Properties of Transpose Matrices

$A^T \Rightarrow$ Transpose of matrix m x n creates a new matrix of n x m that has the 1st row of A as its first column, and 2nd row of A as its 2nd column.

A First row

A 2nd row

$$\text{if } A = \underset{2\times3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}} \Rightarrow \underset{3\times2}{A^T} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix}$$

1st Colum

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix}^T \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix} \Rightarrow A$$

The Transpose of a Transposed Matrix $\rightarrow$ returns to the same matrix

Properties of transpose Matrices

$A^T \Rightarrow$ Transpose of matrix m x n creates a new matrix of n x m that has the 1st row of A as its first column, and 2nd row of A as its 2nd column.

$$(A+B)^T = A^T + B^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 5 & 6 \end{bmatrix}$$

2×3 $\qquad$ 2×3

$$\Rightarrow A+B = \begin{bmatrix} 1+0 & 2+2 & 3+4 \\ 2+1 & 1+5 & 3+6 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 6 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} \; 3 \times 2$$

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & 5 \\ 4 & 6 \end{bmatrix} \; 3 \times 2$$

$$[A+B]^T = \begin{bmatrix} 1 & 3 \\ 4 & 6 \\ 7 & 9 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1+0 & 2+1 \\ 2+2 & 1+5 \\ 3+4 & 3+6 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 6 \\ 7 & 9 \end{bmatrix}$$

3×2 $\qquad$ 3×2

Same

Properties of Transpose Matrices

3-Transpose of the product equals the product of their transpose in reverse order.

$$(A \cdot B)^T = B^T \cdot A^T$$

$$B^T = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 3 & 5 \end{bmatrix} \quad 2\times3$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix} \times B = \begin{bmatrix} 0 & 2 \\ 4 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} \quad 3\times2$$

A^T

$$2\times3 \qquad 3\times2 \Rightarrow 2\times2$$

$$A \cdot B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 0+8+3 & 2+6+15 \\ 0+4+3 & 4+3+15 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 11 & 23 \\ 7 & 22 \end{bmatrix} \Rightarrow (A \cdot B)^T = \begin{bmatrix} 11 & 7 \\ 23 & 22 \end{bmatrix}$$

$$2\times2$$

$$B^T = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 3 & 5 \end{bmatrix} 2 \times 3 \Rightarrow \begin{bmatrix} 0(1) + 4(2) + (1)(3) & 0(2) + 4(1) + 1(3) \\ 2(1) + 3(2) + 5(3) & 2(2) + 3(1) + 5(3) \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} 3 \times 2 \qquad B^T A^T$$

$$B^T \cdot A^T = \begin{bmatrix} 0 + 8 + 3 & 0 + 4 + 3 \\ 2 + 6 + 15 & 4 + 3 + 15 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 7 \\ 23 & 22 \end{bmatrix} \Rightarrow (A \cdot B)^T$$

Example 1.29

Find the transpose of the following matrices:

(i) $A = \begin{pmatrix} -9 & 2 & 3 \\ 7 & -2 & 9 \\ 6 & -1 & 5 \end{pmatrix}$ (ii) $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (iii) $C = \begin{pmatrix} -1 & 3 & 4 \\ 7 & 9 & 0 \end{pmatrix}$ (iv) $D = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Part (i): $A = \begin{pmatrix} -9 & 2 & 3 \\ 7 & -2 & 9 \\ 6 & -1 & 5 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} -9 & 7 & 6 \\ 2 & -2 & -1 \\ 3 & 9 & 5 \end{pmatrix}$

$\hookrightarrow$ To be as first row

Part (ii): $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \Rightarrow B^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Prepared by Eng.Maged Kamel.

Example 1.29

Find the transpose of the following matrices:

(i) $\mathbf{A} = \begin{pmatrix} -9 & 2 & 3 \\ 7 & -2 & 9 \\ 6 & -1 & 5 \end{pmatrix}$ **(ii)** $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ **(iii)** $\mathbf{C} = \begin{pmatrix} -1 & 3 & 4 \\ 7 & 9 & 0 \end{pmatrix}$ **(iv)** $\mathbf{D} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Part-(iii): $C = \begin{pmatrix} -1 & 3 & 4 \\ 7 & 9 & 0 \end{pmatrix} \rightarrow C^T = \begin{pmatrix} -1 & 7 \\ 3 & 9 \\ 4 & 0 \end{pmatrix}$

(2×3) $\quad \hookrightarrow$ To be as first row $\quad 3\times2$

Part (IV): $D = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow D^T = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

(3×1) $\quad (1\times3)$

Prepared by Eng.Maged Kamel.

Example 1.30

Let $\mathbf{A} = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$. Determine

(a) $(\mathbf{A}^T)^T$ **(b)** $(2\mathbf{A})^T - (3\mathbf{B})^T$ **(c)** $(\mathbf{A} + \mathbf{B})^T$ **(d)** $\mathbf{A}^T + \mathbf{B}^T$ **(e)** $(\mathbf{AB})^T$

$\boxed{1.3}$

Part (a) $A = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$

2×3

$\underset{3\times 2}{A^T} = \begin{bmatrix} 3 & 5 \\ -4 & 2 \\ 1 & 6 \end{bmatrix}$ $(A^T)^T \Rightarrow A$

$\begin{pmatrix} -3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ 2×3

Example 1.30

Let $\mathbf{A} = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$. Determine

(a) $(\mathbf{A}^T)^T$ **(b)** $(2\mathbf{A})^T - (3\mathbf{B})^T$ **(c)** $(\mathbf{A}+\mathbf{B})^T$ **(d)** $\mathbf{A}^T + \mathbf{B}^T$ **(e)** $(\mathbf{AB})^T$

$\boxed{1.3}$

Part (b) $2A = 2\begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix} = \begin{pmatrix} 6 & -8 & 2 \\ 10 & 4 & 12 \end{pmatrix}$

$B = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix} \Rightarrow 3B = \begin{pmatrix} -6 & 21 & 15 \\ 3 & 9 & -27 \end{pmatrix}$

$2A^T - 3B^T = \begin{pmatrix} 6 & 10 \\ -8 & 4 \\ 2 & 12 \end{pmatrix} - \begin{pmatrix} -6 & 3 \\ 21 & 9 \\ 15 & -27 \end{pmatrix} = \begin{pmatrix} 12 & 7 \\ -29 & -5 \\ -13 & 39 \end{pmatrix}$

Example 1.30

Let $\mathbf{A} = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$. Determine

(a) $(\mathbf{A}^T)^T$ **(b)** $(2\mathbf{A})^T - (3\mathbf{B})^T$ **(c)** $(\mathbf{A} + \mathbf{B})^T$ **(d)** $\mathbf{A}^T + \mathbf{B}^T$ **(e)** $(\mathbf{AB})^T$

Part (c) $A = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$; $B = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$

2×3

$$A + B = \begin{pmatrix} 3-2 & -4+7 & 1+5 \\ 5+1 & 2+3 & 6-9 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 6 \\ 6 & 5 & -3 \end{pmatrix}$$

$$(A+B)^T = \begin{pmatrix} 1 & 6 \\ 3 & 5 \\ 6 & -3 \end{pmatrix}$$

Example 1.30

Let $\mathbf{A} = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$. Determine

(a) $(\mathbf{A}^T)^T$ **(b)** $(2\mathbf{A})^T - (3\mathbf{B})^T$ **(c)** $(\mathbf{A}+\mathbf{B})^T$ **(d)** $\mathbf{A}^T + \mathbf{B}^T$ **(e)** $(\mathbf{AB})^T$

Part (d)

$$A = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix} \quad 2\times 3$$

$$A^T = \begin{bmatrix} 3 & 5 \\ -4 & 2 \\ 1 & 6 \end{bmatrix}$$

$$B = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix} \quad 2\times 3$$

$$B^T \quad 3\times 2 = \begin{bmatrix} -2 & 1 \\ 7 & 3 \\ 5 & -9 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3-2 & 5+1 \\ -4+7 & 2+3 \\ 1+5 & 6-9 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & 5 \\ 6 & -3 \end{bmatrix}$$

Example 1.30

Let $\mathbf{A} = \begin{pmatrix} 3 & -4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$. Determine

(a) $(\mathbf{A}^T)^T$ **(b)** $(2\mathbf{A})^T - (3\mathbf{B})^T$ **(c)** $(\mathbf{A}+\mathbf{B})^T$ **(d)** $\mathbf{A}^T + \mathbf{B}^T$ **(e)** $(\mathbf{AB})^T$

$$A = \begin{pmatrix} 3 & -4 \\ 5 & 2 \end{pmatrix} \quad \& \quad B = \begin{pmatrix} -2 & 7 & 5 \\ 1 & 3 & -9 \end{pmatrix}$$

2×3 $\qquad$ 2×3

do not match

A & B Cannot be multiplied