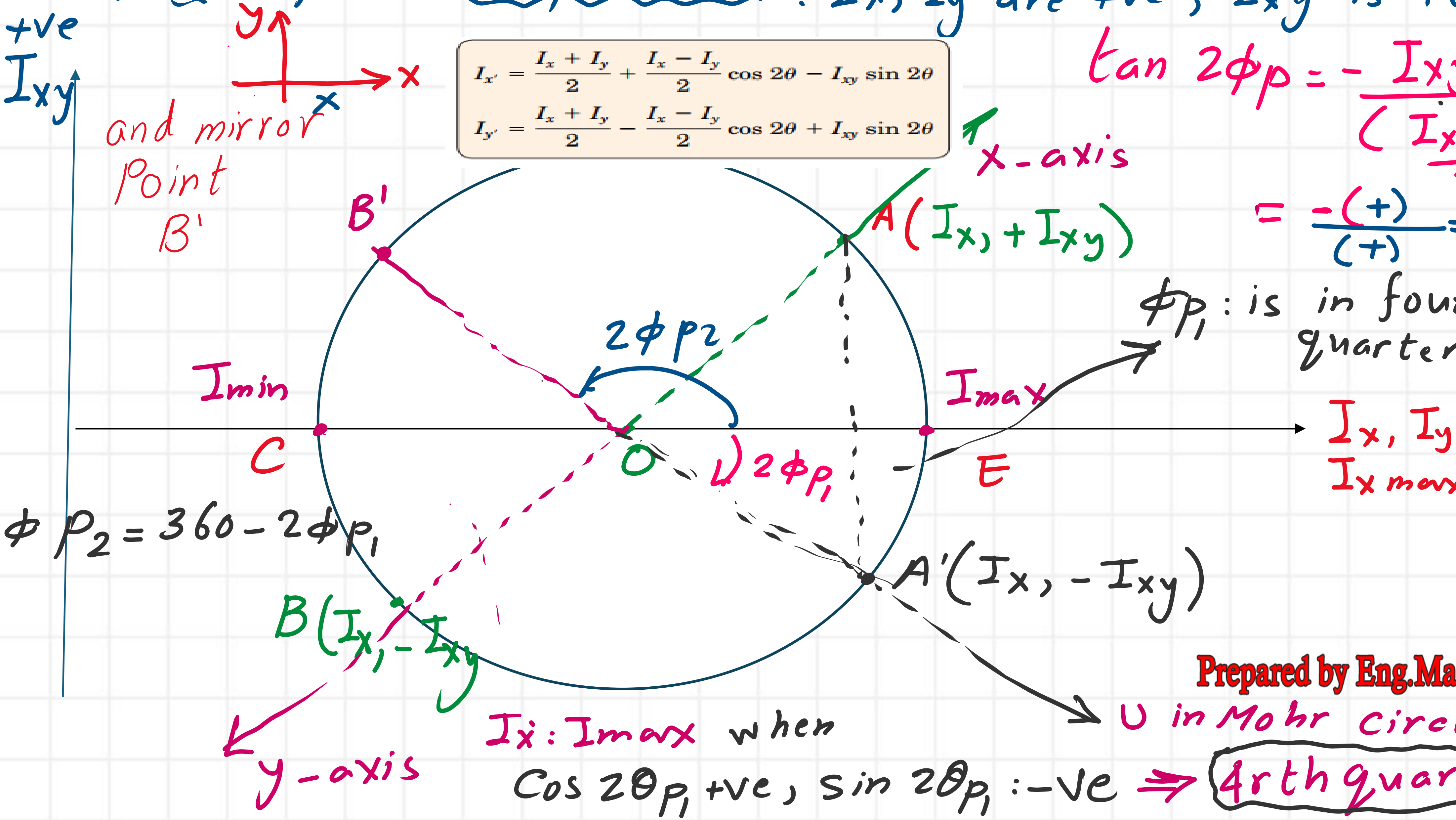
and mirror Point B'

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$\tan 2\phi p = -\frac{I_{xy}}{\frac{(I_x - I_y)}{2}} = \frac{-(+)}{(+)} = -ve$$

ϕp_1 : is in fourth quarter



$2\phi P_2 = 360 - 2\phi p_1$

U in Mohr circle

I_x : I_{max} when

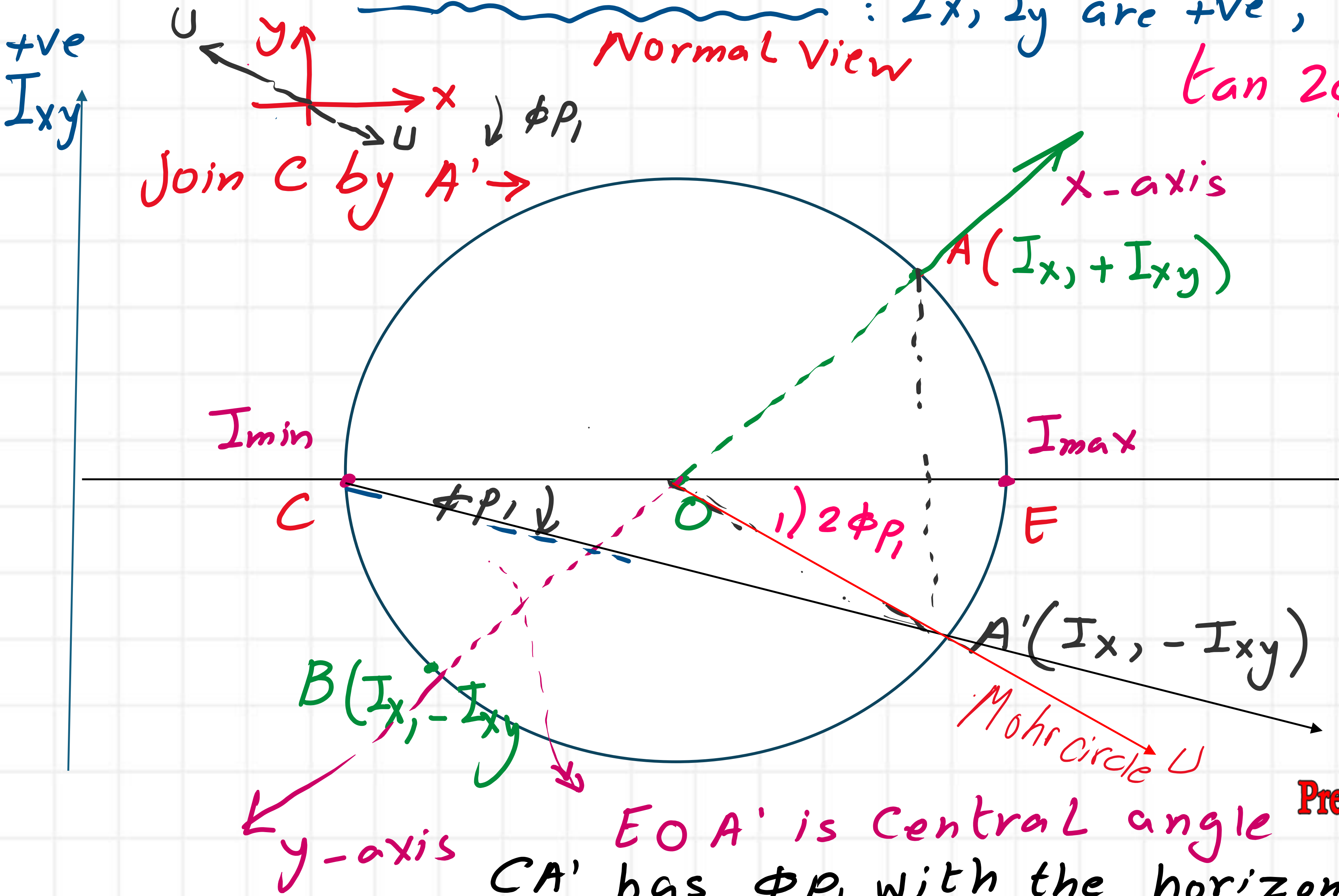
$\cos 2\theta_{p_1}$ +ve , $\sin 2\theta_{p_1}$: −ve ⇒ 4rth quarter

Prepared by Eng.Maged Kamel.

U-direction in Normal View : I_x, I_y are +ve , I_{xy} is +ve

$$\tan 2\phi_p = -\frac{I_{xy}}{\left(\frac{I_x - I_y}{2}\right)} = \frac{-(+)}{(+)} = -ve$$

Join C by A' →



I_x, I_y

I_x max, min

U Normal view

EOA' is central angle

CA' has ϕp_1 with the horizontal direction

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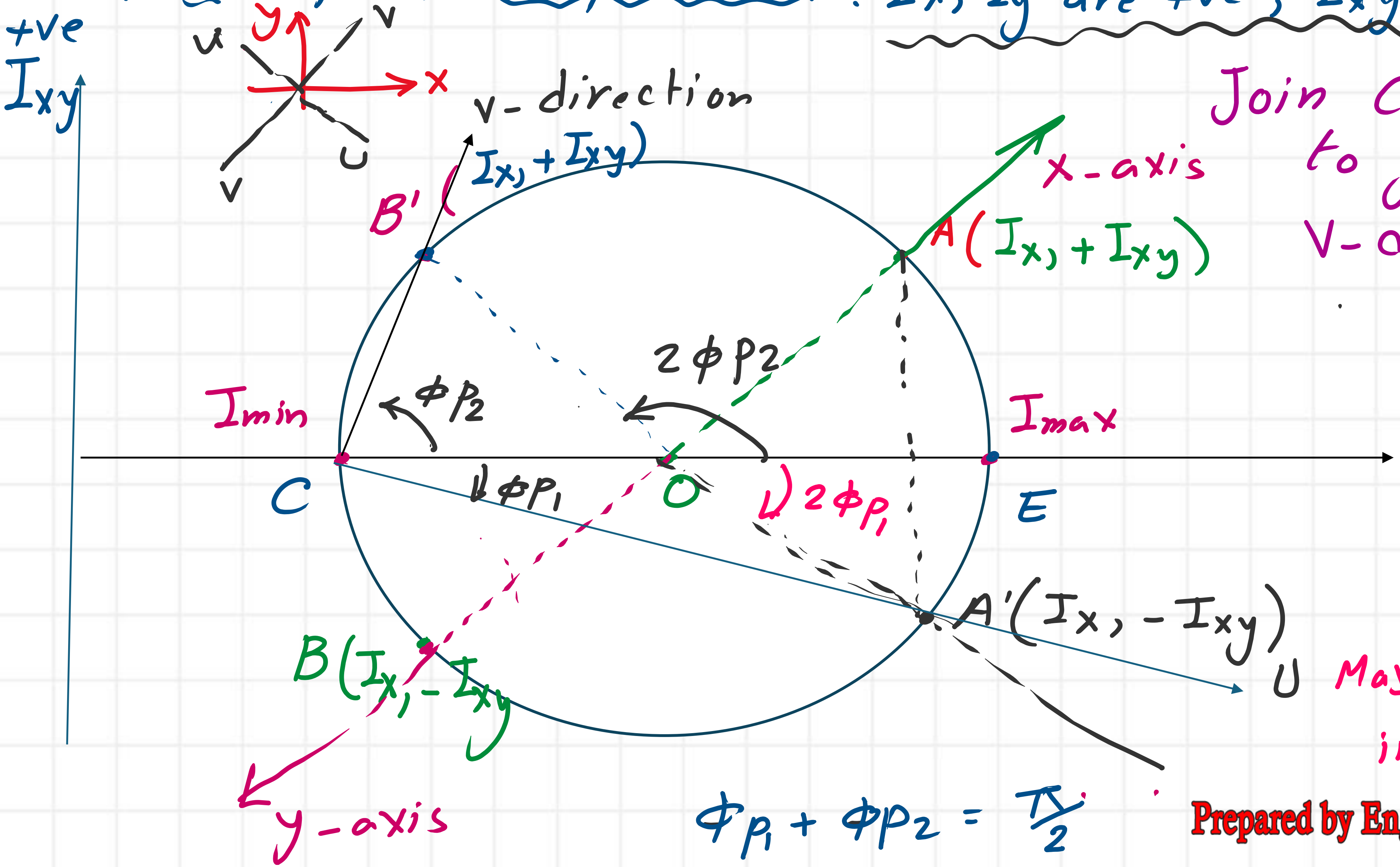
Need of mirror point B' : I_x, I_y are +ve , I_{xy} is +ve



I_x, I_y
$I_{x\,max}$, min

Join C with B' to get V-direction (minor direction)

Major direction in normal view

$$\phi p_1 + \phi p_2 = \frac{\pi}{2}$$

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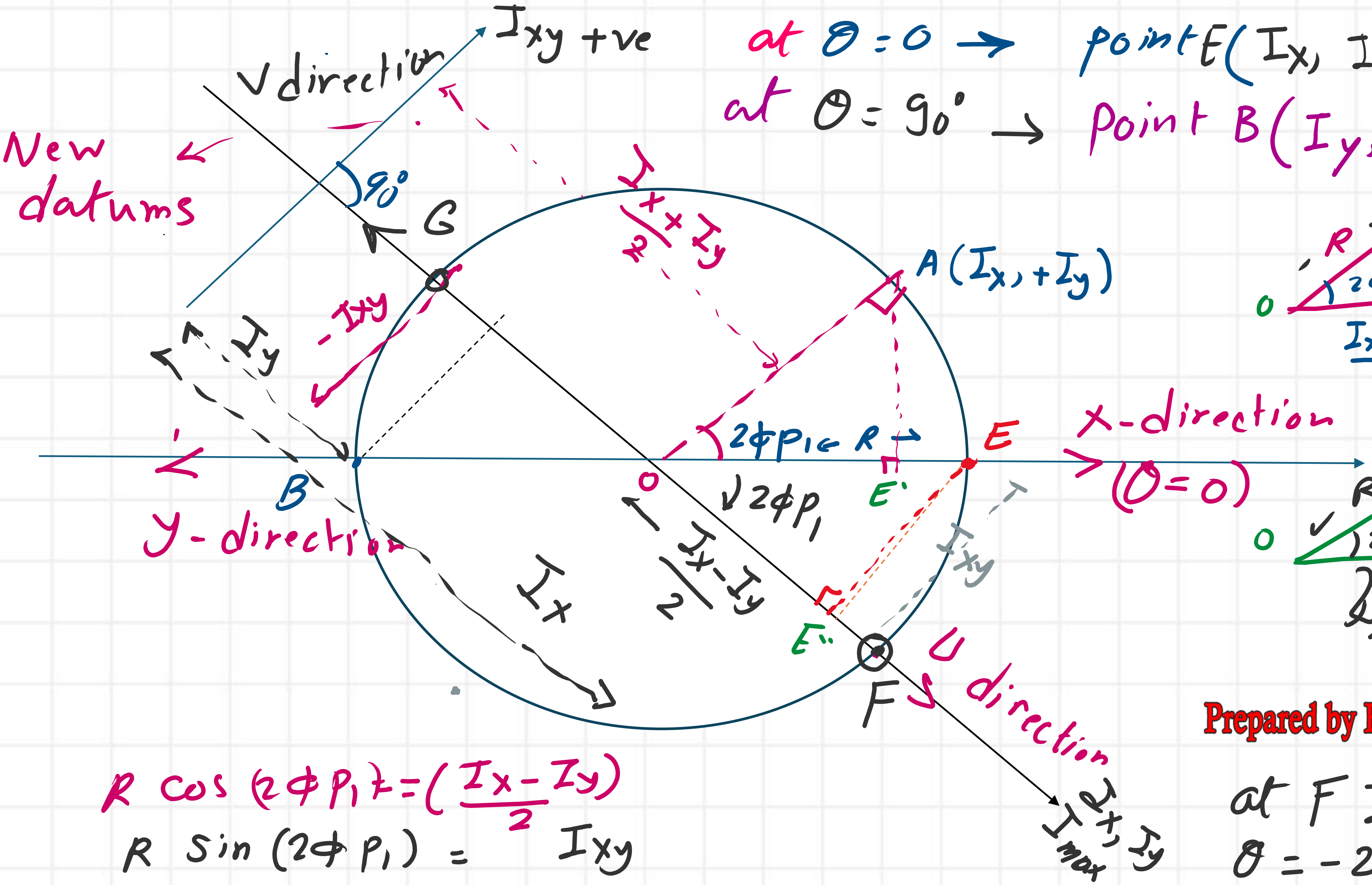
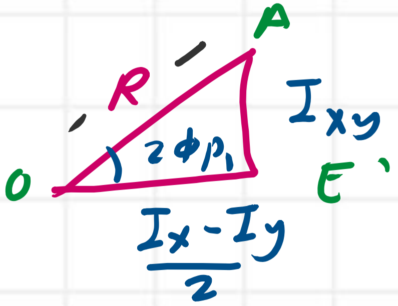
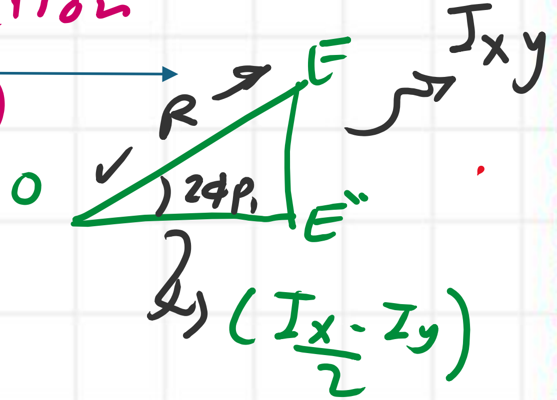
at $\theta = 0 \rightarrow$ point $E(I_x, I_{xy})$

at $\theta = 90^\circ \rightarrow$ point $B(I_y, -I_{xy})$







$R \cos(2\phi p_1) = \left(\frac{I_x - I_y}{2}\right)$

$R \sin(2\phi p_1) = I_{xy}$

at F $I_x = I_{max}$

$\theta = -2\phi p_1$

Prepared by Eng.Maged Kamel.